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Memory parameterized FIR filtering for digital phase-locked loop

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Abstract: In this letter, a new filtering method for digital phase-locked loops (DPLLs) is proposed. The proposed method is based on finite impulse response (FIR) filtering, which estimates the state variables of a system using recent finite measurements. FIR filtering requires the optimal selection of a design parameter, the memory size, which has been cumbersome. A method to compute the optimal memory size has been proposed; however, it is ineffective when the noise information is uncertain. Thus, in this letter, a memory parameterized FIR filter (MPFF) is proposed to solve this problem, and the DPLL simulation results are presented for performance demonstration.

Keywords: digital phase-locked loop (DPLL), finite impulse response (FIR) filtering, memory parameterized FIR filter (MPFF)
Classification: Circuits and modules for electronic instrumentation

References

[1] W. C. Lindsey and C. M. Chie: "A survey of digital phase-locked loops," Proc. IEEE **69** (1981) 410 (DOI: 10.1109/PROC.1981.11986).
[2] A. Patapoutian: "On phase-locked loops and Kalman filters," IEEE Trans. Commun. **47** (1999) 670 (DOI: 10.1109/26.768758).
[3] P. F. Driessen: "DPLL bit synchronizer with rapid acquisition using adaptive Kalman filtering techniques," IEEE Trans. Commun. **42** (1994) 2673 (DOI: 10.1109/26.317406).
[4] B. Kim: "Dual-loop DPLL gear-shifting algorithm for fast synchronization," IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process. **44** (1997) 577 (DOI: 10.1109/82.598428).
[5] S. H. You, *et al.*: "Unbiased finite-memory digital phase-locked loop," IEEE Trans. Circuits Syst. II, Exp. Briefs **63** (2016) 798 (DOI: 10.1109/TCSII.2016.2531138).
[6] S. H. You, *et al.*: "Optimal horizon size for unbiased finite memory digital phase-locked loop," IEICE Electron. Express **14** (2017) 20161184 (DOI: 10.1587/elex.14.20161184).
[7] J. M. Pak, *et al.*: "Distributed hybrid particle/FIR filtering for mitigating NLOS effects in TOA-based localization using wireless sensor networks," IEEE Trans. Ind. Electron. **64** (2017) 5182 (DOI: 10.1109/TIE.2016.2608897).
[8] J. M. Pak, *et al.*: "Accurate and reliable human localization using composite particle/FIR filtering," IEEE Trans. Human-Mach. Syst. **47** (2017) 332 (DOI: 10.1109/THMS.2016.2611826).

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[9] J. M. Pak, *et al.*: "Maximum likelihood FIR filter for visual object tracking," Neurocomputing **216** (2016) 543 (DOI: 10.1016/j.neucom.2016.07.047).
[10] J. M. Pak, *et al.*: "Self-recovering extended Kalman filtering algorithm based on model-based diagnosis and resetting using an assisting FIR filter," Neurocomputing **173** (2016) 645 (DOI: 10.1016/j.neucom.2015.08.011).
[11] J. M. Pak, *et al.*: "Improving reliability of particle filter-based localization in wireless sensor networks via hybrid particle/FIR filtering," IEEE Trans. Ind. Informat. **11** (2015) 1089 (DOI: 10.1109/TII.2015.2462771).
[12] H. W. Sorenson and D. L. Alspach: "Recursive Bayesian estimation using Gaussian sums," Automatica **7** (1971) 465 (DOI: 10.1016/0005-1098(71)90097-5).
[13] D. Alspach and H. Sorenson: "Nonlinear Bayesian estimation using Gaussian sum approximations," IEEE Trans. Automat. Control **17** (1972) 439 (DOI: 10.1109/TAC.1972.1100034).
[14] B. Ristic, *et al.*: *Beyond the Kalman Filter: Particle Filters for Tracking Applications* (Arctech House, Norwood, MA, 2004).

