ELectronics EXpress

IEICE Electronics Express, Vol.2, No.6, 176–181

Performance evaluation of tap selection based MMSE equalization for UWB systems

Lin Zhiwei,[a)] **A. B. Premkumar,**[b)] **and A. S. Madhukumar**[c)]
School of Computer Engineering, Nanyang Technological University
Block N4, Nanyang Avenue, Singapore 639798
a) *zhiweilin@pmail.ntu.edu.sg*
b) *asannamalai@ntu.edu.sg*
c) *asmadhukumar@ntu.edu.sg*

Abstract: In this work, the performance of a tap selection based Minimum Mean Square Error (MMSE) equalization technique for high data rate Ultra Wideband (UWB) systems is evaluated for the first time using greedy method. This technique is shown to significantly outperform the conventional uniformly spaced equalizer with the same number of taps. In addition, the performance of strongest paths based tap selection method is compared with the greedy method. Larger performance gap is observed in the presence of Multiple Access Interference (MAI) and with increased SNR.

Keywords: MMSE, UWB, greedy method
Classification: Microwave and millimeter wave devices, circuits, and systems

References

[1] R. Scholtz, "Multiple Access with Time-Hopping Impulse Modulation," *IEEE Milit. Commun. Conf.*, vol. 2, pp. 447–450, 1993.
[2] Frederick K. H. Lee and Peter J. McLane, "Design of Nonuniformly Spaced Tapped-Delay-Line Equalizers for Sparse Multipath Channels," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 530–535, April 2004.
[3] Sirikiat Ariyavisitakul, Nelson R. Sollenberger, and Larry J. Greenstein, "Tap-Selectable Decision-Feedback Equalization," *IEEE Trans. Commun.*, vol. 45, no. 12, pp. 1497–1500, Dec. 1997.
[4] Lin Zhiwei, Benjamin Premkumar, and A. S. Madhukumar, "A Combined MMSE Receiver for High Data Rate UWB Systems," *Proc. 9th IEEE Int. Conf. on Communication Systems*, pp. 1–5, 2004.
[5] N. Al-Dhahir and J. M. Cioffi, "Fast Computation of Channel-Estimate Based Equalizers in Packet Data Transmission," *IEEE Trans. Signal Processing*, vol. 43, no. 11, pp. 2462–2473, Nov. 1995.
[6] Richard Johnsonbaugh and Marcus Schaefer, *Algorithms*, Pearson Prentice Hall, 2003.
[7] S. Verdu, *Multiuser Detection*, Cambridge, U.K.: Cambridge University Press, 1998.

DOI: 10.1587/elex.2.176
Received February 02, 2005
Accepted February 28, 2005
Published March 25, 2005

ELectronics EXpress
IEICE Electronics Express, Vol.2, No.6, 176–181

[8] IEEE P802.15-02/490r1-SG3a Channel-Modeling-Subcommittee-Report-Final.doc, Feb. 2003. http://grouper.ieee.org/groups/802/15/pub/

1 Introduction

For high data rate Impulse Radio (IR) [1] based multiple access UWB systems, channel response may span over multiple symbol frames. Conventional RAKE receiver suffers from performance degradation due to severe Intersymbol Interference (ISI) and MAI. An MMSE equalization based receiver with large number of taps is superior for ISI and MAI mitigation. But this involves large computational complexity and requires large amount of training symbols due to the large delay spread in UWB channel.

Non-uniformly spaced equalizer has been discussed for sparse multipath channels [2, 3] with reduced complexity. In these works, the performance of the equalization is discussed using simulations and is based on various intuitive tap selection techniques. But how good a near optimal performance may be achieved by tap selection based equalization has not been discussed. In addition, the UWB indoor channel is not as sparse as those channels discussed in the above works. Therefore, it is important to have instructive performance evaluation to find out the effectiveness of using tap selection based equalization for UWB channels.

