

Mainlobe interference suppression via eigenprojection processing and covariance matrix sparse reconstruction

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Abstract: In this paper, a novel mainlobe interference suppression method via eigen-projection processing and covariance matrix sparse reconstruction is proposed, which is able to work when the desired signal is present in the training data. Firstly, the proposed method uses the spatial spectrum algorithm to estimate the direction of arrival (DOA) of sources and the power of sources can be estimated by compressive sensing (CS). Then, the eigen-projection matrix is calculated via the result of DOA to suppress the mainlobe interference in echo data. Finally, adaptive weight vector is obtained by SINCM reconstruction. Compared with other methods, the proposed method can achieve better performance and stronger robustness. **Keywords:** mainlobe interference, covariance matrix sparse reconstruction, eigen-projection processing, adaptive beamforming

Classification: Electromagnetic theory

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1 Introduction

In recent years, adaptive beamforming has been deeply researched and widely applied in radar, wireless communication, microphone and other fields. Sidelobe interferences can be suppressed by adaptive beamforming, effectively [1, 2, 3]. However, when the interference falls into mainlobe area, the conventional adaptive beamforming may lead to the pattern distortion and the output signal to interference plus noise ratio (SINR) decreased, et al. [4].

In order to overcome this problem, some methods have been proposed, such as a large aperture auxiliary array [5], the block matrix processing (BMP) method [6], the eigen-projection matrix processing (EMP) method [7]. Nevertheless, the auxiliary array method must work together with the auxiliary array. The BMP method would cause the effective degrees of freedom (DOF) decreasing. The EMP method doesn't reduce the degrees of freedom and is widely developed. The modified EMP method based on covariance matrix reconstruction (CMR) is proposed for main beam peak offset [8]. However, The EMP method relies on the absence of the desired signal in the training data. In addition, the pattern peak offset cannot be conquered in both BMP and EMP method. In order to overcome these problems, a modified method is proposed by utilizing Capon power spectrum for covariance matrix reconstruction [9], but it suffers from spectrum energy leakage. When the DOA of the desired signal and the mainlobe interference are closed and the power of mainlobe interference is large, it is not possible to distinguish the desired signal and the mainlobe interference through searching the space spectrum. Another modified EMP method [10] is proposed to reconstruct covariance matrix by iterative adaptive approach (IAA), which has high accuracy of DOA and power spectrum estimation. However, IAA method uses the Frobenius norm at each iteration, which costs a lot of computing resources.

In this paper, a novel mainlobe interference suppression method is proposed, which bases on the covariance matrix sparse reconstruction. The problem of interference-plus-noise covariance matrix (INCM) reconstruction can be transformed into a compressed sensing (CS) problem, and the power spectrum estimation can be given by closed-form solution. The proposed method can achieve high estimation accuracy of power spectrum and high output SINR at lower snapshots.





Finally, simulation results demonstrate the effectiveness of the proposed method even when the signal of interest (SOI) is present in the training data.

Notation: $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{-1}$, $\|\cdot\|_{F}$, $\|\cdot\|_{0}$, $\|\cdot\|_{1}$, and $E[\cdot]$ denote conjugate transpose, transpose, inverse, Frobenius norm, L0 norm, L1 norm, and mathematical expectation operator, respectively.

2 Problem formulation

We consider a far-field narrowband array signal model composed of N sensors, which are half-wavelength spaced uniform linear array (ULA). Hypothetically, there is a desired signal and P interferences signal and one mainlobe interference, where P + 1 < N. The signal can be expressed as

$$\mathbf{x}(k) = \mathbf{a}(\theta_0)\mathbf{s}_0(k) + \sum_{p=1}^{P+1} \mathbf{a}(\theta_p)\mathbf{s}_p(k) + \mathbf{n}(k)$$

= $\mathbf{x}_{s0}(k) + \mathbf{x}_{int}(k) + \mathbf{n}(k)$ (1)

where \mathbf{s}_p , $\mathbf{a}(\theta_p)$, θ_p and $\mathbf{n}(k)$ denote the *p*-th interference signal complex envelope, the corresponding steering vector (SV), *p*-th interference signal DOA and Gaussian noise, respectively. $\mathbf{a}(\theta_0)$ and $\mathbf{s}_0(k)$ denote the corresponding steering vector and the complex envelope of desired signal. $\mathbf{a}(\theta_p)$ and $\mathbf{s}_p(k)$ stand for the *p*-th sidelobe interference. And $\mathbf{s}_{P+1}(k)$ denotes the mainlobe interference signal waveform. Furthermore, all signals are not related to each other.

