Equivalent circuits of multi-conductor transmission lines above lossy ground excited by external electromagnetic fields

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Abstract: The equivalent circuit of multi-conductor transmission lines above lossy ground will be more complex compared with one single transmission line above lossy ground, and many problems of matrices need to be solved. In this paper, for multi-conductor transmission lines above lossy ground, the frequency-domain sections of ground impedance matrix and exciting voltage matrix are approximated as the rational function form by the vector fitting (VF) method. So the time domain results can be obtained conveniently by the Laplace element in HSPICE. The DEPACT macro-models and equivalent sources of external electromagnetic fields are built based on voltage controlled current source and voltage controlled voltage source. Then, combined with these equivalent circuits, the equivalent circuits for multi-conductor transmission lines above lossy ground excited by external electromagnetic fields are presented. The simulation results are compared with the finite difference time domain (FDTD) method and good agreement is obtained. Using this approach, the transient responses for multi-conductor transmission lines above lossy ground with nonlinear or frequency-dependent terminations excited by external electromagnetic fields could be obtained quickly and accurately.

Keywords: equivalent circuit, electromagnetic field, lossy ground, multi-conductor transmission lines, matrix

Classification: Electromagnetic theory

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1   Introduction

When the termination is linear load, the transient analysis for single line above lossy ground excited by external electromagnetic fields can be performed by the inverse Fourier transform of the results obtained in the frequency domain, and the finite difference time domain (FDTD) method can be used to solve the problem for multi-conductor transmission lines above lossy ground [1, 2, 3]. However, it will become very complex if the termination load is nonlinear [4, 5]. Macro-model technique can solve the problem by building the equivalent circuit of the transmission line, which can directly linked to nonlinear SPICE-like circuit simulators [6, 7, 8]. The method in [9] can get the transient responses of the transmission lines above lossless ground excited by external electromagnetic fields. The delay extraction-based passive compact transmission-line (DEPACT) macro-modelling algorithm can be used to get the coupled responses of lossy multi-conductor transmission lines in [10]. However, the equivalent circuit of multi-conductor transmission lines above lossy ground excited by external electromagnetic fields is not found.

The method of DEPACT macro-modelling algorithm is that the equivalent circuit of transmission line is viewed as a cascade of some transmission line sub-networks, and each sub-network is represented by a cascade of one lossy section and two lossless sections [11]. The authors have built the equivalent circuit of a single transmission line above lossy ground excited by the external electromagnetic fields in [12]. The equivalent circuit of multi-conductor transmission lines above lossy ground will be more complex compared with single transmission line above lossy ground, and many problems of matrices need to be solved. In this paper, for multi-conductor transmission lines above lossy ground, the frequency-domain sections of ground impedance matrix and exciting voltage matrix will be approximated as the rational function form by the vector fitting (VF) method [13]. So the time domain results can be obtained conveniently by the Laplace element in HSPICE. The DEPACT macro-models and equivalent sources of external electromagnetic fields can be built based on voltage controlled current source and voltage controlled voltage source. Then, combined with these equivalent circuits, the equivalent circuits for multi-conductor transmission lines above lossy ground excited by external electromagnetic fields will be presented.

2 Proposed equivalent circuit

The multi-conductor transmission lines above lossy ground excited by external electromagnetic fields can be represented by Telegrapher's equations in the frequency-domain as

$$\frac{d}{dx}\begin{bmatrix} V^{sca}(x,s) \\ I(x,s) \end{bmatrix} = Q(s)\begin{bmatrix} V^{sca}(x,s) \\ I(x,s) \end{bmatrix} + \tilde{F}^{sca}(x,s) \tag{1}$$

where $\quad Q(s) = (A(s) + sB), \quad B = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix}, \quad A(s) = \begin{bmatrix} 0 & -Z(s) \\ 0 & 0 \end{bmatrix}, \quad \tilde{F}^{sca}(x,s) =$

$$\begin{bmatrix} E_{x1}^{inc}(x,s) + E_{x1}^{ref}(x,s) \\ \vdots \\ E_{xn}^{inc}(x,s) + E_{xn}^{ref}(x,s) \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