2 System model and BER performance of MMSE detection

Let the transmitted UWB signal with bipolar modulation be $s(t) = \sum_{n=-\infty}^{\infty} x(n) w(t - nT_f)$, where $x(n) \in \{\pm 1\}$ is the data bit stream, $w(t)$ is the pulse waveform and T_f is the symbol duration. Applying a tapped-delay-line channel model $c(t) = \sum_{p=1}^{n_L} \alpha_p \, \delta(t - (p-1)\Delta\tau - \tau)$, where $\Delta\tau$ is the sampling duration, τ is the channel delay and n_L is the channel length in samples. For simplicity, assume that the sampling rate is an integer multiple of the symbol repetition rate, that is, let $n_\tau = T_f/\Delta\tau$ be an interger. Consider an observation window of n_{sym} frames and let the number of symbols affected by ISI be $n_{ISI} = \lceil n_L/n_\tau \rceil$. A discrete-time UWB system model can then be expressed as [4],

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{m} \tag{1}$$

where $\mathbf{y} = [\mathbf{y}^T(n), \cdots, \mathbf{y}^T(n+n_{sym}-1)]^T$, $\mathbf{y}(n) = [y^{(1)}(n), \cdots, y^{(n_\tau)}(n)]^T$ denotes the over sampled received signal vector and $\mathbf{m}$ is the corresponding AWGN signal vector with covariance $\mathbf{R}_m = \sigma_m^2 \mathbf{I}$. The transmitted signal is $\mathbf{x} = [x(n-n_{ISI}+1), \cdots, x(n), \cdots, x(n+n_{sym}-1)]^T$ and the channel transmission matrix in block Toeplitz form is represented as,

$$\mathbf{H} = \begin{pmatrix} \mathbf{h}(n_{ISI}-1) & \cdots & \mathbf{h}(0) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \mathbf{h}(n_{ISI}-1) & \cdots & \mathbf{h}(0) \end{pmatrix}_{N_R \times N_T} \tag{2}$$

DOI: 10.1587/elex.2.176
Received February 02, 2005
Accepted February 28, 2005
Published March 25, 2005

IEICE Electronics Express, Vol.2, No.6, 176–181

where $N_R = n_\tau n_{sym}$, $N_T = n_{ISI} + n_{sym} - 1$. In addition, $\mathbf{h}(k) = [h^{(kn_\tau+1)}, \cdots, h^{(kn_\tau+n_\tau)}]^T_{1\times n_\tau}$, where $h^{(p)}$ denotes the p-*th* path of the sampled generalized Channel Impulse Response (CIR) $h(t) = c(t) * w(t)$.

In the presence of MAI, system model is given as follows, which is in the same form as Eq. (1),

$$\mathbf{y} = \mathbf{H}_{(1)}\mathbf{x}_{(1)} + \mathbf{H}_{(mai)}\mathbf{x}_{(mai)} + \mathbf{m} = \mathbf{H}\mathbf{x} + \mathbf{m} \tag{3}$$

where $\mathbf{H} = [\mathbf{H}_{(1)}\,\mathbf{H}_{(mai)}]$ and $\mathbf{x} = [\mathbf{x}^T_{(1)}\,\mathbf{x}^T_{(mai)}]^T$, subscript (1) is for the desired user and (mai) denotes the terms corresponding to the MAI users. For simplicity, the self spreading multipath channels of UWB are utilized for multiple access without adding extra spreading sequence. In addition, strict channel synchronization is not necessary since the tap selection technique will automatically choose the right paths for the desired symbol equalization.

Without loss of generality, let the desired symbol $x(n)$ be the first element in $\mathbf{x}$. This can be done by simply reordering the transmission symbols in vector $\mathbf{x}$.