For instance, for a ULA, the mainlobe width BW₀ can be calculated as

$$BW_0 = 2 \arcsin\left(\frac{\lambda}{Nd} + \sin\theta_0\right)$$
(2)

where λ and *d* denote wavelength and array element distance, respectively. Therefore, the mainlobe angle area Θ can be expressed as

$$\Theta = \left[\theta_0 - \frac{BW_0}{2}, \theta_0 + \frac{BW_0}{2}\right] \tag{3}$$

And $\overline{\Theta}$ denotes the complement sector of Θ , where $\Theta \cap \overline{\Theta} = \emptyset$ and $\Theta \cup \overline{\Theta} = [-\frac{\pi}{2}, \frac{\pi}{2}]$. Apart from desired signal, the signal in Θ is the mainlobe interference, and other interference signals are sidelobe interferences. Assuming that $\mathbf{w} = [w_1, w_2, \dots, w_N]^T$ is an adaptive beamformer, and the beamforming output can be calculated as

$$\mathbf{y} = \mathbf{w}^{\mathrm{H}} \mathbf{X} \tag{4}$$

where $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \dots, \mathbf{x}(K)]$ is the echo signal and *K* is the number of training data snapshots. The problem of beamforming is the extraction of the weight vector. Here, Minimum Variance Distortionless Response (MVDR) beamformer is taken as an example. The MVDR beamformer weight vector can be expressed by the following formula

$$\min_{\mathbf{w}} \mathbf{w}^{\mathsf{H}} \mathbf{R}_{i+n} \mathbf{w} \ s.t. \ \mathbf{w}^{\mathsf{H}} \mathbf{a}(\theta_0) = 1$$
(5)

where \mathbf{R}_{i+n} is INCM, and it can be calculated as

$$\mathbf{R}_{i+n} = E[(\mathbf{x}_{\text{int}} + \mathbf{n})(\mathbf{x}_{\text{int}} + \mathbf{n})^{\text{H}}]$$
(6)



The solution of (5) is

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^{\text{H}} \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_0)}$$
(7)

However, the INCM is hardly obtained, especially under the nonideal conditions, such as array mismatch, SOI appearing in the training data, the limited number of snapshots and so on [9]. Consequently, INCM is regular substituted by the sample covariance matrix in practical.

$$\hat{\mathbf{R}}_{i+n} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k) \mathbf{x}^{\mathrm{H}}(k)$$
(8)

The beamformer can be written as

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$$\mathbf{w}_{\text{SMI}} = \frac{\hat{\mathbf{R}}_{i+n}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^{\text{H}} \hat{\mathbf{R}}_{i+n}^{-1} \mathbf{a}(\theta_0)}$$
(9)

where \mathbf{w}_{SMI} is the Sample Matrix Inversion (SMI) beamformer. Adopting this method, the MVDR beamformer can be transformed into SMI beamformer. In addition, the SMI beamformer will form the self-null in the mainlobe when there is a mainlobe interference in the training data, and the desired signal power will be suppressed. Unfortunately, the performance of beamformer will deteriorate further if the desired signal mix together with the training samples.

3 Eigen-projection processing

EMP method has been proposed to suppress the mainlobe interference. The sample covariance matrix is eigen-decomposed as

$$\hat{\mathbf{R}}_{i+n} = \sum_{i=1}^{N} \lambda_i \mathbf{u}_i \mathbf{u}_i^{\mathrm{H}} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^{\mathrm{H}} + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^{\mathrm{H}}$$
(10)

where \mathbf{u}_i and λ_i denote *i*-th eigenvector and eigenvalue, respectively. And $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{P+1}]$ spans the interference subspace, and the corresponding eigenvalue space is $\mathbf{\Lambda}_s = diag[\lambda_1, \lambda_2, \dots, \lambda_{P+1}]$. $\mathbf{U}_n = [\mathbf{u}_{P+2}, \mathbf{u}_{P+3}, \dots, \mathbf{u}_N]$ spans the noise subspace, which eigenvalue space is $\mathbf{\Lambda}_n = diag[\lambda_{P+2}, \lambda_{P+3}, \dots, \lambda_N]$.

