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Optimal horizon size for unbiased finite memory digital phase-locked loop

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Abstract: Digital phase locked-loop (DPLL) is a circuit system for frequency synchronization, and unbiased finite memory DPLL (UFMDPLL) is DPLL using a finite impulse response (FIR) filter for phase detection. This letter proposes a novel method for finding the optimal horizon size, which is a key design parameter of UFMDPLL, based on the minimization of the estimation error variance. The effectiveness and efficiency of the proposed method are demonstrated in comparisons using the conventional Monte Carlo simulation method.

Keywords: digital phase-locked loop (DPLL), finite impulse response (FIR) filter, horizon size, unbiased finite memory DPLL (UFMDPLL)

Classification: Circuits and modules for electronic instrumentation

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1 Introduction

A phased-locked loop (PLL) is an electronic circuit system that synchronizes its output signal frequency with its input signal frequency [1]. PLLs have been widely used for various purposes, such as clock synchronization in computer systems and

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demodulation in communications systems [1, 2, 3, 4, 5]. PLL consists of two main components: a phase detector and a variable frequency oscillator [6]. The phase detector compares the phase of the output signal with that of the input signal; frequency synchronization is achieved by keeping the input and output phases the same, which is performed by the variable frequency oscillator by adjusting the output frequency.

A Kalman filter (KF) is used in the digital PLL (DPLL) for phase detection [7, 8, 9]. The KF is a mathematical algorithm that estimates unknown parameters from noisy measurements. DPLL based on the KF has shown superior performance that conventional DPLL [7, 8, 9]. However, KF-based DPLL is vulnerable to computational errors such as round-off error and quantization error, because KF has an infinite impulse response (IIR) structure that uses all past measurements and accumulates computational errors over time [10, 11]. Unbiased finite memory DPLL (UFMDPLL) was proposed in [10] to overcome the drawbacks of KF-based DPLL. UFMDPLL is based on the finite impulse response (FIR) filter [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] using only recent finite measurements, and it showed superior robustness against computational errors than KF-based DPLL.

The FIR filter has an important design parameter called the horizon size, which is the number of measurements used for estimation. Because the horizon size, usually denoted N, is a critical problem in FIR filtering [20, 22, 23]. Various approaches to finding an optimal N (denoted N_{opt}) for general FIR filters has been proposed based on the minimum mean square value [16], bank of FIR filters [20], and Monte Carlo simulation [14, 15], but an analytic method to calculate N_{opt} has not yet been found. A method to find N_{opt} for a moving average DPLL was proposed in [24]. However, the method is only applicable to scalar DPLL systems that consider a single unknown parameter, the timing offset. No method of finding N_{opt} for a complete DPLL system has yet been proposed to the best of the authors knowledge.

We propose a novel method to find N_{opt} for a complete DPLL system in this letter in which two unknown parameters, the timing offset and the zero-crossing point, are considered. First, we show that the FIR filter gain of the UFRDPLL can be represented by a function of N. Then, we derive the estimation error variance equation as a function of N. Lastly, we propose a method to find N_{opt} based on the partial derivatives of the estimation error variance equation. Simulation results demonstrate that the proposed method is effective and more efficient than conventional Monte Carlo simulation.

2 Main results

Consider the following state-space models of the zero-crossing DPLL [7]:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k, \tag{1}$$

$$y_k = \mathbf{C}\mathbf{x}_k + v_k, \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

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In (1) and (2), $\mathbf{x}_k$ and y_k are respectively the state and the measurement at discrete time index k. The state vector $\mathbf{x}_k$ is defined as $\mathbf{x}_k = [\alpha_k\ \beta_k]^T$, where $\alpha_k = t_0 + k(T_1 - T_0)$ is the zero crossing point, $\beta_k = T_1 - T_0$ is the timing offset, and T_0 and T_1 are the respective sampling periods of the receiver and transmitter. $\mathbf{w}_k$ and v_k are respectively the process and measurement noises; they are assume to be zero-mean white Gaussian and their covariances are defined as

$$\mathbf{Q} \triangleq diag[q_1^2, q_2^2], \quad R \triangleq r^2, \tag{3}$$

where $diag(\cdot)$ indicates a diagonal matrix. The UFMDPLL [10] for estimating $\mathbf{x}_k$ is given as

$$\hat{\mathbf{x}}_k = \mathbf{H}\mathbf{Y}_{k-1}, \tag{4}$$

where $\mathbf{H}$ is the gain matrix and $\mathbf{Y}_{k-1}$ is the augmented measurement matrix defined as