Using MMSE detection criterion, the Signal-to-Interference-plus-Noise Ratio (SINR) is given by [5],

$$SINR^{(1)} = \frac{1}{\sigma_m^2\left\{(\mathbf{H}^T\mathbf{H} + \sigma_m^2\mathbf{I}_{N_T})^{-1}\right\}_{1,1}} - 1 \tag{4}$$

where the subscript (i,j) in $\{\mathbf{A}\}_{i,j}$ denotes the i-*th* row and j-*th* column of the matrix $\mathbf{A}$. Bit Error Rate (BER) is then obtained by,

$$P_b^{(1)} = Q(\sqrt{SINR^{(1)}}) \tag{5}$$

3 Greedy tap selection method

For optimal selection of n_S non-uniformly spaced taps out of total n_L taps, an exhaustive search is needed and this requires the MMSE detection performance to be evaluated for $\binom{n_L}{n_S}$ possible combination of the tap subsets. This is not feasible for the large n_L. In fact, this is an NP-hard problem similar to the travelling salesman problem. Greedy algorithm [6] is a practical method for finding an optimal solution by starting from an optimal solution to some component or small part of the data structure and extending it, by considering additional components of the data structure one by one, although in many cases there is no guarantee that making locally optimal improvements in a locally optimal solution yields the optimal global solution.

For performance evaluation purpose, a greedy algorithm based method for near optimal tap selection is described as follows.

Tap selection can be considered as forming a transmission matrix $\mathbf{H}_S$ by choosing a subset of rows from $\mathbf{H}$ to maximize the $SINR^{(1)}_{\mathbf{H}_S}$. From Eq. (4), this is equivalent to minimizing $\{\mathbf{A}_S\}_{1,1} = \left\{(\mathbf{H}_S{}^T\mathbf{H}_S + \sigma_m^2\mathbf{I}_{N_T})^{-1}\right\}_{1,1}$.

Suppose that at the $(n-1)$-*th* step, a selected transmission matrix with $(n-1)$ rows is denoted as $\mathbf{H}_{n-1}$. Define,

$$\mathbf{A}_{n-1} = (\mathbf{H}^T_{n-1}\mathbf{H}_{n-1} + \sigma_m^2\mathbf{I}_{N_T})^{-1} \tag{6}$$

DOI: 10.1587/elex.2.176
Received February 02, 2005
Accepted February 28, 2005
Published March 25, 2005

IEICE Electronics Express, Vol.2, No.6, 176–181

To obtain $\mathbf{H}_n$ at the n-th step, one more row denoted as $\mathbf{h}_k$ will be selected and appended to $\mathbf{H}_{n-1}$. Also notice that the rows in $\mathbf{H}_S$ may be re-ordered. Thus we have,

$$\mathbf{A}_n = (\mathbf{H}_n^T\mathbf{H}_n + \sigma_m^2\mathbf{I}_{N_T})^{-1} = (\mathbf{A}_{n-1}^{-1} + \mathbf{h}_k^T\mathbf{h}_k)^{-1} \tag{7}$$

where $\mathbf{H}_n^T\mathbf{H}_n = \mathbf{H}_{n-1}^T\mathbf{H}_{n-1} + \mathbf{h}_k^T\mathbf{h}_k$. Applying matrix inversion lemma, an iterative formula is obtained with reduced computation cost for matrix inversion as,

$$\mathbf{A}_n = \mathbf{A}_{n-1} - \mathbf{A}_{n-1}\mathbf{h}_k^T(1 + \mathbf{h}_k\mathbf{A}_{n-1}\mathbf{h}_k^T)^{-1}\mathbf{h}_k\mathbf{A}_{n-1} \tag{8}$$

Thus, an incremental iterative algorithm is developed for tap selection as follows.

1. Start from $n = 0$, set $\mathbf{A}_0 = \frac{1}{\sigma_m^2}\mathbf{I}_{N_T}$.
2. Set $n = n + 1$, using Eq. (8), one more row $\mathbf{h}_s$ is added by selecting $s = \arg\min_k \left\{\{\mathbf{A}_n\}_{1,1}\right\}$,
   where k denotes the unselected row index for $\mathbf{H}$.
3. Continue iteration to select the taps until $n = n_S$.