$V^{sca}(x,s)$ and $I(x,s)$ are the vector matrices of scattered voltages and currents along the lines in frequency domain. L and C are the matrices of per-unit-length inductance and conductance. $C = \varepsilon_0\mu_0/L, \quad L_{ii} \approx \mu_0/(2\pi)\ln 2h_i/r_i, \quad L_{ij} \approx \mu_0/(2\pi)\ln(\sqrt{d_{ij}^2 + (h_i + h_j)^2}/\sqrt{d_{ij}^2 + (h_i - h_j)^2})$, where h_i and h_j are the height of the conductors i and j above the ground. d_{ij} is horizontal distance between two conductors i and j. r_i is the radius of conductor i. $Z(s)$ is the matrix of the ground impedance, $Z(s)_{ii} \approx s\mu/(2\pi)\ln((1 + \gamma_g h_i)/\gamma_g h_i), \quad Z_{ij}(s) \approx s\mu_0/(4\pi)\ln[((1 + \gamma_g(h_i + h_j)/2)^2 + (\gamma_g d_{ij}/2)^2)/((\gamma_g(h_i + h_j)/2)^2 + (\gamma_g d_{ij}/2)^2)], \quad r_g = \sqrt{s\mu_0(\sigma_g + s\varepsilon_r\varepsilon_0)}$, in which σ_g and ε_r represent the ground conductivity and relative permittivity, respectively. $Z(s)_{ii}$ and $Z_{ij}(s)$ are self and mutual ground impedance. $\tilde{F}^{sca}(x,s)$ represents the effect matrix of external exciting field. $E_{xi}^{inc}(x,s)$ is the horizontal component of incident electric field, and $E_{xi}^{ref}(x,s)$ is the horizontal component of reflecting field along the wire. The solutions of Eq. (1) can be expressed as:

$$\begin{bmatrix} \boldsymbol{V}^{sca}(l,s) \\ \boldsymbol{I}(l,s) \end{bmatrix} = e^{\boldsymbol{Q}(s)l} \begin{bmatrix} \boldsymbol{V}^{sca}(0,s) \\ \boldsymbol{I}(0,s) \end{bmatrix} + \boldsymbol{J}^{sca}(s) \tag{2}$$

$$\boldsymbol{J}^{sca}(s) = \begin{bmatrix} \boldsymbol{V}^f(s) \\ \boldsymbol{I}^f(s) \end{bmatrix} = \int_0^l e^{\boldsymbol{Q}(s)(l-x)} \tilde{\boldsymbol{F}}^{sca}(x,s) \mathrm{d}x \tag{3}$$

To build the equivalent circuits for Eq. (2), first the DEPACT macro-model is built when $\boldsymbol{J}^{sca}(s)$ is ignored. Second, the equivalent sources of $\boldsymbol{J}^{sca}(s)$ are built. Then, the equivalent circuits for Eq. (2) can be build by combining the equivalent sources and the DEPACT macro-model.

The DEPACT macro-model is composed of some transmission line sub-networks, and each sub-network is represented by a cascade of one lossy section and two lossless sections [11]. Supposing that the number of sub-networks is m, the length of the lossless section will be $d/2m$, and the equivalent circuit for the lossless section can be built as [8]. The length of the lossy section will be d/m, and the exponential stamp matrix can be written as

$$e^{A(s)l/m} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{Z}(s)l/m \\ 0 & \boldsymbol{I} \end{bmatrix} \tag{4}$$

The voltage and current matrices on both ends of lossy sections are related as

$$\begin{bmatrix} \boldsymbol{V}(l_{k+1},s) \\ -\boldsymbol{I}(l_{k+1},s) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{Z}(s)l/m \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}(l_k,s) \\ \boldsymbol{I}(l_k,s) \end{bmatrix} \tag{5}$$

For the conductor i, expand Eq. (5) and yield the following expression:

$$\begin{aligned} V_i(l_k,s) &- V_i(l_{k+1},s) \\ &= (Z(s)_{i1}/Z(s)_{11})Z(s)_{11}(d/m)I_1(l_k,s) \\ &\quad + \cdots + Z(s)_{ii}(d/m)I_i(l_k,s) + \cdots + (Z(s)_{in}/Z(s)_{nn})Z(s)_{nn}(d/m)I_n(l_k,s) \\ &= (Z(s)_{i1}/Z(s)_{11})V_{11}(l_k,s) + \cdots + V_{ii}(l_k,s) + \cdots + (Z(s)_{in}/Z(s)_{nn})V_{nn}(l_k,s) \end{aligned} \tag{6}$$