$$\mathbf{Y}_{k-1} \triangleq [y_{k-N}^T\ y_{k-N+1}^T\ \cdots\ y_{k-1}^T]^T, \tag{5}$$

where N is the number of measurements on the time horizon $[k-N, k-1]$ and is called the horizon size. $\mathbf{Y}_{k-1}$ can be represented as a function of state vector $\mathbf{x}_k$ as follows:

$$\mathbf{Y}_{k-1} = \bar{\mathbf{C}}_N\mathbf{x}_k + \bar{\mathbf{G}}_N\mathbf{W}_{k-1} + \mathbf{V}_{k-1}, \tag{6}$$

where

$$\bar{\mathbf{C}}_N \triangleq \begin{bmatrix} \mathbf{C}\mathbf{A}^{-N} \\ \mathbf{C}\mathbf{A}^{1-N} \\ \mathbf{C}\mathbf{A}^{2-N} \\ \vdots \\ \mathbf{C}\mathbf{A}^{-1} \end{bmatrix}, \tag{7}$$

$$\bar{\mathbf{G}}_N \triangleq \begin{bmatrix} \mathbf{C}\mathbf{A}^{-1} & \mathbf{C}\mathbf{A}^{-2} & \cdots & \mathbf{C}\mathbf{A}^{-N} \\ 0 & \mathbf{C}\mathbf{A}^{-1} & \cdots & \mathbf{C}\mathbf{A}^{1-N} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{C}\mathbf{A}^{-1} \end{bmatrix}, \tag{8}$$

$$\mathbf{W}_{k-1} \triangleq \begin{bmatrix} \mathbf{w}_{k-N}^T & \mathbf{w}_{k-N+1}^T & \cdots & \mathbf{w}_{k-1}^T \end{bmatrix}^T, \tag{9}$$

$$\mathbf{V}_{k-1} \triangleq \begin{bmatrix} v_{k-N}^T & v_{k-N+1}^T & \cdots & v_{k-1}^T \end{bmatrix}^T. \tag{10}$$

By substituting (6) into (4), we obtain

$$\begin{aligned} \hat{\mathbf{x}}_k &= \mathbf{H}(\bar{\mathbf{C}}_N\mathbf{x}_k + \bar{\mathbf{G}}_N\mathbf{W}_{k-1} + \mathbf{V}_{k-1}) \\ &= \mathbf{H}\bar{\mathbf{C}}_N\mathbf{x}_k + \mathbf{H}(\bar{\mathbf{G}}_N\mathbf{W}_{k-1} + \mathbf{V}_{k-1}). \end{aligned} \tag{11}$$

Thanks to the unbiasedness property, the UFMDPLL [10] satisfies $\mathbf{H}\bar{\mathbf{C}}_N = \mathbf{I}$. Thus, equation (11) can be written as

$$\hat{\mathbf{x}}_k = \mathbf{x}_k + \mathbf{H}(\bar{\mathbf{G}}_N\mathbf{W}_{k-1} + \mathbf{V}_{k-1}). \tag{12}$$

We define the estimation error $\mathbf{e}_k$ as $\mathbf{e}_k = \hat{\mathbf{x}}_k - \mathbf{x}_k$. Then, we obtain

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$$\begin{aligned}\mathbf{e}_k &= \hat{\mathbf{x}}_k - \mathbf{x}_k \\ &= \mathbf{H}(\bar{\mathbf{G}}_N \mathbf{W}_{k-1} + \mathbf{V}_{k-1}).\end{aligned} \tag{13}$$

The optimal gain matrix $\mathbf{H}_{\text{opt}}$ is obtained by minimizing the variance of the estimation error as

$$\begin{aligned}\mathbf{H}_{\text{opt}} &= arg \min_{\mathbf{H}} E[\mathbf{e}_k^T \mathbf{e}_k] \\ &= arg \min_{\mathbf{H}} E[tr(\mathbf{e}_k \mathbf{e}_k^T)],\end{aligned} \tag{14}$$

where $tr(\cdot)$ indicates the trace of a matrix. Substituting (13) into (14), we obtain

$$\mathbf{H}_{\text{opt}} = arg \min_{\mathbf{H}} tr[\mathbf{H}\bar{\mathbf{G}}_N \mathbf{Q}_N \bar{\mathbf{G}}_N^T \mathbf{H}^T + \mathbf{H}\mathbf{R}_N \mathbf{H}^T], \tag{15}$$

where $\mathbf{Q}_N$ and $\mathbf{R}_N$ are defined as

$$\mathbf{Q}_N \triangleq diag[\mathbf{Q}, \cdots, \mathbf{Q}], \quad \mathbf{R}_N \triangleq diag[R, \cdots, R]. \tag{16}$$