However, the problem of determining the eigenvector of the mainlobe interference is unavoidable in EMP method. In general, the correlation coefficient method is used to determine the eigenvector of the mainlobe interference. Assuming \mathbf{u}_m stand for the eigenvector of the mainlobe interference, and the eigenprojection matrix **B** is expressed as

$$\mathbf{B} = \mathbf{I} - \mathbf{u}_m (\mathbf{u}_m^{\mathrm{H}} \mathbf{u}_m)^{-1} \mathbf{u}_m^{\mathrm{H}}$$
(11)

where **I** is the $N \times N$ identity matrix. The \mathbf{u}_m is estimated by

$$|\mathbf{u}_m^{\mathrm{H}} \mathbf{a}(\theta_0)|^2 \ge c |\mathbf{a}(\theta_0)|^2 \tag{12}$$

where c(c > 0) is a scaling coefficient, which rely on empirical values. In [8], \mathbf{u}_m can be determined by utilizing the correlation coefficient method, which can be obtained by

$$|\rho(\mathbf{u}_m, \mathbf{a}(\theta_0))| = \max_{\mathbf{u}_i} |\rho(\mathbf{u}_i, \mathbf{a}(\theta_0))|$$
(13)

where ρ is correlation coefficient.





Then, utilizing the eigen-projection matrix \mathbf{B} to remove the mainlobe interference in echo data \mathbf{x} as follows

$$\mathbf{Y} = \mathbf{B}\mathbf{X} \tag{14}$$

In this approach, the mainlobe interference can be removed effectively.

4 The proposed method

Consider sampling the whole space, and $\{\theta_1, \theta_2, \dots, \theta_M\}$ are the grid points, where $M \gg P + 1$. Overcomplete basement can be written as

$$\mathbf{M} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_M)]$$
(15)

where $\mathbf{a}(\theta_m)$ denotes the SV of θ_m . Then, the array received signals are rewritten as

$$\mathbf{x}(t) = \mathbf{M}\tilde{\mathbf{s}}(t) + \mathbf{n}(t) \tag{16}$$

where $\tilde{\mathbf{s}}(t)$ is the $M \times 1$ sparse vector, and the nonzero elements correspond to the location of the signal grid. Therefore, the covariance matrix can be expressed as

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}(t)^{\mathrm{H}}]$$

$$= \sum_{m=1}^{M} p_{m} \mathbf{a}(\theta_{m}) \mathbf{a}(\theta_{m})^{\mathrm{H}} + \begin{bmatrix} \sigma_{1} & 0 & \cdots & 0 \\ 0 & \sigma_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_{N} \end{bmatrix}$$
(17)
$$= \mathbf{MPM}^{\mathrm{H}} + \Lambda_{\sigma}$$

where Λ_{σ} and p_m denote the noise covariance matrix and the power of θ_m signal, where $\mathbf{P} = diag\{p_m\}$. In [11], the signal locations and their powers can be formulated as

$$\min_{\mathbf{P},\sigma_n^2} \|\hat{\mathbf{R}} - \mathbf{D}\mathbf{P}\mathbf{D}^{\mathrm{H}} - \sigma_n^2 \mathbf{I}\|_F^2 + \kappa \|\mathbf{p}\|_0$$

$$s.t. \ \mathbf{p} \ge \mathbf{0}, \quad \sigma_n^2 \ge 0$$
(18)

where **p** is the spatial spectrum distribution on the sample grids of all locations of $[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_M)]$, $\mathbf{P} = diag\{\mathbf{p}\}$ is the corresponding diagonal matrix, $\mathbf{D} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_{P+1})]$ is the array manifold matrix, and σ_n^2 is the noise power.

