

Analysis of signal detection rates in Optical CDMA

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Abstract: The estimation of OCDMA system performance under the condition of code phase alignment between received and locally generated code signals is demonstrated in dual homodyne correlation receiver using balanced detector. Based on the analysis of auto correlation property for prime codes, distinctive compared to the conventional CDMA codes, the developed model provides a method for interpreting the effect of code synchronization state on the false alarm probability and the corresponding bit error probability. The performance influenced by the chip pulse power and noises is also examined with this model for the cases of single and multiple users.

Keywords: Code division multiaccess, synchronization, correlation, homodyne detection, communication system performance

Classification: Photonics devices, circuits, and systems

References

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1 Introduction

In CDMA systems, code synchronization plays a fundamental role in the system performance. This synchronization process is composed of acquisi-

tion and tracking. Code acquisition corresponds to achieving a coarse alignment between the received and the locally generated sequences, while fine alignment is demanded for the code tracking. When communications are bursty, or intermittent, and characterized by frequent periods of silence, a transmitter emits its CDMA signal that may or may not be acquired at a receiver. In many of these cases, the detection probability and the false alarm probability are more appropriate criteria to be considered in the particular environment [1].

So far, the system performance, especially the probability of bit error was extensively studied in the field of Optical CDMA (OCDMA) [2], [3]. Code synchronization was assumed to be perfect for most recently reported OCDMA works before the receiver begins to recover the transmitted data bits. The cross correlation property was just considered for the multi-user (MUI) analysis whereas the auto correlation property for prime codes determining the performance of code phase synchronization and ultimately determines that of the system was not emphasized. However, because of the optical signal processing based on fiber delay lines, the auto correlation property for prime codes, and the very short chip pulse duration, precise code synchronization is so difficult that it could be achieved under the non-aligned condition.

In this work, the code correlation properties are observed for the chosen prime code OCDMA systems. The influence of the code phase alignment condition in its synchronizing process on system performance is analyzed for intensity modulated OCDMA system with dual homodyne correlation receiver using balanced detector. The receiver noises are included to obtain the generality for the receiver model as well as MUI. Rather than concentrating the study on the strategy for the code synchronization procedure followed by the computation of the average acquisition time as the performance of interest [4], the probability of false alarm (P_{fa}) and probability of bit error (P_{be}) depending on the state of code phase alignment are on focus.

2 Periodic correlation properties for prime code sequences

Code sequences in CDMA systems require the following two properties: 1) each code in the set is easy to distinguish from a time-shifted version of itself and 2) each code in the set is easy to distinguish from every other code in the set [5]. The first property is important for the code auto correlation function determining the performance of code synchronization process, while the second for the cross correlation characteristics suggesting the influence of multiuser interference.

The auto correlation function for code C_x is

$$R_{xx}(\tau) = \sum_{j=0}^{N-1} x_j \cdot x_{j \oplus \tau}, 0 \leq \tau \leq N - 1 \quad (1)$$

where \oplus is the modulo N addition (N : code length), and the cross correlation

function for codes C_x and C_y is

$$R_{xy}(\tau) = \sum_{j=0}^{N-1} x_j \cdot y_{j \oplus \tau}, 0 \leq \tau \leq N - 1 \quad (2)$$

The prime code shows the difference in the property of code correlation functions compared to those of the conventional orthogonal one, e.g., maximal length sequence (m-sequence). Unlike the code properties of m-sequence, which has a distinctive auto correlation trait, ‘thumb tack’, showing only the peak value when the two signals are correctly aligned and the minimum level when the both signals’ mismatches (Δ) are larger than T_{chip} (unit chip pulse time width), the prime code has a large value of sidelobes even when $|\Delta| > T_{\text{chip}}$ as shown in Fig. 1. The used code sequences are based on the prime number $p = 17$. When the code phases are perfectly aligned, the auto correlation shows the peak S_0 , while the sidelobes are represented by S_1 for the largest value and so forth. Additionally, the cross correlation has only 3 levels of values. This determines the level of MUI and the discriminating ability of intended code from other different codes.

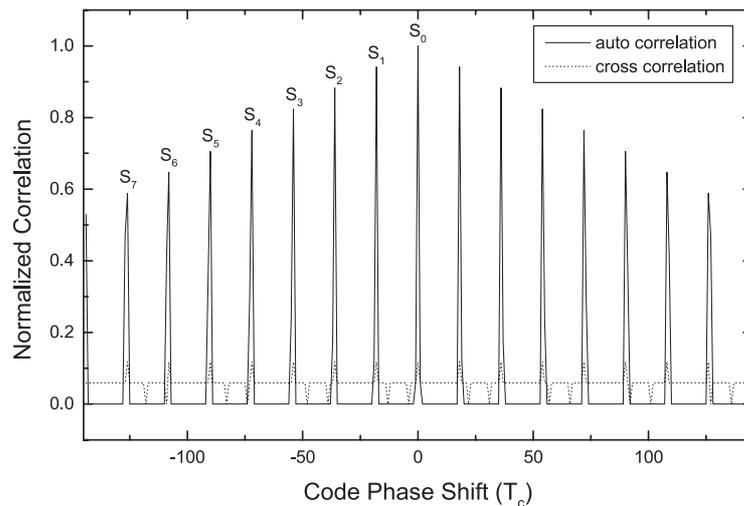


Fig. 1. Correlation properties for prime codes.