1 Introduction

Phase locked-loop (PLL) is a control system circuit used to generate an output signal, that tracks the phase of an input signal. A PLL includes a phase detector, a voltage-controlled oscillator, and a loop filter [1]. Currently, digital PLLs (DPLLs) can use software implementing state estimation algorithms as phase detectors. The Kalman filter (KF) [2, 3, 4] and finite impulse response (FIR) filter [5, 6] have been used for phase detection in the DPLL. While the KF uses all the measurements from the initial time to the current time, the FIR filter uses only recent finite measurements. Thus, the FIR filter is also called the finite memory filter and has better robustness against modeling and computational errors than the KF [7, 8, 9, 10, 11]. However, the FIR filter requires accomplishing a cumbersome task, which is the selection of a design parameter, the memory size (or horizon size). The memory size is the number of measurements used for a single FIR estimation and is a key parameter affecting the FIR filter performance. In [6], a method to compute the optimal memory size, N_{opt}, was proposed. Using this method, N_{opt} can be computed when the noise covariance is known. The measurement noise covariances of sensors are available from experiments, but the process noise covariance remains uncertain and should be estimated. An incorrect estimate of the process noise covariance results in the incorrect computation of N_{opt}. Thus, an alternative to compute the memory size has been proposed in this letter. The proposed method is called the memory parameterized FIR filter (MPFF), which incorporates various memory sizes using the Gaussian sum approximation technique [12, 13] instead of selecting an N_{opt} value. Using this method, the MPFF can offer a reliable performance despite the process noise covariance remaining uncertain. The performance of the MPFF is better than both the KF and the conventional FIR filter, as demonstrated by the DPLL simulations.

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2 Main results

The utilization of the MPFF requires the state space models of a system to estimate its state variables. The state space models of the DPLL have been presented in [3, 5, 6], where the two state variables, the zero crossing point α and the timing offset β at a discrete time k are defined as $\alpha_k = t_0 + k(T_1 - T_0)$ and $\beta_k = T_1 - T_0$, where t_0 is the initial timing offset, T_0 and T_1 are the sampling periods of the receiver and transmitter, respectively. The state vector at a discrete time k is defined as $\mathbf{x}_k = [\alpha_k \ \beta_k]^T$, and the state equation is written as

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k, \tag{1}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \tag{2}$$

where $\mathbf{w}_k$ is the process noise vector and is assumed to be Gaussian noise with a covariance $\mathbf{Q}$. The measurement system obtains noisy measurements of the first state, α_k. The measurement equation is written as

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + v_k, \tag{3}$$

$$\mathbf{C} = [1 \ 0], \tag{4}$$

where v_k is the Gaussian measurement noise with a covariance $\mathbf{R}$.

The MPFF estimates the state vector, $\mathbf{x}_k$, from the noise measurement $\mathbf{y}_k$. The MPFF is an FIR filter that uses multiple memory sizes. We adopt the minimum variance FIR filter (MVFF) as a component of the MPFF. The MVFF equation for the state space models (2)–(4) is

$$\hat{\mathbf{x}}_k = \mathbf{H}_N \mathbf{Y}_N, \tag{5}$$

where $\hat{\mathbf{x}}_k$, $\mathbf{H}_N$, and $\mathbf{Y}_N$ are the estimated state, the gain matrix, and the augmented measurement matrix, respectively. $\mathbf{H}_N$ and $\mathbf{Y}_N$ are defined as

$$\mathbf{H}_N \triangleq \mathbf{J}_N \begin{bmatrix} \mathbf{W}_{1,1} & \mathbf{W}_{1,2} \\ \mathbf{W}_{1,2}^T & \mathbf{W}_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\mathbf{C}}_N^T \\ \tilde{\mathbf{G}}_N^T \end{bmatrix} \mathbf{R}_N^{-1},$$

$$\mathbf{J}_N \triangleq [\mathbf{A}^N \ \mathbf{A}^{N-1} \ \mathbf{A}^{N-2} \ \cdots \ \mathbf{A} \ I],$$

$$\mathbf{W}_{1,1} \triangleq \tilde{\mathbf{C}}_N^T \mathbf{R}_N^{-1} \tilde{\mathbf{C}}_N,$$

$$\mathbf{W}_{1,2} \triangleq \tilde{\mathbf{C}}_N^T \mathbf{R}_N^{-1} \tilde{\mathbf{G}}_N,$$

$$\mathbf{W}_{2,2} \triangleq \tilde{\mathbf{G}}_N^T \mathbf{R}_N^{-1} \tilde{\mathbf{G}}_N + \mathbf{Q}_N^{-1},$$

$$\tilde{\mathbf{C}}_N \triangleq \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A} \\ \mathbf{C}\mathbf{A}^2 \\ \vdots \\ \mathbf{C}\mathbf{A}^{N-1} \end{bmatrix},$$

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$$\tilde{\mathbf{G}}_N \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{CA} & \mathbf{C} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{CA}^{N-2} & \mathbf{CA}^{N-3} & \dots & \mathbf{C} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{R}_N \triangleq [\mathrm{diag}(\overbrace{\mathbf{R}_f\ \mathbf{R}_f\ \cdots\ \mathbf{R}_f}^{N})],$$

$$\mathbf{Q}_N \triangleq [\mathrm{diag}(\overbrace{\mathbf{Q}_f\ \mathbf{Q}_f\ \cdots\ \mathbf{Q}_f}^{N})],$$

$$\mathbf{Y}_N \triangleq [\mathbf{y}_{k-N}^T\ \mathbf{y}_{k-N+1}^T\ \cdots\ \mathbf{y}_{k-1}^T]^T, \tag{6}$$

where $\mathbf{Q}_f$ and $\mathbf{R}_f$ are the process and measurement noise covariances respectively, used for filter design. In (5) and (6), the subscript N indicates the memory size. $\mathbf{H}_N$ and $\mathbf{Y}_N$ change according to the memory size N, which implies that $\hat{\mathbf{x}}_k$ also changes according to N.