Eq. (18) is a CS problem. Fortunately, when the signal support set is obtained, the L_0 norm problem can be transformed into a L_1 norm problem, and the detailed derivation can be found in [11]. The signal support is obtained by MUSIC algorithm, which can be formulated as

$$\tilde{\theta} = peak\left(\frac{1}{\mathbf{a}(\theta)^{\mathrm{H}}\mathbf{U}_{n}\mathbf{U}_{n}^{\mathrm{H}}\mathbf{a}(\theta)}\right)$$
(19)

Eq. (18) can be transformed into

$$\min_{\mathbf{P}(\tilde{\theta})} \|\hat{\mathbf{R}} - \mathbf{D}(\tilde{\theta})\mathbf{P}(\tilde{\theta})\mathbf{D}(\tilde{\theta})^{\mathrm{H}} - \hat{\sigma}_{n}^{2}\mathbf{I}\|_{F}^{2}$$
s.t. $\mathbf{p}(\tilde{\theta}) \ge \mathbf{0}$
(20)

where $\mathbf{D}(\tilde{\theta}) = [\mathbf{a}(\tilde{\theta}_1), \mathbf{a}(\tilde{\theta}_2), \cdots, \mathbf{a}(\tilde{\theta}_{P+1})]$ is the array manifold matrix. $\mathbf{p}(\tilde{\theta})$ is the $\tilde{\theta}$ signal power, and $\mathbf{P}(\tilde{\theta}) = diag\{\mathbf{p}(\tilde{\theta})\}$. $\hat{\sigma}_n^2$ is the minimum eigenvalue of $\hat{\mathbf{R}}$ because





adaptive beamforming is not sensitive to noise error. Through simplified calculation, $\mathbf{p}(\tilde{\theta})$ can be formulated as

$$\mathbf{p}(\tilde{\theta}) = (\mathbf{G}\mathbf{G}^{\mathrm{H}})^{-1}\mathbf{G}^{\mathrm{H}}\mathbf{r}$$
(21)

where **G** and **r** are obtained by stacking the array responses and the sample covariance matrix is subtracted by a noise covariance matrix, respectively. **G** and **r** can be formulated as

$$\mathbf{G} \equiv [vec(\mathbf{a}(\tilde{\theta}_1)\mathbf{a}^{\mathrm{H}}(\tilde{\theta}_1)), \cdots, vec(\mathbf{a}(\tilde{\theta}_{P+1})\mathbf{a}^{\mathrm{H}}(\tilde{\theta}_{p+1}))$$
(22)

$$\mathbf{r} = vec(\hat{\mathbf{R}} - \hat{\sigma}_n^2 \mathbf{I}) \tag{23}$$

3.7

The covariance matrix of the mainlobe interference signal can be expressed as

$$\tilde{\mathbf{R}}_{P+1} = \mathbf{p}(\tilde{\theta}_{P+1})\mathbf{a}(\tilde{\theta}_{P+1})\mathbf{a}(\tilde{\theta}_{P+1})^{\mathrm{H}} = \sum_{i=1}^{N} \hat{\lambda}_{i} \mathbf{u}_{i} \mathbf{u}_{i}^{\mathrm{H}}$$
(24)

Because the neglect of the noise, the maximum eigenvalue corresponds to the mainlobe interference SV $\tilde{\mathbf{u}}_m$. The eigen-projection matrix can be formulated as

$$\mathbf{B} = \mathbf{I} - \tilde{\mathbf{u}}_m (\tilde{\mathbf{u}}_m^{\mathrm{H}} \tilde{\mathbf{u}}_m)^{-1} \tilde{\mathbf{u}}_m^{\mathrm{H}}$$
(25)

The sidelobe-INCM (SINCM) can be expressed as

$$\tilde{\mathbf{R}}_{i+n} = \sum_{i=1}^{P} \mathbf{p}(\tilde{\theta}_i) \mathbf{a}(\tilde{\theta}_i) \mathbf{a}(\tilde{\theta}_i)^{\mathrm{H}} + \hat{\sigma}_n^2 \mathbf{I}$$
(26)