where

$$I_i(l_k,s) = [1/(Z(s)_{ii}d/m)]V_{ii}(l_k,s) \tag{7}$$

$$V_{cij}(l_k,s) = (Z(s)_{ij}/Z(s)_{jj})V_{jj}(l_k,s) \tag{8}$$

The Eq. (7) and Eq. (8) can be expressed in the time domain as:

$$I_i(l_k,t) = [1/(Z(t)_{ii}d/m)] * V_{ii}(l_k,t) \tag{9}$$

$$V_{cij}(l_k,t) = (Z(t)_{ij}/Z(t)_{jj}) * V_{jj}(l_k,t) \tag{10}$$

The $1/(Z(s)_{ii}d/m)$ and $Z(s)_{ij}/Z(s)_{jj}$ can be written as rational function form by the vector fitting method, then the time domain results can be obtained conveniently and quickly by the Laplace element in HSPICE. The equivalent circuit of lossy section can be represented using the Laplace element in HSPICE, and is realized using voltage controlled current source and voltage controlled voltage source.

When the external electromagnetic field is uniform plane wave, the geometry for conductor i is shown in Fig. 1. α is electric field polarization angle, ϕ is azimuthal angle, and ψ is elevation angle. Z_{i1} and Z_{i2} are the termination loads. $E_v = E_0 \cos \alpha$ and $E_h = E_0 \sin \alpha$ are vertical and horizontal electric field, respectively.
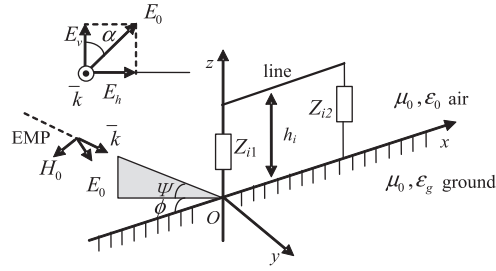
Fig. 1. The geometry of external electromagnetic field and the conductor i above lossy ground.

The $\boldsymbol{J}^{sca}(s)$ can be expressed as

$$\boldsymbol{J}^{sca}(s) = E_0(s)\boldsymbol{f}(s) \tag{11}$$

where

$$
\boldsymbol{f}(s) = \begin{bmatrix} \boldsymbol{f}^V(s) \\ \boldsymbol{f}^I(s) \end{bmatrix}
$$

$$
= \int_0^l e^{\boldsymbol{Q}(s)(l-x)} e^{-s\frac{\cos\psi\cos\phi x}{c}} dx \begin{bmatrix} \cos\alpha\sin\psi\cos\phi(e^{s\frac{\sin\psi h_1}{c}} - R_{ver}e^{-s\frac{\sin\psi h_1}{c}}) + \sin\alpha\sin\phi(e^{s\frac{\sin\psi h_1}{c}} + R_{lev}e^{-s\frac{\sin\psi h_1}{c}}) \\ \vdots \\ \cos\alpha\sin\psi\cos\phi(e^{s\frac{\sin\psi h_n}{c}} - R_{ver}e^{-s\frac{\sin\psi h_n}{c}}) + \sin\alpha\sin\phi(e^{s\frac{\sin\psi h_n}{c}} + R_{lev}e^{-s\frac{\sin\psi h_n}{c}}) \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

$$\tag{12}$$

The Eq. (11) can be expressed in the time domain as:

$$
\boldsymbol{J}^{sca}(t) = \begin{bmatrix} \boldsymbol{V}^f(t) \\ \boldsymbol{I}^f(t) \end{bmatrix} = \begin{bmatrix} E_0(t) * f_1^V(t) \\ \vdots \\ E_0(t) * f_n^V(t) \\ E_0(t) * f_1^I(t) \\ \vdots \\ E_0(t) * f_n^I(t) \end{bmatrix} \tag{13}
$$

$f_i^V(s)$ and $f_i^I(s)$ can be written as rational function form by the vector fitting method, then the time domain results can be obtained conveniently and quickly by the Laplace element in HSPICE. $V_i^f(t)$ and $I_i^f(t)$ are realized using voltage controlled voltage source and voltage controlled current source, respectively. Therefore, the equivalent circuit of scatted voltages is as Fig. 2.