$\mathbf{H}_{\text{opt}}$ is obtained by solving the trace optimization problem (15). In [10], $\mathbf{H}$ was found that satisfies the unbiased condition, which is represented by a function of the horizon size N as follows:

$$\begin{aligned}&\mathbf{H} = \begin{bmatrix} a_1 & a_2 & \cdots & a_N \\ b_1 & b_2 & \cdots & b_N \end{bmatrix}, \\ &a_i = \frac{-2N - 4 + 6i}{(N-1)N}, \quad b_i = \frac{-6N - 6 + 12i}{(N-1)N(N+1)}, \quad (i = 1, 2, \cdots, N).\end{aligned} \tag{17}$$

Using (17), the trace term in (15) can be represented by a function of N. Through simple algebraic computation, we thus obtain

$$tr[\mathbf{H}\bar{\mathbf{G}}_N \mathbf{Q}_N \bar{\mathbf{G}}_N^T \mathbf{H}^T + \mathbf{H}\mathbf{R}_N \mathbf{H}^T] = q_1^2 f_{q_1} + q_2^2 f_{q_2} + r^2 f_r, \tag{18}$$

where q_1, q_2, and r are the components of the noise covariance matrices defined in (3), and f_{q_1}, f_{q_2}, and f_r are as follows:

$$\begin{aligned}f_{q_1} &\triangleq \frac{2N^4 + 9N^3 + 32N^2 + 9N + 20}{15N(N^2 - 1)}, \\ f_{q_2} &\triangleq \frac{2N^6 + 11N^5 + 103N^4 + 242N^3 + 19N^2 - 199N + 38}{210N(N^2 - 1)}, \\ f_r &\triangleq \frac{2(2N^2 + 3N + 7)}{N^3 - N}.\end{aligned} \tag{19}$$

$\mathbf{H}_{\text{opt}}$ can be obtain by minimizing (18). We see that (18) is a function of N, q_1, q_2, and r. Thus, (18) can be represented as

$$f(N, q_1, q_2, r) = q_1^2 f_{q_1} + q_2^2 f_{q_2} + r^2 f_r. \tag{20}$$

When given q_1, q_2, and r, (18) only depends on N. The gain matrix, $\mathbf{H}$, depends on the horizon size N, and the optimal horizon size, N_{opt}, is the horizon size that corresponds to $\mathbf{H}_{\text{opt}}$. Thus, N_{opt} can be found by minimizing (20).

Now, we show that $f(N, q_1, q_2, r)$ is a convex function with respect to N and N_{opt} is unique. The first order partial derivative of $f(N, q_1, q_2, r)$ with respect to N is obtained as follows:

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$$\frac{\partial f(N,q_1,q_2,r)}{\partial N} = q_1^2\frac{\partial f_{q_1}}{\partial N} + q_2^2\frac{\partial f_{q_2}}{\partial N} + r^2\frac{\partial f_r}{\partial N}, \tag{21}$$

where

$$\frac{\partial f_{q_1}}{\partial N} = \frac{2(N^6 - 19N^4 - 18N^3 - 46N^2 - 10)}{15N^2(N^2-1)^2},$$
$$\frac{\partial f_{q_2}}{\partial N} = \frac{6N^8 + 22N^7 + 93N^6 - 44N^5 - 328N^4 - 86N^3 - 133N^2 + 38}{210N^2(N^2-1)^2},$$
$$\frac{\partial f_r}{\partial N} = \frac{2(2N^4 - 6N^3 - 23N^2 + 7)}{N^2(N^2-1)^2}.$$

The second order partial derivative of $f(N,q_1,q_2,r)$ with respect to N is

$$\frac{\partial^2 f(N,q_1,q_2,r)}{\partial N^2} = q_1^2\frac{\partial^2 f_{q_1}}{\partial N^2} + q_2^2\frac{\partial^2 f_{q_2}}{\partial N^2} + r^2\frac{\partial^2 f_r}{\partial N^2}, \tag{22}$$