Due to the existence of array error, the desired signal steering vector need to be corrected in practical situation. The SOI can be espressed as $\theta_0 \pm 2^\circ$. By utilizing the result of signal support, and the modified steering vector $\tilde{\theta}_0$ can be updated as

$$\mathbf{a}(\hat{\theta}_0) = \mathbf{a}(\theta_0) \tag{27}$$

The proposed method beamformer can be formulated as

$$\mathbf{w} = \frac{\tilde{\mathbf{R}}_{i+n}^{-1} \mathbf{a}(\tilde{\theta}_0)}{\mathbf{a}(\tilde{\theta}_0)^{\mathrm{H}} \tilde{\mathbf{R}}_{i+n}^{-1} \mathbf{a}(\tilde{\theta}_0)}$$
(28)

The output of adaptive beamforming is calculated as

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$$\mathbf{Z} = \mathbf{w}^{\mathrm{H}} \mathbf{Y} = \mathbf{w}^{\mathrm{H}} \mathbf{B} \mathbf{X}$$
(29)

5 Simulation results

The effectiveness of the proposed method is verified by simulations. A uniform linear array with M = 16 omnidirectional sensors spaced half-wavelength is considered in simulations. The desired signal direction is 0°, and two sidelobe interferences impinge from -25° and 35° with interference-to-noise ratio (INR) 30 and 40 dB. A mainlobe interference impinge from -3° , which INR is 5 dB. The hypothesis that noise is Gaussian white additive noise with SNR 0 dB. The proposed method is compared to EMP, the method proposed in [8] (EMP-CMR) and the method proposed in [9] (EMP-CMIR). When the EMP and EMP-CMR methods are simulated, there is no signal in SOI. The adaptive array pattern is obtained from one Monte Carlo simulation, and the output SINR is calculated by the average of 100 independent Monte Carlo simulations.





In Fig. 1, the beam patterns of EMP, EMP-CMR, EMP-CMIR and the proposed method are shown, and compared with the quiescent (QUI) beam pattern, respectively. It is directly found that the distortion of pattern has been corrected by the proposed method, and the deepest nulls are formed to the sidelobe interference suppression. However, EMP method suffers from the peak offset, EMP-CMR and EMP-CMIR methods suffer from the limited depth of sidelobe interference nulls.



Fig. 1. Adaptive beam patterns comparison.

In Fig. 2, the output SINRs versus the number of snapshots are investigated by the proposed method and compared to Optimum, SMI, EMP, EMP-CMR and EMP-CMIR methods. Compared with other methods, the proposed method can achieve higher output SINR, also can maintain high performance in the low number of snapshots. In addition, the proposed method is lower in complexity and smaller in calculation compared with [9] and the convergence of the proposed algorithm is much faster than other methods.



Fig. 2. Output SINRs versus the number of snapshots.

In Fig. 3, we depict the output SINRs curves when the INR of the mainlobe interference varies from -20 to 20 dB. The number of snapshots is fixed at 100. It is obvious that the proposed method output performance is better than other methods in the INR of mainlobe interference. Under the high INR condition, the proposed method has a great performance advantage over [9], which means that the proposed method is more effective in suppressing mainlobe and sidelobe interferences.







Fig. 3. Output SINRs versus the INR of mainlobe interference.

Finally, we examine the robustness of the beamformers to array SV error. We assume that there is an error in the desired signal guidance vector. Suppose that the desired signal SV is located at -1° , but the true SV is located at 0° . The proposed method is compared with other methods when the INR varies from -20 to 20 dB. Fig. 4 shows the performance of proposed method is higher than other methods at the region of INR.



Fig. 4. Output SINRs with SV error versus INR of mainlobe interference.

6 Conclusion

In this article, we have proposed a novel mainlobe interference suppression method based on eigen-projection processing and covariance matrix sparse reconstruction. The position and the power of the interference in the proposed method can be easily estimated and is more accurate than other methods. In addition, the novel method can maintain the performance, especially when the desired signal exists in the training data. Compared with other methods, the simulation results have demonstrated that the proposed method can achieve better performance and stronger robustness.