3 Estimation of false alarm and bit error probabilities

When the noises are included in the receiver model, the probability density function (PDF) of the signal output in dual homodyne correlation receiver is [3]

$$f_x(u) = f(u/S_x) = \frac{1}{2\sigma^2} e^{-\frac{(u+K_x)}{2\sigma^2}} I_0 \left(\frac{(uK_x)^{1/2}}{\sigma^2} \right) \quad (3)$$

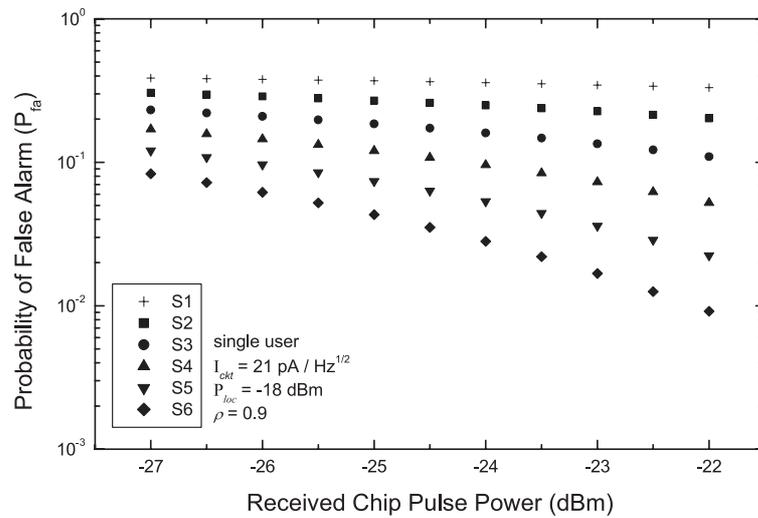
where σ^2 stands for the total noise including shot noise, receiver circuit noise, and MUI, modeled into the Gaussian distribution, $K_x = \rho^2 P_1 P_{loc} R_{1,1}(\tau_x)/p^4$, and τ_x the code phase difference between the incident desired signal and the locally generated one. P_1 is the received chip pulse power, P_{loc} the

local chip pulse power, $R_{1,1}$ the auto correlation for the code C_1 , and ρ the photodetector responsivity.

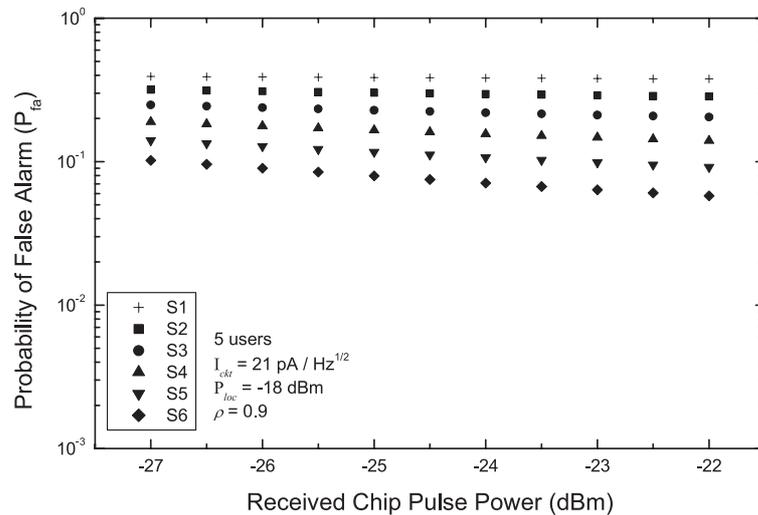
False alarm is the status when the receiver in the code synchronization mode decides the code phase is aligned though it is actually not. MUI and other receiver noises affect this probability as well as the auto correlation sidelobes. From the given receiver model, the false alarm probability, P_{fa} can be calculated as follows

$$P_{fa} = \int_{\xi}^{\infty} f_x(u) du \quad (4)$$

where the optimum threshold ξ is obtained from $f_0(\xi) = f_x(\xi)$.