where

$$\frac{\partial f_{q_1}^2}{\partial N^2} = \frac{4}{15N^3(N^2-1)^3} \times (17N^6 + 27N^5 + 111N^4 + 9N^3 - 30N^2 + 10),$$
$$\frac{\partial f_{q_2}^2}{\partial N^2} = \frac{1}{105N^3(N^2-1)^3} \times (6N^{10} + 11N^9 - 18N^8 - 33N^7 + 142N^6 + 195N^5 + 594N^4 + 43N^3 - 114N^2 + 38), \tag{23}$$
$$\frac{\partial f_r^2}{\partial N^2} = \frac{4}{N^3(N^2-1)^3} \times (2N^6 + 9N^5 + 48N^4 + 3N^3 - 21N^2 + 7), \tag{24}$$

where $\frac{\partial f_{q_1}^2}{\partial N^2}$, $\frac{\partial f_{q_2}^2}{\partial N^2}$, and $\frac{\partial f_r^2}{\partial N^2}$ are all positive because of $N \geq 2$. The horizon size must be equal to or greater than the dimension of the state vector, which is a basic condition of FIR filtering. q_1, q_2, and r are all positive because they are standard deviations of Gaussian noises. q_1, q_2, and r cannot be zeros in real systems. Consequently, we obtain $\frac{\partial^2 f(N,q_1,q_2,r)}{\partial N^2} > 0$. Because the second-order partial derivative $f(N,q_1,q_2,r)$ with respect to N is positive, $f(N,q_1,q_2,r)$ is a convex function with respect to N. Therefore, there exists a unique N_{opt}, which is the global minimum.

Lastly, we derive the equation to obtain N_{opt}. Setting (21) to zero, we obtain

$$\frac{\partial f(N,q_1,q_2,r)}{\partial N} = \frac{1}{210N^2(N^2-1)^2} \times g(N,q_1,q_2,r) = 0,$$

where

$$\begin{aligned} g(N,q_1,q_2,r) = {} & 6q_2^2N^8 + 22q_2^2N^7 + (28q_1^2 + 93q_2^2)N^6 - 44N^5q_2^2 \\ & - (532q_1^2 + 328q_2^2 + 840r^2)N^4 - (504q_1^2 + 86q_2^2 + 2520r^2)N^3 \\ & - (1288q_1^2 + 133q_2^2 + 9660r^2)N^2 + 280q_1^2 + 38q_2^2 + 2940r^2. \end{aligned} \tag{25}$$

From (25), we can find N_{opt} by solving (25). However, (25) is a high-order equation and is difficult to solve directly. Thus, the solution of (25) is obtained using a numeric solver (e.g., MATLAB). Given q_1^2, q_2^2, and r^2, we can solve (25). Because the horizon size must be a natural number in the discrete-time case, N_{opt} is the natural number closest to the solution of (25) that satisfies the condition $N \geq 2$.

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3 Numerical example

The proposed method is an analytic approach to N_{opt}. Here, we demonstrate the effectiveness and efficiency of the proposed method by comparing it with the experimental approach, Monte Carlo (MC) simulation. We set the DPLL system parameters as follows [7, 8, 9]. The process noise covariances were set as $q_1^2 = q_2^2 = T_0^2/12$, where $T_0 = 0.001$. The measurement noise covariance, r^2, varies according to the signal-to-noise ratio (SNR) values. We consider nine cases of SNR in the range 10–90 dB. The SNR is computed as $10log_{10}(T_0^2/r^2)$.

First, we find N_{opt} using the proposed method. Fig. 1 shows the absolute value of $\frac{\partial f(N,q_1,q_2,r)}{\partial N}$ for the horizon sizes in the interval $2 \leq N \leq 250$, when the SNR is 10 dB. The horizon size producing $\frac{\partial f(N,q_1,q_2,r)}{\partial N}$ closest to zero is the optimal horizon size; thus, $N_{\text{opt}} = 212$ in this case, as shown in Fig. 1. In this way, we can find N_{opt} for the nine SNR values; the results are presented in Table I.

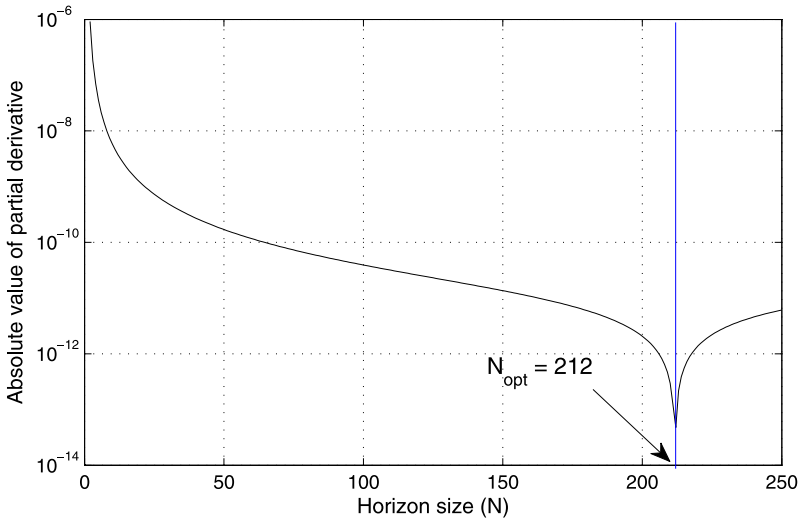

Fig. 1. Absolute value of $\frac{\partial f(N,q_1,q_2,r)}{\partial N}$ for different N in case of 10 dB measurement noise