The results can be expressed back in terms of total voltages, as

$$
\begin{bmatrix} V(l,s) \\ I(l,s) \end{bmatrix} = e^{\boldsymbol{Q}(s)l}\left\{ \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} - \begin{bmatrix} V_z^{ex}(0,s) \\ 0 \end{bmatrix} \right\} + \boldsymbol{J}^{scax}(s) + \begin{bmatrix} V_z^{ex}(l,s) \\ 0 \end{bmatrix} \tag{14}
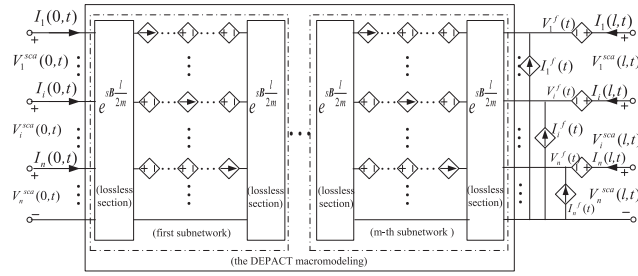$$

where

Fig. 2. Equivalent circuits of scatted voltages.

$$V_{zi}{}^{ex}(x,s) = -\int_{-\infty}^{h} E_{zi}^{ex}(x,z)dz$$

$$= E_0(s)\left\{ \begin{array}{l} -\cos\alpha T_{ver}\cos\psi_t e^{-\gamma_g\cos\psi_t\cos\phi x}/\gamma_g\sin\psi_t - \cos\alpha\cos\psi e^{-s\cos\psi\cos\phi x/c} \\ [c/(s\sin\psi)(e^{s\sin\psi h_i/c}-1) - R_{ver}c/(s\sin\psi)(e^{-s\sin\psi h_i/c}-1)] \end{array} \right\}$$

(15)

The solution can be obtained as

$$V_{zi}{}^{ex}(0,s) = E_0(s)f^0(s)$$

$$= E_0(s)\left\{ \begin{array}{l} -\cos\alpha T_{ver}\cos\psi_t/(\gamma_g\sin\psi_t) - \cos\alpha\cos\psi \\ [c/(s\sin\psi)(e^{s\sin\psi h_i/c}-1) - R_{ver}c/(s\sin\psi)(e^{-s\sin\psi h_i/c}-1)] \end{array} \right\}$$

(16)

$$V_{zi}{}^{ex}(l,s) = E_0(s)f^l(s)$$

$$= E_0(s)\left\{ \begin{array}{l} -\cos\alpha T_{ver}\cos\psi_t e^{-\gamma_g\cos\psi_t\cos\phi l}/(\gamma_g\sin\psi_t) - \cos\alpha\cos\psi e^{-s\cos\psi\cos\phi l/c} \\ [c/(s\sin\psi)(e^{s\sin\psi h_i/c}-1) - R_{ver}c/(s\sin\psi)(e^{-s\sin\psi h_i/c}-1)] \end{array} \right\}$$

(17)

The Eq. (16) and Eq. (17) can be expressed in the time domain as:

$$V_{zi}{}^{ex}(0,t) = E_0(t) * f_i^0(t) \tag{18}$$

$$V_{zi}{}^{ex}(l,t) = E_0(t) * f_i^l(t) \tag{19}$$

$f_i^0(s)$ and $f_i^l(s)$ can also be written as rational function form by the vector fitting method. $V_{zi}{}^{ex}(0,t)$ and $V_{zi}{}^{ex}(l,t)$ are realized using voltage controlled voltage sources.

The equivalent circuit of total voltages is as Fig. 3. In the following, we will verify the equivalent circuit.
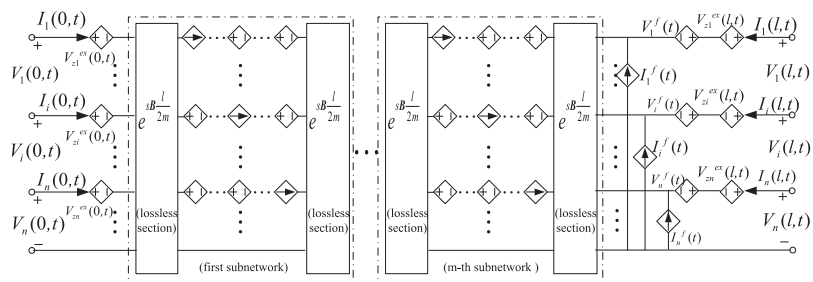

Fig. 3. Equivalent circuits of total voltages.

