

LETTER

Equivalent circuits of transmission lines sag above lossy ground excited by external electromagnetic fields

Qingxi Yang^{1a)}, Xing Zhou¹, Kai Yao¹, Tianpeng Li¹, Yan Zhang², and Min Zhao¹

Abstract To obtain actual responses of nonlinear terminations for the transmission lines sag above lossy ground, it imperatively need to build the equivalent circuits of transmission lines sag. In this paper, the multi-conductor transmission lines sag above lossy ground are divided into a sequence of smaller segments, each of which is approximate equal height. Equivalent circuit of the transmission lines sag is composed of each segment in series. And the simplified equivalent circuit of transmission lines sag above lossy ground excited by the external electromagnetic fields is built by combined the neighboring elements. The proposed method is compared with the result obtained by the FDTD method, a good agreement is observed. And the amplitude of the response is less than the results when the transmission lines are considered as the straight lines.

Keywords: equivalent circuit, electromagnetic field, lossy ground, transmission lines, sag

Classification: Electromagnetic theory

1. Introduction

By building equivalent circuits of transmission lines, we can obtain the time domain response of nonlinear termination loads [1, 2, 3, 4, 5, 6, 7]. For the transmission lines above lossy ground excited by external electromagnetic fields, the equivalent circuits have been built in [8, 9]. The overhead transmission lines are considered as the straight lines in these papers [10, 11, 12, 13, 14, 15, 16]. Actually, the line is not straight and does not parallel to the ground level due to the sag produced by the weight of the line, etc [17, 18, 19, 20, 21, 22]. To obtain actual responses of nonlinear terminations for the transmission lines sag above lossy ground, it imperatively need to build the equivalent circuits of transmission lines sag.

In this paper, the multi-conductor transmission lines above lossy ground with sag are divided into a sequence of smaller segments, each of which is approximate equal height. Equivalent circuit of the transmission lines sag is composed of each segment in series. And the simplified equivalent circuit of transmission lines sag above the lossy ground excited by the external electromagnetic fields is built by combined the neighboring elements.

2. Sag calculated

The sag for one line of multi-conductor transmission lines above lossy ground is shown in Fig. 1 [17]. The sag height difference $h'(x)$ with respect to the point x can be calculated as:

$$h'(x) = 2 \frac{\sigma_0}{g} sh\left(\frac{g}{2\sigma_0} x\right) sh\left(\frac{g}{2\sigma_0} (L - x)\right) \quad (1)$$

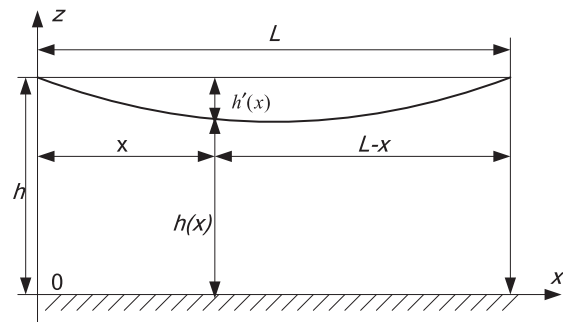


Fig. 1. Geometry of sag

Where σ_0 is the horizontal stress of the line at the lowest point, g is the load of the line per unit area, L is the length of line, x is the horizontal distances of the point x to the left neighboring towers, h is the height of the tower. So the actual height of line sag at the point x given by:

$$h(x) = h - h'(x) \quad (2)$$

3. Proposed equivalent circuit

From [9], we can see that the per-unit-length inductance matrix \mathbf{L} , the per-unit-length conductance matrix \mathbf{C} , and ground impedance $\mathbf{Z}(s)$ will be vary with the actual height $h(x)$, for the multi-conductor transmission lines sag above lossy ground. The multi-conductor transmission lines sag above lossy ground can be divided into a sequence of smaller segments, each of which is approximate equal height [23, 24, 25, 26]. The per-unit-length parameter matrices of every segment are regarded as constant value. Assumed that the multi-conductor transmission lines sag are considered as a cascade of N segments. For the k segment, the multi-conductor transmission lines above lossy ground excited by external electromagnetic fields can be represented by Telegrapher's equations in the frequency-domain as:

¹Army Engineering University, ShiJiaZhuang 050003, China

²School of Electrical Engineering, Hebei University of Science and Technology, Shijiazhuang, China

a) ruoshui2015666@163.com

DOI: 10.1587/elex.16.20190102

Received February 22, 2019

Accepted February 26, 2019

Publicized March 11, 2019

Copyrighted April 10, 2019

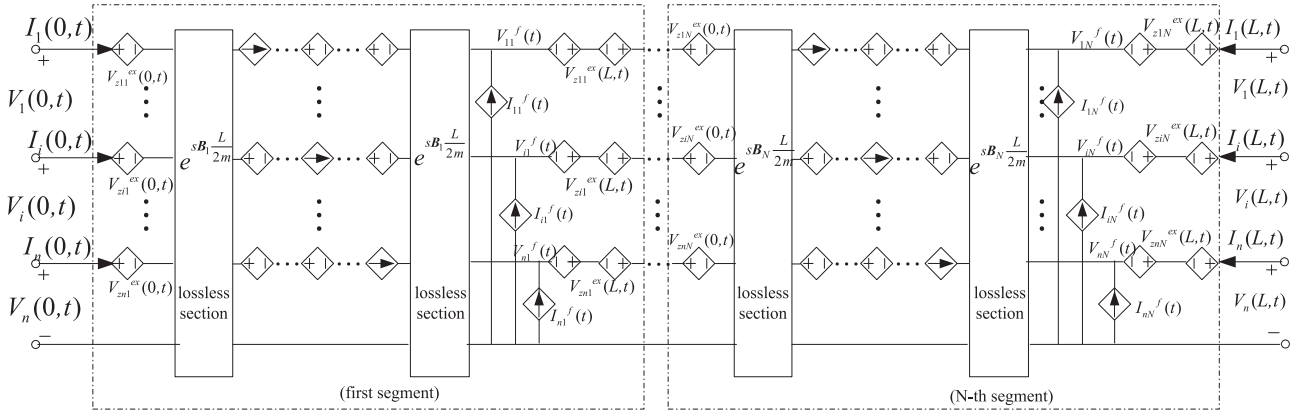


Fig. 2. Equivalent circuit of the multiconductor transmission lines sag above the lossy ground excited by the external electromagnetic fields

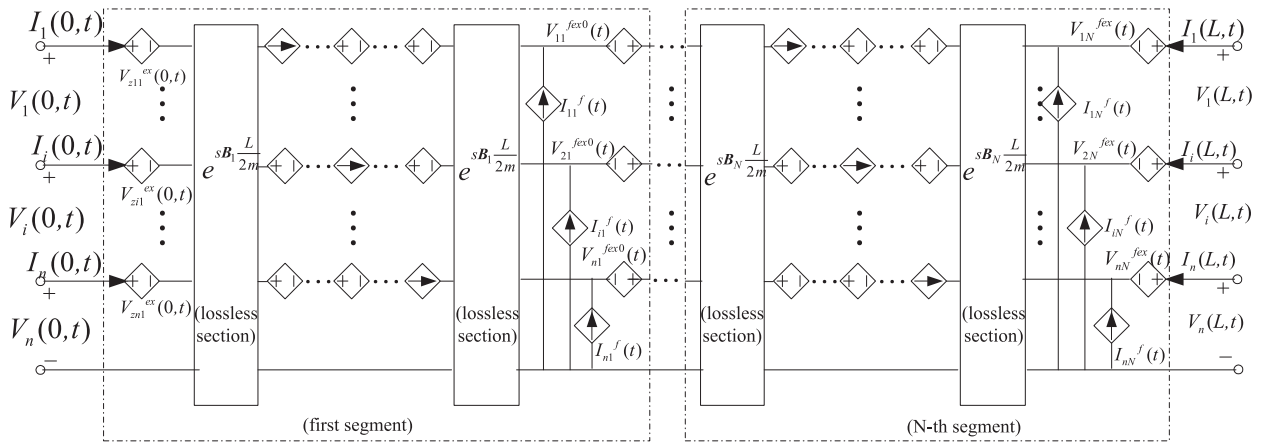


Fig. 3. Simplified equivalent circuit of the multiconductor transmission lines with sag above the lossy ground excited by the external electromagnetic fields

$$\begin{bmatrix} V_k(L, s) \\ I_k(L, s) \end{bmatrix} = e^{\mathbf{Q}_k(s)L} \left\{ \begin{bmatrix} V_k(0, s) \\ I_k(0, s) \end{bmatrix} - \begin{bmatrix} V_{zk}^{ex}(0, s) \\ 0 \end{bmatrix} \right\} + \mathbf{J}_k^{sca}(s) + \begin{bmatrix} V_{zk}^{ex}(L, s) \\ 0 \end{bmatrix} \quad (3)$$

Equivalent circuit of the multiconductor transmission lines sag is composed of every segment in series, as Fig. 2.