Next, we find N_{opt} using MC simulation. We conducted a 100 MC run for each N in the interval $2 \leq N \leq 250$ and computed the mean square error (MSE) values for each case. Fig. 2 shows the MSE as a function of N in the case of 50 dB SNR. MSEs for only $2 \leq N \leq 100$ are shown in Fig. 2 to show the position of N_{opt} clearly, since the horizon size producing the minimum MSE is $N_{\text{opt}} = 21$ in this case. Following the same way, we obtained N_{opt} for the nine cases, which are presented in Table I.

In Table I, we see that N_{opt} values obtained by the proposed method are almost the same as those obtained via MC simulation. Thus, the effectiveness of the proposed method is verified experimentally. In the low SNR (i.e., high measurement noise) cases such as 10- and 20-dB cases, N_{opt} values of the proposed method slightly differ from those of the MC simulation. This is because running 100 MC is insufficient for low SNR cases. High noise cases require more MC runs to obtain reliable results compared to low noise cases.

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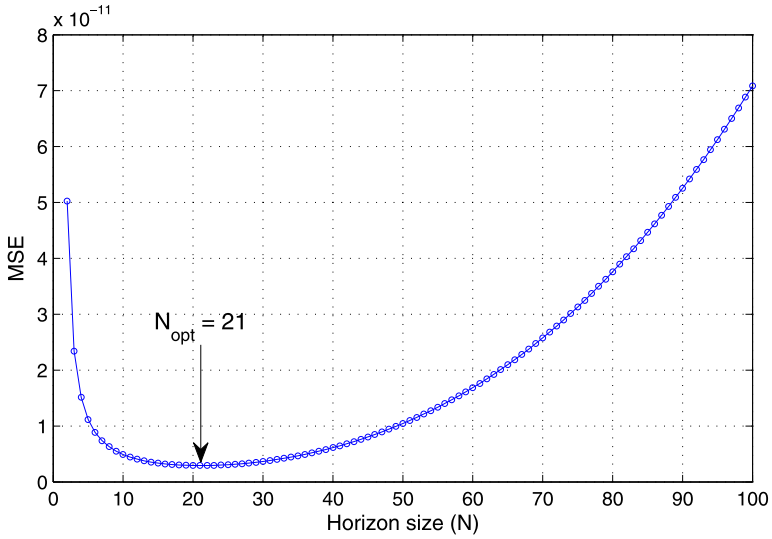

Fig. 2. Mean squared errors for different N obtained via Monte Carlo simulation in case of 50 dB measurement noise

Table I. N_{opt} obtained using proposed method and MC simulation

SNR (dB)	proposed method	MC simulation
10	212	208
20	119	122
30	67	67
40	37	37
50	21	21
60	12	12
70	7	7
80	4	4
90	3	3

Table II. Computation time of proposed method and MC simulation

Proposed method	MC simulation
434 ms	2098 s

Table II compares the times required for the proposed method and the MC simulation. The simulations were carried out using MATLAB on a personal computer with an Intel Core i5-6200U CPU at 2.3 GHz. The time for the proposed method was only 434 ms, which is much smaller than the 2098 s required by the MC simulation. Decreasing the number of the MC run to 10 allows the computation time to be reduced to approximately 210 s, which is still notably larger than that of the proposed method. However, 10 MC run is insufficient to obtain reliable MC simulation results. The proposed method is very efficient compared to the conventional MC simulation method.

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4 Conclusion

In this letter, we have proposed a novel method for finding N_{opt} for UFMDPLL. The estimation error variance of the UFMDPLL was represented by a function of N. We have proved that the function of N is convex and derived from the equation for obtaining N_{opt} via the partial derivative with respect to N. In the numerical example, the proposed method provided N_{opt} with a remarkably short computation time compared to conventional MC simulation. Using the proposed method, the N_{opt} of UFMDPLL can be obtained correctly and efficiently.

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