LETTER

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A widely amplitude-adjustable chaotic oscillator based on a physical model of HP memristor

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Abstract: We design a novel chaotic oscillator based on a new boundary-restricted HP memristor model. This system is mathematically simple, while dynamics rich. It is found that the system keeps chaotic within a wide range of the parameter, and the amplitude of chaotic sequences can be modified into infinite or infinitesimal by changing the parameter value of the memristor model linearly. Moreover, initial values of the width of doped region do not have influence on system's stability. The two features give the memristor advantages in generating a large key space and adjustable chaotic signals continuously. Circuit simulation of the system which shows good agreement with numerical results is also given.

Keywords: HP memristor, Chua's oscillator, chaotic dynamics

Classification: Electron devices, circuits and modules

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1 Introduction

Memristors, the fourth electrical elements which were theoretically postulated by Chua, have a typical property of memorizing the flowing charge [1]. Since they were first physically realized in a Pt/TiO2/Pt device by HP Labs [2], memristors, and memristive devices have attracted extensive research attention for their prospective applications on nonvolatile memories, neuromorphic devices, logic devices, and chaotic circuits [3].

During all the research on memristive chaos, chaotic systems characterized by memristor models like cubic and piecewise linearity were thoroughly investigated [4, 5, 6, 7]. However, such hypothetical memristors cannot be physically realized. As we know, HP memristor is the most promising one to be fabricated now, but investigations of chaotic circuit with HP memristor are still rarely reported. In [8], Buscarino et al. derived an oscillator by replacing Chua's diode with HP memristors in a canonical Chua's circuit topology, but two anti-parallel memristors are required in that, so circuit complexity increases. Later, an oscillator contains only one HP memristor was reported [9]. They assumed HP memristor system to be a smooth system, but a memristor-based system should be a non-smooth one due to the boundary limits of the device, and the derivation process for flux-charge relation in their paper is rather complicate. In our work, a novel HP memristor-based non-smooth collision system derived from the dual circuit of canonical Chua's circuit is proposed. We adopted a different HP memristor modeled by linking the voltage across the device with state variable for the first time, rather than characterized by present window functions. More importantly, this chaotic system shows strong robustness within a very wide range of the memristor parameter, and the amplitude of chaotic sequences can be infinite or infinitesimal by adjusting this parameter, which benefits chaos-based applications. Besides, Its stability is insensitive to the initial state of the memristor, which means chaos can be generated under any conditions, and such phenomenon is still rarely reported in most memristive chaotic systems [10]. It is expected that this paper would reinforce the argument in favor of memristor-based oscillator from a practical view.

2 Physical model of HP memristor

As shown in Fig. 1(a), a HP TiO2 thin film memristor is composed of a layer doped with oxygen vacancies and an undoped layer. D is the length of the device, and x is the width of the doped region. Letting $w = x/\mathrm{D}$ $(0 < w < 1)$, the resistance of this memristor can be calculated as:

$$M(w) = M_{ON}w + M_{OFF}(1 - w) \tag{1}$$

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Where M_{ON} (M_{OFF}) is the limit value of resistance for $x = D$ ($x = 0$).

The derivative of w in time is in proportion to the current i through the device:

$$dw/dt = \mu_v R_{ON} i/D^2 \tag{2}$$

Where constant μ_v is dopant mobility.

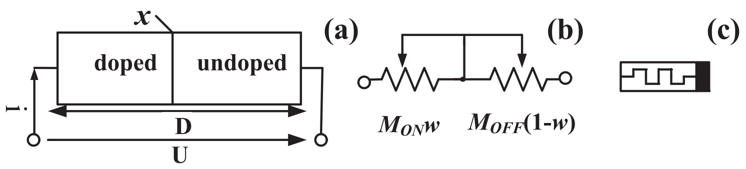

Fig. 1. HP memristor. (a) Structure. (b) Equivalent circuit. (c) Electrical symbol.