The numerical result in following section shows that the equalization performance of tap selection by this greedy algorithm (local optimal solution) is close to the full rank (512 taps) equalization performance, with only one eighth of the CIR samples (64 taps).

4 Numerical Analysis

For simplicity, following performance evaluation is based on the channel model CM2 from IEEE UWB channel model recommendation [8], where the sampling duration is set as $\Delta\tau = 0.167\,ns$ and total number of samples for the channel is $n_L = 512$.

Fig. 1 illustrates the BER performance of tap selection based MMSE equalization. The symbol rate for UWB transmission is set at $374.25\,MHz$, in the presence of severe ISI ($n_{ISI} = 32$). The observation window size is set as ($n_{sym} = 32$) symbol frames over totally 512 samples. It is observed that the greedy tap selection method with only ($n_S = 64$) taps is able to achieve near optimal equalization performance as compared to the optimal performance when applying all 512 taps.

On the other hand, the tap selection based MMSE equalization significantly outperforms the conventional uniformly spaced MMSE equalization with either 64 equally spaced taps over the entire channel length or 64 earliest taps of the channel, as shown in Fig. 1.

Moreover, the simplest way to make the tap selection is to choose n_S strongest paths from CIR. It is observed in Fig. 1 that the performance gap between strongest paths based simple tap selection and greedy method based near optimal tap selection is quite limited.

DOI: 10.1587/elex.2.176
Received February 02, 2005
Accepted February 28, 2005
Published March 25, 2005

IEICE Electronics Express, Vol.2, No.6, 176–181

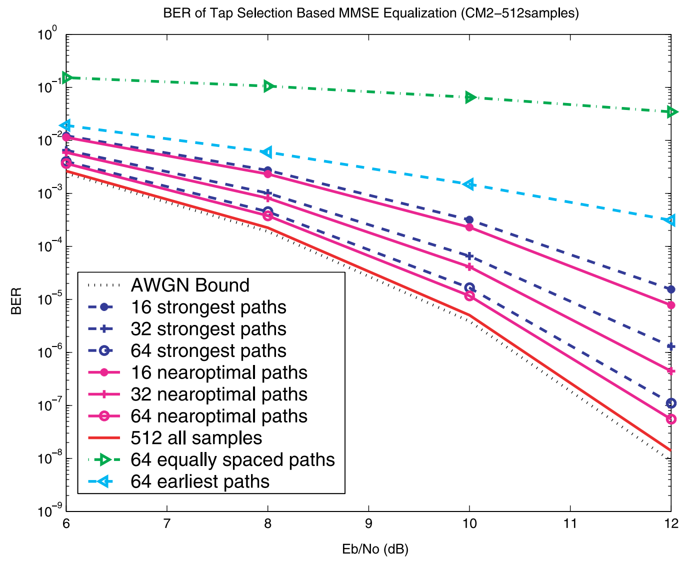

Fig. 1. Tap selection based MMSE detection performance evaluation (BER vs. SNR)

In the presence of MAI transmitters, the BER performance of tap selection based MMSE equalization is illustrated in Fig. 2, where the symbol rate for UWB transmission is set at 93.56 MHz, in the presence of ISI ($n_{ISI} = 8$) and MAI ($n_{mai} = 0 - 3$). It is observed that the performance gap between strongest paths based simple tap selection and greedy method based near optimal tap selection becomes larger with increased number of MAI transmitters, as well as with increased SNR level. This can be explained as follows.