The MPFF operates multiple MVFFs in parallel using different memory sizes and obtains multiple estimates. The multiple estimates are then combined using the Gaussian sum technique [12, 13]. We have assumed that the process and measurement noises are Gaussian, and that the estimated state is also Gaussian. The probability density function (PDF) of the estimated state is represented by the conditional PDF, $p(\mathbf{x}_k|\mathbf{Y}_k)$, where $\mathbf{Y}_k \triangleq \{\mathbf{y}_i,\ i = 1, \cdots, k\}$ indicates the sequence of measurements up to the current time k. Thus, the Gaussian sum approximation to combine multiple Gaussian PDFs is

$$p(\mathbf{x}_k|\mathbf{Y}_k) \approx \sum_{i=1}^{N_F} w_k^i \mathcal{N}(\mathbf{x}_k^i; \mathbf{x}_{k|k}^i, \mathbf{P}_{k|k}^i) \tag{7}$$

where w_k^i is the weight, $\mathcal{N}(\mathbf{x}_k^i; \mathbf{x}_{k|k}^i, \mathbf{P}_{k|k}^i)$ is the Gaussian PDF with the mean $\mathbf{x}_{k|k}^i$ and the variance $\mathbf{P}_{k|k}^i$, and M is the number of Gaussian PDFs [14]. For the MPFF, the weights in (7) are initialized as

$$w_0^i = \frac{1}{N_F}, \quad i = 1, 2, \dots, N_F, \tag{8}$$

where N_F is the number of MVFFs operating in parallel. The initialized weights are updated at each time step using the following update rule:

$$w_k^i = \frac{p(\mathbf{y}_k|i) w_{k-1}^i}{\sum_{j=1}^{N_F} p(\mathbf{y}_k|j) w_{k-1}^i}, \tag{9}$$

where $p(\mathbf{y}_k|i)$ is the likelihood of measurement $\mathbf{y}_k$ given the state estimate $\hat{\mathbf{x}}_k^i$ obtained from the i-th MVFF. The likelihood is computed as

$$p(\mathbf{y}_k|i) = \frac{1}{\sqrt{2\pi \mathbf{R}^2}} \exp\left[\frac{(\mathbf{y}_\mathrm{k} - \hat{\mathbf{y}}_\mathrm{k}^\mathrm{i})(\mathbf{y}_\mathrm{k} - \hat{\mathbf{y}}_\mathrm{k}^\mathrm{i})}{\mathbf{R}}\right]. \tag{10}$$

$$\hat{\mathbf{y}}_k^i = \mathbf{C}\hat{\mathbf{x}}_k^i, \tag{11}$$

Finally, the MPFF combines the multiple estimates using the weights as

$$\hat{\mathbf{x}}_k = \sum_{i=1}^{N_F} w_k^i \hat{\mathbf{x}}_k^i. \tag{12}$$

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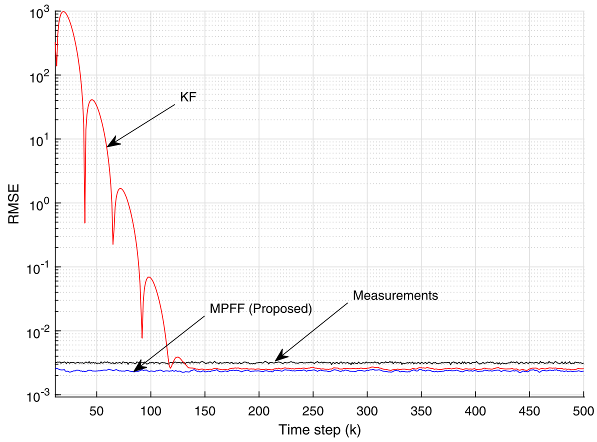
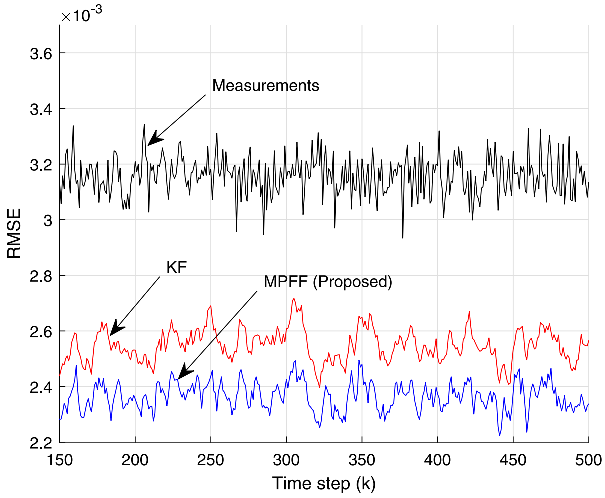
3 Simulation