3 Application of equivalent circuit

We choose two lines located at a height $h = 3.3\,\mathrm{m}$ above lossy ground as the example. The length is $40\,\mathrm{m}$, and the radius is $2.5\,\mathrm{mm}$. The parameters of the ground are $\sigma_g = 0.016\,\mathrm{S/m}$, $\varepsilon_r = 10$ and $\mu_r = 1$. $Z_{11} = Z_{12} = Z_{21} = Z_{22} = 50\,\Omega$. The waveform of the electric field is described by a double exponential $E_0(t) = 1000(e^{-10000t} - e^{-400000t})\,\mathrm{V/m}$ with $\alpha = 0$, $\varphi = 0$ and $\psi = \pi/6$. Fig. 4 depicts transient responses of voltage induced at the line end by the proposed method and the FDTD method when the m is 20.
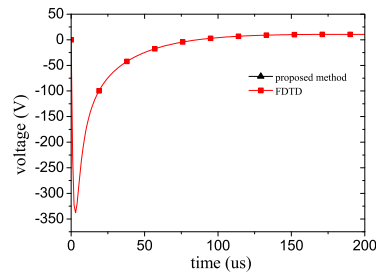

Fig. 4. Transient response of voltage induced at the line end when the termination resistors are all 50.

From Fig. 4, we can see that the transient responses obtained by the proposed method and the FDTD method are excellent agreement. The computational time of the FDTD method is $120\,\mathrm{s}$ when the time step is $1\,\mu\mathrm{s}$. Moreover, the computational time of the proposed method is $50\,\mathrm{s}$, which is less than the half of the FDTD method.

When a transient voltage suppressor is connected in parallel with one of the load resistance, the transient response of the voltage induced at the line end by the proposed method is shown in Fig. 5. The type of the transient voltage suppressor is 1.5KE39CA.
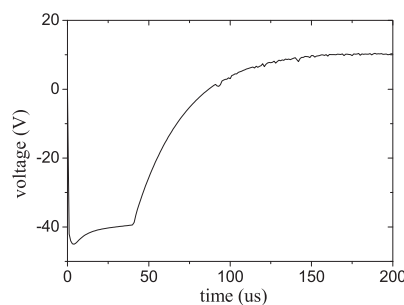

Fig. 5. Transient response of the voltage induced at the line end when a transient voltage suppressor is connected in parallel with one of the load resistance.

From Fig. 5 we can see that the amplitude of the response is less than the maximum clamping voltage of the transient voltage suppressor, also agrees well with theoretical analysis result. When four different capacitances are connected in parallel with one of the load resistance, respectively, the transient responses are as Fig. 6.
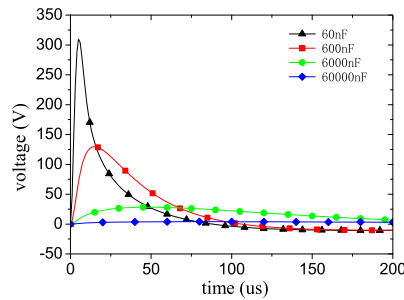
Fig. 6. Transient responses of the voltage induced at the line end when four different capacitances are connected in parallel with one of the load resistance, respectively.

From Fig. 6 we can see that the amplitude of the response is lower when the capacitance is bigger. That is because the cut-off frequency will be lower when the capacitance becomes bigger, and more compositions of the response will discharge to the earth.

4 Conclusion

For multi-conductor transmission lines above lossy ground, the frequency-domain sections of ground impedance matrix and exciting voltage matrix are approximated as the rational function form by the vector fitting (VF) method. The equivalent circuits for multi-conductor transmission lines above lossy ground excited by external electromagnetic fields are presented based on voltage controlled current source and voltage controlled voltage source. When the termination loads are resistances, the transient response of voltage induced at a load simulated by the proposed method is compared with the result obtained by the FDTD method, a good agreement is observed, and the computational time of proposed method is less than the half of the FDTD method. When a transient voltage suppressor and three different capacitances are connected in parallel with one of the load resistances, respectively, the transient responses are also agreed with the theoretical analysis results. Consequently, using this approach, the transient response for multi-conductor transmission lines above lossy ground with nonlinear or frequency-dependent terminations excited by the external electromagnetic fields can be obtained quickly and accurately.

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