Where:

$$\mathbf{J}_k^{sca}(s) = \begin{bmatrix} V_k^f(s) \\ I_k^f(s) \end{bmatrix} = \int_0^L e^{\mathbf{Q}_k(s)(L-x)} \tilde{\mathbf{F}}_k^{sca}(x, s) dx \quad (4)$$

The Eq. (3) and Eq. (4) can be combined and sorted as:

$$\begin{bmatrix} V_k(L, s) \\ I_k(L, s) \end{bmatrix} = e^{\mathbf{Q}_k(s)L} \left\{ \begin{bmatrix} V_k(0, s) \\ I_k(0, s) \end{bmatrix} - \begin{bmatrix} V_{zk}^{ex}(0, s) \\ 0 \end{bmatrix} \right\} + \begin{bmatrix} V_k^{fex}(s) \\ I_k^f(s) \end{bmatrix} \quad (5)$$

Where:

$$V_k^{fex}(s) = V_k^f(s) + V_{zk}^{ex}(L, s) \quad (6)$$

For the conductor i , the time domain $V_{ik}^{fex}(t)$ can be obtained by the vector fitting method [27, 28], and be realized using voltage controlled voltage source also [29, 30, 31]. The neighboring $V_k^{fex}(t)$ and $V_{z(k+1)}^{ex}(0, t)$ can be combined as $V_k^{fex0}(t)$, $1 \leq k < N$.

So the simplified Equivalent circuit as Fig. 3. In the following, we will verify the equivalent circuit.

4. Application of equivalent circuit

We choose two lines sag located at a height $h = 3.3$ m above lossy ground as the example. The length is 40 m, and the radius is 2.5 mm. The parameters of the ground are $\sigma_g = 0.016$ S/m, $\epsilon_r = 10$ and $\mu_r = 1$, $\sigma_0 = 20$ Mpa $g = 0.057$ N/m·mm², $Z_{11} = Z_{12} = Z_{21} = Z_{22} = 50 \Omega$. The waveform of the electric field is described by a double exponential $E_0(t) = 1000(e^{-10000t} - e^{-400000t})$ V/m with $\alpha = 0$, $\phi = 0$ and $\psi = \pi/6$. Fig. 4 depicts transient responses of voltage induced at the line end by the proposed method and the FDTD method.

From Fig. 4, we can see that the transient responses obtained by the proposed method and the FDTD method are excellent agreement. Contrast with the results without sag, as Fig. 5.

From Fig. 5 we can see that the amplitude of the response with sag is less than the without sag.

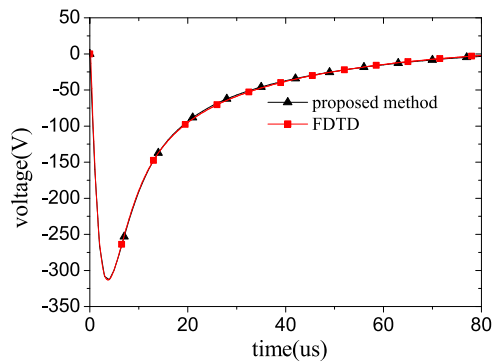


Fig. 4. Transient response of voltage induced at the line end.

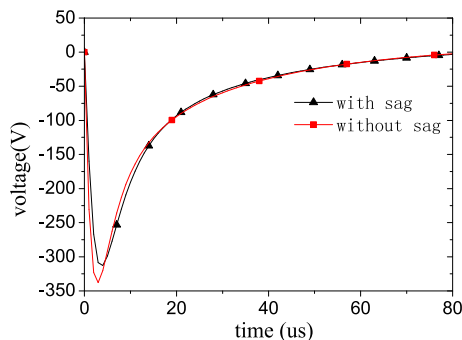


Fig. 5. Transient responses for the lines with and without sag.

5. Conclusion

For the transmission lines sag above lossy ground, the equivalent circuit model is built in this paper. The multiconductor transmission lines sag above lossy ground are divided into a sequence of smaller segments, each of which is approximate equal height. Equivalent circuit of the transmission lines sag is composed of each segment in series. And the simplified equivalent circuit of transmission lines sag above the lossy ground excited by the external electromagnetic fields is built by combined the neighboring elements. The proposed method is compared with the result obtained by the FDTD method, a good agreement is observed. And the amplitude of the response is less than the results when the transmission lines are considered as the straight lines.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (NO. 50977091).