The use of the MPFF is reliable despite the uncertainty of the process noise covariance. If an incorrect covariance is used, the method proposed in [6] produces an incorrect N_{opt} value and the KF performance worsens. We simulated this situation and compared the MPFF, KF, and the conventional FIR filter (MVFF). Using the state space models (2)–(4), the state and measurement sequences from the time step $k = 1$ to $k = 500$ are obtained. The initial state and noise covariances are determined by setting $\hat{\mathbf{x}}_0 = [0.0005\ 0.0001]^T$, $\mathbf{Q} = 8.33 \times 10^{-8}\mathbf{I}_2$, and $\mathbf{R} = 1.0 \times 10^{-5}$ [5, 6], where $\mathbf{I}_2$ is the 2×2 identity matrix. The noise covariances are required for filtering as well, but we assumed the process noise covariance to be uncertain. Thus, we set $\mathbf{Q}_f = 0.1\mathbf{Q}$ and $\mathbf{R}_f = \mathbf{R}$. The design parameters of the MPFF were chosen as $N_F = 3$, $N_1 = 10$, $N_2 = 20$, and $N_3 = 30$. We then compared the MPFF with the three MVFFs using the three memory sizes. For performance comparison, we computed the root mean square error (RMSE) of the estimation for the first state variable α_k.



(a)



(b)

Fig. 1. Comparison of proposed algorithm (MPFF) and Kalman filter (KF) (a) full picture and (b) zoomed in picture (interval $[150 \leq k \leq 500]$).

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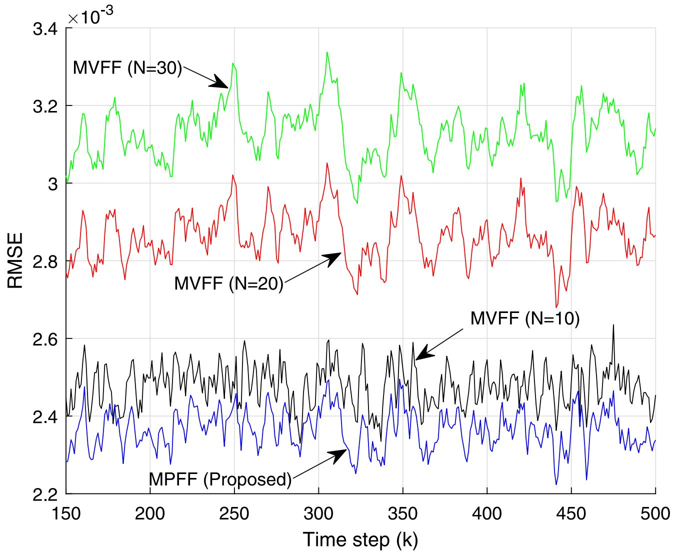

Fig. 2. Comparison of the proposed algorithm (MPFF) and conventional FIR filter (MVFF)

Fig. 1 shows the RMSE of the KF, MPFF, and the measurements. The filters should produce a more accurate estimate than that of the noisy measurements. If the RMSE of a filter is larger than that of the measurement, the filter is unsuitable for use. In Fig. 1, the KF produces significantly larger RMSEs than those produced by the measurements in the early stages of the time steps. The MPFF constantly produces smaller RMSEs than those produced by both, the measurements and the KF. Thus, the MPFF is more accurate and reliable than the KF in this DPLL simulation.

Fig. 2 compares the MPFF with the conventional MVFFs. We see that the RMSE of the MVFF is smaller than those of the three MVFFs using $N = 10$, 20, and 30. This demonstrates that combining the memory sizes by the Gaussian sum technique presents a more accurate estimation than using the memory sizes individually.

4 Conclusion

In this letter, a new filtering method called the MPFF was proposed for the DPLL. The KF, one of the most renowned filters, exhibited large initial errors when the process noise information was uncertain. However, the MPFF constantly exhibited smaller errors than not only the KF but also the conventional MVFF. Thus, the proposed MPFF can present an accurate and reliable phase detection performance in DPLLs.

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