For restricting the value of w between 0 and 1, we adopted a new function *trunc* by linking w with the voltage v across the device:

$$trunc(w, v) = (step(w) + step(v))(step(1 - w) + step(-v))/2 \tag{3}$$

Where *step* represents step function, and v is voltage applied on the memristor, Compared with previous window functions, *trunc* can mimic the effect of physical boundaries of a two-terminal memristor, and do not interfere with the moving of the interface between the doped and undoped regions. To be specific,

$$trunc(w, v) = \begin{cases} 1, & 0 < w < 1 \\ 0, & w > 1, v > 0 \textbf{ or } w < 0, v < 0 \end{cases} \tag{4}$$

Eq. (2) can be rewritten as:

$$dw/dt = \mu_v R_{ON} trunc(w, v) i/D^2 \tag{5}$$

3 Chaotic oscillator and its dynamics

Replacing Chua's diode with the above-mentioned model in a dual circuit of canonical Chua's circuit as shown in Fig. 2, the following set of differential equation can be obtained:

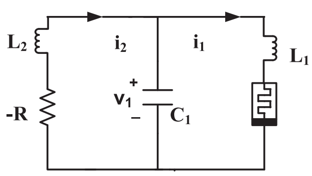

Fig. 2. Chaotic oscillator with a physical model of HP memristor.

$$\begin{cases} dx/dt = j(z - y) \\ dy/dt = k(x - M(w)y) \\ dz/dt = mz - nx \\ dw/dt = ptrunc(w, x)y \end{cases} \tag{6}$$

Where

$$x = v_1,\ y = i_1,\ z = i_2,\ j = 1/C_1,\ k = 1/L_1,\ m = R/L_2,\ n = 1/L_2,\ p = \mu_v R_{ON}/D^2$$

This oscillator has an equilibrium set $S = \{(x, y, z, w) \mid x = y = z = 0, w = c\}$,

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where c is a constant within the range of w. The Jacobian matrix of (6) at the equilibrium set is given by:

$$J_S = \begin{bmatrix} 0 & -j & j & 0 \\ k & -k(R_{ON}w - R_{OFF}(w-1)) & 0 & ky(R_{OFF} - R_{ON}) \\ -n & 0 & m & 0 \\ 0 & p & 0 & 0 \end{bmatrix} \tag{7}$$

If we set $j = 2.5$ $k = 4$ $m = 1.3$ $n = 0.75$ $p = 2 * 10^7$ $R_{ON} = 1.8$ and $R_{OFF} = 10$, for $c = 0.1$, the four eigenvalues are calculated as $\lambda_{1,2} = 0.5103 \pm 1.1266\mathrm{j}$, $\lambda_3 = 0$, $\lambda_4 = -36.4460$. The result implies S is an unstable saddle-focus. Four Lyapunov exponents are $\mathrm{LE}_1 = 0.6876$, $\mathrm{LE}_2 = -0.0061$, $\mathrm{LE}_3 = -0.5579$, $\mathrm{LE}_4 = -16.2367$. Thus a chaotic attractor is formed, as shown in Fig. 3. Fig. 3(b) shows special boundary-restricted attractor formed by the physical limits of a memristor, which is different from previous chaotic oscillators.

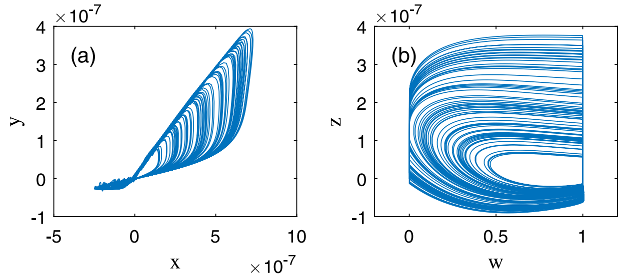

Fig. 3. Chaotic attractor with initial value $(0, -1 * 10^{-7}, 0, 0.1)$.

This oscillator has unique ability of keeping chaotic within a very wide range of parameter, and such phenomenon never appeared in previous chaotic oscillators. Fig. 4 shows time-domain waveform of z and the attractor when $p = 1 * 10^{-300}$. As can been seen, the amplitude of chaotic can be up to 10^{300}. Indeed, the chaotic amplitude can be infinite as long as the value of p approaches 0 infinitely, but it is worth mentioning that chaotic sequences are not generated in the early stage, as reported in Fig. 4(a).