References

- [1] N. M. Nakhla and R. Achar: "Simplified delay extraction-based passive transmission line macromodeling algorithm," *IEEE Trans. Adv. Packag.* **33** (2010) 498 (DOI: [10.1109/TADVP.2009.2032157](https://doi.org/10.1109/TADVP.2009.2032157)).
- [2] A. Dounavis, *et al.*: "A general class of passive macromodels for lossy multiconductor transmission lines," *IEEE Trans. Microw. Theory Techn.* **49** (2001) 1686 (DOI: [10.1109/22.954772](https://doi.org/10.1109/22.954772)).
- [3] C. R. Paul: *Introduction to Electromagnetic Compatibility*

- (Hoboken, A John Wiley & Sons, 2006) 310.
- [4] C. R. Paul: *Analysis of Multiconductor Transmission Lines* (New York, Wiley Inter Science, 2007) 230.
- [5] Y. Qingxi, *et al.*: "Verification and analysis of lumped-circuit approximate model of multiconductor transmission line excited by fast rising pulse," *High Voltage Engineering* **41** (2015) 327.
- [6] R. Achar and M. S. Nakhla: "Simulation of high-speed interconnects," *Proc. IEEE* **89** (2001) 693 (DOI: [10.1109/5.929650](https://doi.org/10.1109/5.929650)).
- [7] H. Xie, *et al.*: "A hybrid FDTD-SPICE method for transmission lines excited by a nonuniform incident wave," *IEEE Trans. Electromagn. Compat.* **51** (2009) 811 (DOI: [10.1109/TEMC.2009.2020913](https://doi.org/10.1109/TEMC.2009.2020913)).
- [8] Q. Yang, *et al.*: "Equivalent circuits of multiconductor transmission lines above lossy ground excited by external electromagnetic fields," *IEICE Electron. Express* **15** (2018) 20171261 (DOI: [10.1587/elex.15.20171261](https://doi.org/10.1587/elex.15.20171261)).
- [9] Q. Yang, *et al.*: "Equivalent circuit of external electromagnetic fields coupling to a transmission line above a lossy ground," *IEICE Electron. Express* **12** (2015) 20150474 (DOI: [10.1587/elex.12.20150474](https://doi.org/10.1587/elex.12.20150474)).
- [10] H. K. Hoidalén: "Analytical formulation of lightning-induced voltages on multiconductor overhead lines above lossy ground," *IEEE Trans. Electromagn. Compat.* **45** (2003) 92 (DOI: [10.1109/TEMC.2002.804772](https://doi.org/10.1109/TEMC.2002.804772)).
- [11] P. Dan-Klang and E. Leelarasamee: "Transient simulation of voltage and current distributions within transmission lines," *IEICE Trans. Electron.* **E92.C** (2009) 522 (DOI: [10.1587/transele.E92.C.522](https://doi.org/10.1587/transele.E92.C.522)).
- [12] F. M. Tesche: "On the analysis of a transmission line with nonlinear terminations using the time-dependent BLT equation," *IEEE Trans. Electromagn. Compat.* **49** (2007) 427 (DOI: [10.1109/TEMC.2007.897141](https://doi.org/10.1109/TEMC.2007.897141)).
- [13] S. Ogawa, *et al.*: "Millimeter-wave transmission line with through-silicon via for RF-MEMS devices," *IEICE Electron. Express* **10** (2013) 20130565 (DOI: [10.1587/elex.10.20130565](https://doi.org/10.1587/elex.10.20130565)).
- [14] T. Suzuki and K. Shiraishi: "Examination of short calibration problem of transmission line pulse," *IEICE Electron. Express* **10** (2013) 20130029 (DOI: [10.1587/elex.10.20130029](https://doi.org/10.1587/elex.10.20130029)).
- [15] V. A. Rakov and F. Rachidi: "Overview of recent progress in lightning research and lightning protection," *IEEE Trans. Electromagn. Compat.* **51** (2009) 428 (DOI: [10.1109/TEMC.2009.2019267](https://doi.org/10.1109/TEMC.2009.2019267)).
- [16] Y. Watanabe and H. Igarashi: "Accelerated FDTD analysis of antennas loaded by electric circuits," *IEEE Trans. Antennas Propag.* **60** (2012) 958 (DOI: [10.1109/TAP.2011.2173148](https://doi.org/10.1109/TAP.2011.2173148)).
- [17] S. Alush, *et al.*: "Transmission line sag influence on lightning stroke probability," *IET Gener. Transm. Distrib.* **6** (2012) 1046 (DOI: [10.1049/iet-gtd.2011.0501](https://doi.org/10.1049/iet-gtd.2011.0501)).
- [18] A. H. Khawaja, *et al.*: "Estimation of current and sag in overhead power transmission lines with optimized magnetic field sensor array placement," *IEEE Trans. Magn.* **53** (2017) 6100210 (DOI: [10.1109/TMAG.2017.2657490](https://doi.org/10.1109/TMAG.2017.2657490)).
- [19] S. Arias-Guzman, *et al.*: "Analysis of voltage sag severity case study in an industrial circuit," *IEEE Trans. Ind. Appl.* **53** (2017) 15 (DOI: [10.1109/TIA.2016.2603470](https://doi.org/10.1109/TIA.2016.2603470)).
- [20] A. H. Khawaja and Q. Huang: "Estimating sag and wind-induced motion of overhead power lines with current and magnetic-flux density measurements," *IEEE Trans. Instrum. Meas.* **66** (2017) 897 (DOI: [10.1109/TIM.2017.2676140](https://doi.org/10.1109/TIM.2017.2676140)).
- [21] T. Kang, *et al.*: "Series voltage regulator for a distribution transformer to compensate voltage sag/swell," *IEEE Trans. Ind. Electron.* **64** (2017) 4501 (DOI: [10.1109/TIE.2017.2668982](https://doi.org/10.1109/TIE.2017.2668982)).
- [22] I. A. Pires, *et al.*: "On the application of single-phase voltage sag compensators in three-phase systems," *IEEE Trans. Ind. Appl.* **53** (2017) 630 (DOI: [10.1109/TIA.2016.2603466](https://doi.org/10.1109/TIA.2016.2603466)).
- [23] L. Li and W. L. Ji: "Transient analysis of unequal length multiconductor transmission lines loaded by nonlinear devices," *Chinese Journal of Radio, Science* **24** (2009) 529.
- [24] K. Afrooz and A. Abdipour: "Efficient method for time-domain analysis of lossy nonuniform multiconductor transmission line driven by a modulated signal using FDTD technique," *IEEE Trans. Electromagn. Compat.* **54** (2012) 482 (DOI: [10.1109/TEMC.2011](https://doi.org/10.1109/TEMC.2011)).