From system model in Eq. (3), the SINR for tap selection based MMSE

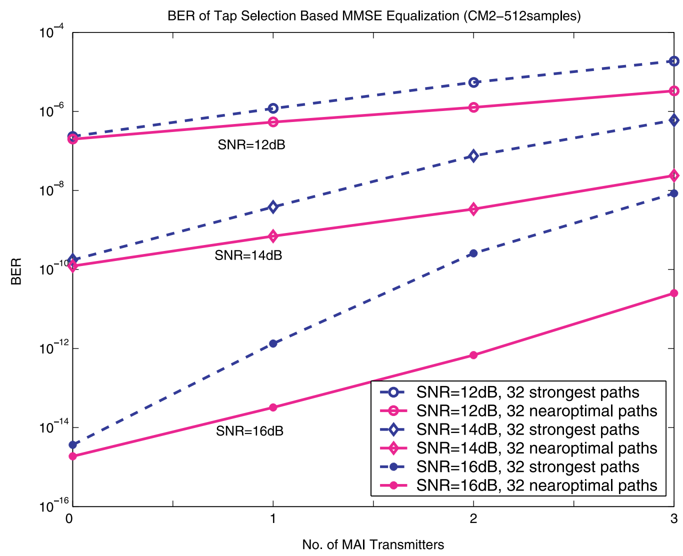

Fig. 2. Tap selection based MMSE detection performance evaluation (BER vs. No. of MAI transmitters)

DOI: 10.1587/elex.2.176
Received February 02, 2005
Accepted February 28, 2005
Published March 25, 2005

IEICE Electronics Express, Vol.2, No.6, 176–181

detection can be written in another form as [7],

$$SINR^{(1)}_{\mathbf{H}_S} = \frac{B_1^2}{\sigma^2_{I^{(isi)}_{residual}} + \sigma^2_{I^{(mai)}_{residual}} + \|\mathbf{f}_{mmse}\|^2\sigma^2_m} \tag{9}$$

where $\mathbf{f}_{mmse}$ is the equalization filter, $B_1 = (\mathbf{f}^T_{mmse}\mathbf{H}_S)_1$ and B_1^2 represents the multipath energy capture for the desired symbol. Gaussian approximation is applied to the terms $I^{(isi)}_{residual}$ and $I^{(mai)}_{residual}$ which represent the residual ISI and MAI respectively. The strongest paths based tap selection is optimal only in the absence of ISI and MAI, in view of Maximum Ratio Combining (MRC) theory. In the presence of ISI and MAI, the greedy algorithm based method carefully selects the taps one by one by trying to minimize the term $\sigma^2_{I^{(isi)}_{residual}} + \sigma^2_{I^{(mai)}_{residual}}$ and maximize the term B_1^2 at the same time. In contrast, the strongest paths based tap selection is less capable of ISI and MAI mitigation compared with greedy algorithm based method. In addition, the effect of ISI resulted by the tail of CIR is less severe than MAI due to the exponentially decaying power law for UWB channel, assuming equal transmission power for all the transmitters. So the performance gap shown in Fig. 2 becomes larger with increased number of MAI transmitters. On the other hand, in the case of higher SNR (lower σ^2_m), the term $\sigma^2_{I^{(isi)}_{residual}} + \sigma^2_{I^{(mai)}_{residual}}$ becomes dominating the interference power in the denominator in Eq. (9). So, increased SNR also results in larger performance gap in Fig. 2.

5 Discussions

The performance evaluation and analysis show that using tap selection of a small portion of the total CIR samples, the complexity of the equalizer is greatly reduced with limited performance degradation when compared to that obtained by equalization using all taps. This indicates that the tap selection based equalization technique is a promising method for high performance UWB receiver design with reduced complexity. In addition, under single user transmission, strongest paths based tap selection method is shown to work well for high data rate UWB channel equalization with limited performance degradation compared to the near optimal tap selection by greedy method. However, in the presence of MAI transmitters, larger performance degradation is observed when using this simple strongest paths based method. This degradation also accelerates with increased SNR. Therefore, it is necessary to develop a fast tap selection algorithm which is suitable for realtime implementation and achieves comparable performance as by greedy method.

DOI: 10.1587/elex.2.176
Received February 02, 2005
Accepted February 28, 2005
Published March 25, 2005