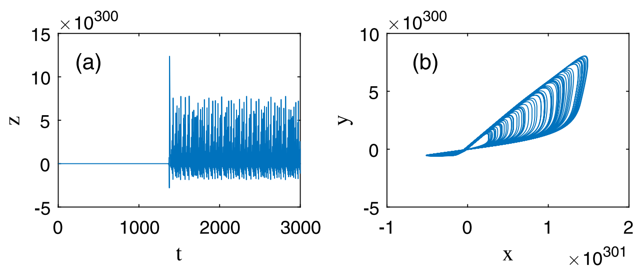

Fig. 4. $p = 1 * 10^{-300}$. (a) Time-domain waveform of z. (b) Chaotic attractor.

Bifurcation diagrams of variable x with respect to the parameter p shown in Fig. 5 also prove the wide range chaotic behavior of the system. More interestingly, amplitudes of the chaotic sequences except for w are inversely proportional to the parameter p, as reported in Fig. 6(a). These features contribute to a large key space and adjustment of chaotic sequences, thus giving the system advantages in

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encryption. In addition, it is found that system keeps chaotic despite initial value w (width of the doped region) changing. Fig. 6(b) shows the largest Lyapunov exponent (always positive) with respect to the initial value of w.

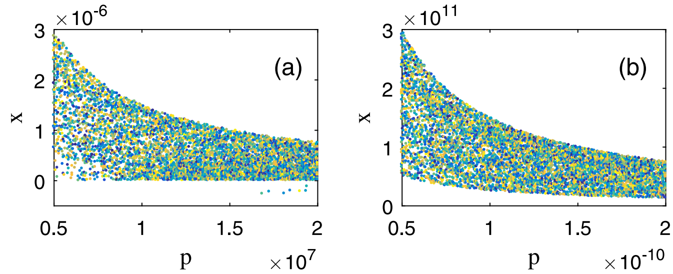

Fig. 5. Bifurcation diagram with value p increasing.

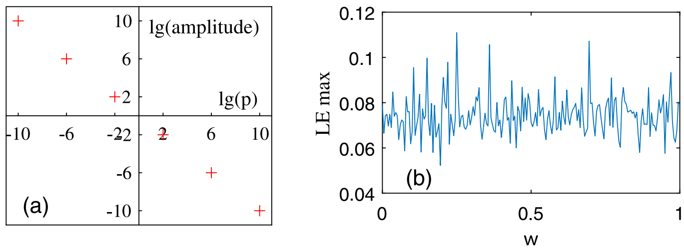

Fig. 6. (a) Relation between orders of magnitude of p and of chaotic signals in log scale. (b) Largest Lyapunov exponent with initial value of w changing.

4 Circuit simulation

Circuit simulation is implemented in NI Multisim. In order to let the circuit devices work in the linear range, set p = 4. The circuit diagrams of function $trunc(w, x)$ and chaotic oscillator are shown in Fig. 7 and Fig. 8 respectively.