- 2161765).
- [25] M. Tang and J. Mao: “A precise time-step integration method for transient analysis of lossy nonuniform transmission lines,” *IEEE Trans. Electromagn. Compat.* **50** (2008) 166 (DOI: [10.1109/TEM.2007.913222](https://doi.org/10.1109/TEM.2007.913222)).
 - [26] L. Dou and J. Dou: “Sensitivity analysis of lossy nonuniform multiconductor transmission lines with nonlinear terminations,” *IEEE Trans. Adv. Packag.* **33** (2010) 492 (DOI: [10.1109/TADVP.2009.2035439](https://doi.org/10.1109/TADVP.2009.2035439)).
 - [27] B. Gustavsen and A. Semlyen: “Rational approximation of frequency domain responses by vector fitting,” *IEEE Trans. Power Deliv.* **14** (1999) 1052 (DOI: [10.1109/61.772353](https://doi.org/10.1109/61.772353)).
 - [28] B. Gustavsen: “Improving the pole relocating properties of vector fitting,” *IEEE Trans. Power Deliv.* **21** (2006) 1587 (DOI: [10.1109/TPWRD.2005.860281](https://doi.org/10.1109/TPWRD.2005.860281)).
 - [29] Q. Yang, *et al.*: “Fast transient analysis method for lossy nonuniform transmission line with nonlinear terminations,” *IEICE Electron. Express* **12** (2015) 20150362 (DOI: [10.1587/ele.12.20150362](https://doi.org/10.1587/ele.12.20150362)).
 - [30] G. S. Shinh, *et al.*: “Simplified Macromodel of MTLs with incident fields (SiMMIF),” *IEEE Trans. Electromagn. Compat.* **50** (2008) 375 (DOI: [10.1109/TEM.2008.922788](https://doi.org/10.1109/TEM.2008.922788)).
 - [31] N. M. Nakhla, *et al.*: “DEPACT: Delay extraction-based passive compact transmission-line macromodeling algorithm,” *IEEE Trans. Adv. Packag.* **28** (2005) 13 (DOI: [10.1109/TADVP.2004.841677](https://doi.org/10.1109/TADVP.2004.841677)).