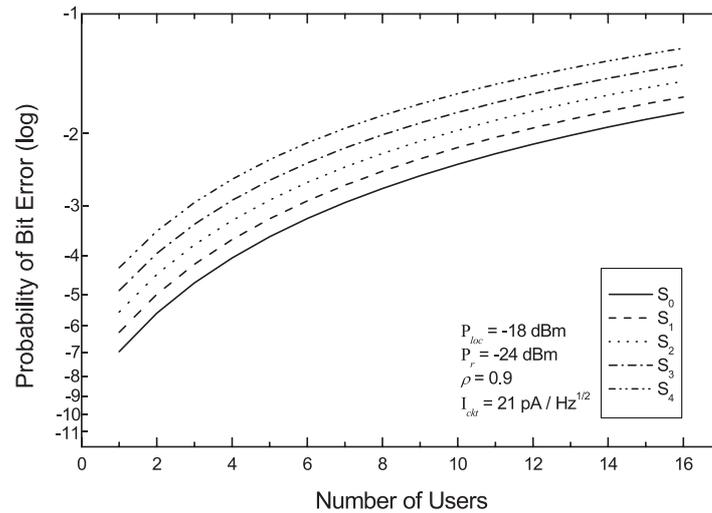


(a)

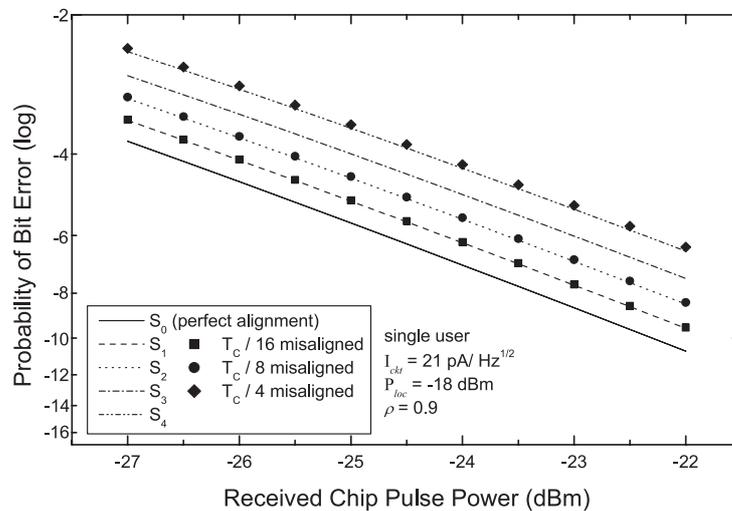


(b)

Fig. 2. Probability of false alarm as a function of received chip pulse power with various code phase alignment states (I_{ckt} represents the receiver circuit noise): (a) single user, and (b) 5 users.



(a)



(b)

Fig. 3. Probability of bit error (I_{ckt} represents the receiver circuit noise): (a) number of simultaneous users with various code phase alignment states, and (b) received chip pulse power with various code phase alignment states.

Fig. 2 shows calculated P_{fa} with various states of misalignment. For the single user case without including MUI, P_{fa} does not much enhance when the sidelobe is large for the increased average chip power, and is getting more insensitive to the average chip power even in smaller sidelobes for the increased MUI due to the increased noise level compared to the receiving signal power. Those large values of P_{fa} mean that the receiver tends to decide that the code phase is aligned and is prone to enter the data bit recovery mode.

The bit error probability P_{be} can be calculated from

$$P_{be} = \frac{1}{2} \left\{ \int_0^n p_1(u/S_x) du + \int_\eta^\infty p_0(u/S_x) du \right\} \quad (5)$$

where $p_1(u/S_x)$ and $p_0(u/S_x)$ are the PDF's of data bit '1' and '0' respectively conditioned on the code phase alignment to state S_x , and η the optimum threshold.

The P_{be} as a function of the number of simultaneous users is shown in Fig. 3 (a). It is more impaired for the decreased level of detected signals when the code is aligned to the smaller sidelobes according to the reduced signal to noise ratio. In Fig. 3 (b), to observe just the effect of misalignment on the performance without MUI, the case of single user is considered. Similar results are also obtained from $|\Delta| < T_{\text{chip}}$ by simulation. These are due to the close detected signal levels between both cases. They correspond to the similar portion of full signal levels for the competent states.

4 Conclusion

Estimation of the probability of false alarm from the analysis of the auto correlation function for prime codes and that of bit error corresponding to the state of alignment between received and locally generated code signals in OCDMA systems is described. With the single user case analysis the pure effect of code phase alignment on system performance is observed and it can be similarly interpreted when the misalignments are both larger or smaller than chip pulse width due to the close detected signal levels at the receiver. Furthermore, even if the probability of false alarm and bit error increase as the number of simultaneous users does, the influence of misalignment decreases as a result of increased MUI and reduces that of received chip pulse power.