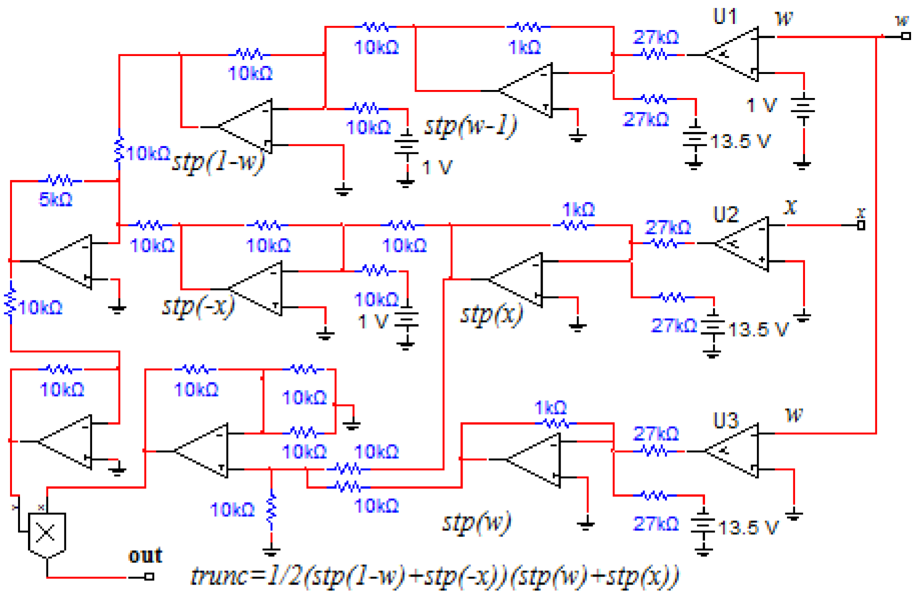

Fig. 7. Circuit diagram of $trunc(w, x)$. U1 U2 U3 are voltage comparators whose output Vo $= -13.5\,\mathrm{sgn}(V_{-} - V_{+})$.

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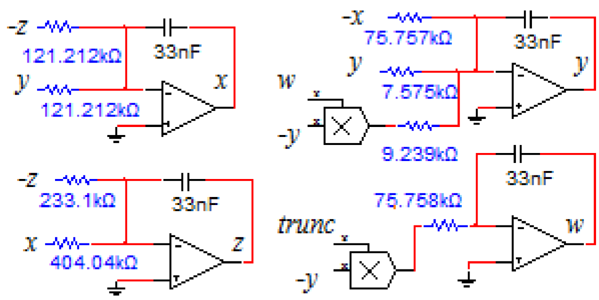

Fig. 8. Circuit diagram of chaotic oscillator. Inverters are omitted for clarity.

Circuit simulation, shown in Fig. 9, shows good agreement with numerical results.

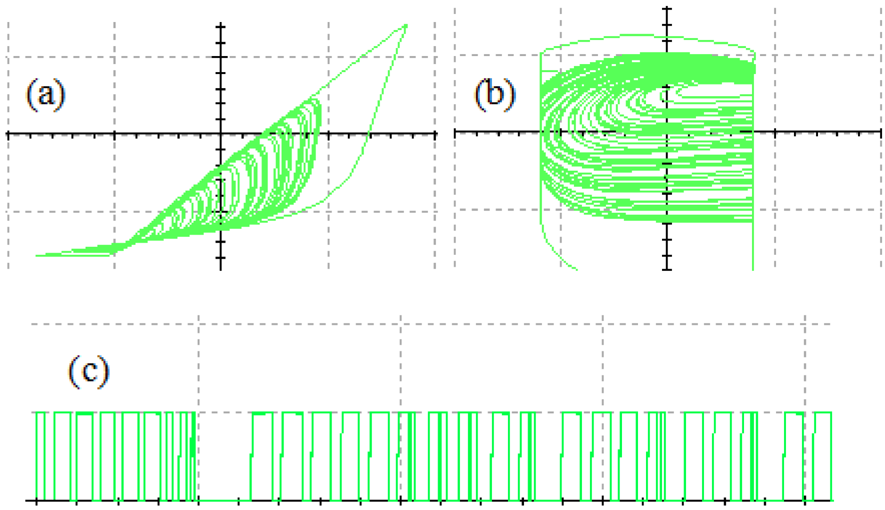

Fig. 9. Circuit simulation results. (a) Phase diagram in x-y plane. (b) Phase diagram in w-z plane. (c) Time-domain waveform of $trunc(w, x)$ (digital sequence).

5 Conclusion

Different from memristor oscillators based on memristors characterized by hypothetical smooth flux-charge characteristics, we defined a physical model of realized HP memristor which are much closer to reality. This model is found to be applicable for chaos, and the amplitude of chaotic signals can be controlled arbitrarily by adjusting the parameter determined by memristor itself. Its strong robustness also reflexes in its insensitivity on the initial value of state variable of the device. These are beneficial to chaos-based applications. Finally, we illustrate circuit simulation according to the mathematical expression of the system, but we do not present real circuit because voltage offset of the amplifiers that cannot be eliminated has impact on system stability. An oscillator based on real memristive devices has advantages over one built with traditional analog devices in circuit simplification, size, power consumption, etc. With manufacturing technology becoming mature, we believe that a practical memristor-based nonlinear oscillator will be realized.