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# An efficient convolution interpolation kernel for digital image scaling

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**Abstract:** This paper presents an efficient extended linear convolution interpolation method for efficient scaling. The kernel of the extended linear convolution interpolation is built up of first-order polynomial and approximates the ideal sinc-function in interval $[-2, 2]$. The approach reduces the computational complexity of interpolation and the interpolation quality is compatible to that of bi-cubic convolution interpolations.

**Keywords:** bi-cubic convolution, interpolation, scaling

**Classification:** Science and engineering for electronics

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## 1 Introduction

Digital image scaling has a variety of applications, such as multimedia, medical image processing, military applications and consumer electronics. Numerous digital image scaling techniques have been presented [1, 2, 3, 4, 5, 6, 7, 8]. The simplest approaches are nearest neighbor and bi-linear interpolation [1]; however, they have undesirable blocks effect and blurring effect. A better quality of interpolation is achieved by using higher order models [1, 2]. The typical method of this category is cubic convolution interpolations [3, 4, 5, 6, 7, 8]; nevertheless, it requires heavy computations. Therefore, Nuno-Maganda [7] decomposed the bi-cubic method into two 1-dimension interpolations on a *Xilinx Virtex II Pro FPGA* for real-time processing.

The kernel of cubic convolution interpolations is built up of third-order polynomials [4, 5, 6, 7] and approximates the ideal sinc-function in interval $[-2, 2]$. The kernel is given by

$$k_C(d) = \begin{cases} (c+2)|d|^3 - (c+3)|d|^2 + 1, & 0 \le |d| < 1 \\ c \times |d|^3 - 5c \times |d|^2 + 8c \times |d| - 4c, & 1 \le |d| < 2 \\ 0, & 2 \le |d|. \end{cases} \tag{1}$$

Bi-cubic [4] interpolation applies a widely used constant $c = -1$ as a compromise approximation to the ideal sinc-function. The kernel of bi-cubic interpolation is given by

$$k_{BC}(d) = \begin{cases} 1 - 2\,|d|^2 + |d|^3\,, & 0 \le |d| < 1 \\ 4 - 8\,|d| + 5\,|d|^2 - |d|^3\,, & 1 \le |d| < 2 \\ 0, & 2 \le |d|\,. \end{cases} \tag{2}$$

This paper presents an efficient image scaling method, extended linear interpolation. The scheme has the advantages of low operation complexity with its interpolation quality compatible to that of bi-cubic convolution interpolation.

## 2 The kernel of extended linear convolution interpolation

Extended linear convolution interpolation improves the quality of linear interpolation and moreover, has the advantage of low complexity as linear interpolation. The number of sampling points of third-order polynomial interpolation [2] is applied in extended linear convolution interpolation, which decomposes 2-dimension 4×4 interpolations as two 1-dimensions. In addition, the sampling points of extended linear convolution interpolation uses 16 points as illustrated in Fig. 1(a). The proposed method approximates the


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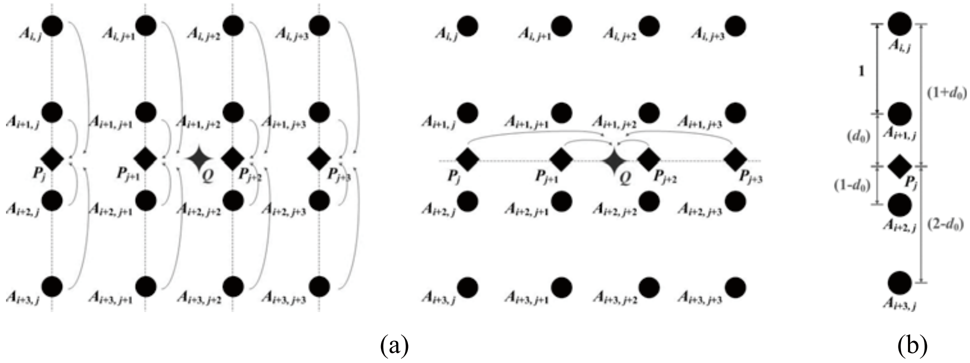

**Fig. 1.** (a) Decomposition of 2-dimension 4×4 interpolations as vertical and horizontal interpolation (b) 1-dimension of third-order polynomial interpolation (vertical interpolation).

ideal sinc-function in the interval $[-2, 2]$. The weighting coefficients are given by

$$k_{EL}(d) = \begin{cases} c_{10}\,|d| + c_{00}, & 0 \leq |d| < 1 \\ c_{11}\,|d| + c_{01}, & 1 \leq |d| < 2 \\ 0, & 2 \leq |d|\,. \end{cases} \tag{3}$$

In eq. (2), bi-cubic interpolation has feature [2] as following to derive the kernel of extended linear interpolation; since $k_{BC}(d)$ is an even function, only values for $d \geq 0$ are discussed.

Separated bi-cubic interpolation is a 1-dimension of third-order polynomial interpolation; the interpolated pixel $(P)$ can be obtained from corresponding source pixels as shown in Fig. 1(b). The feature of weighting coefficients of separated bi-cubic interpolation in both directions can be obtained by the equation:

$$k_{EL}(d_0) + k_{EL}(1 + d_0) = 1 - d_0 \tag{4}$$

1. According to eq. (4), substitute $d = 1$ into eq. (3), then $c_{01} + c_{11} = 0$; assume $c_{11} = \alpha$, then $c_{01} = -\alpha$.

2. According to 1, replace $d = 0$ into eq. (3), then $c_{00} + c_{01} + c_{11} = 1$; that is, $c_{00} = 1$.

3. Based on 1, replace $d = 0.5$ into eq. (3), then $c_{10} = -1 - c_{11}$; that is, $c_{10} = -(\alpha + 1)$.

Thus, eq. (3) can be expressed as

$$k_{EL}(d) = \begin{cases} -(\alpha + 1) \times |d| + 1, & 0 \leq |d| < 1 \\ \alpha \times |d| - \alpha, & 1 \leq |d| < 2 \\ 0, & 2 \leq |d|\,, \end{cases} \tag{5}$$

where $\alpha$ is sharpness factor.


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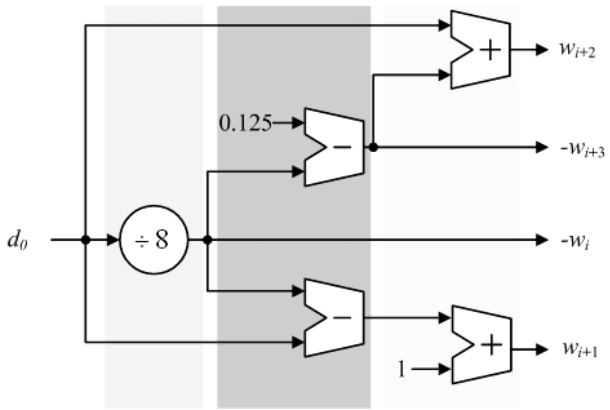


**Fig. 2.** The weighting coefficient generator.

## 3 Optimal sharpness factor

In order to obtain the sharpness factor which has the best interpolation quality, the sum of standard deviation is applied to determine the value of $\alpha$, between $-1$ and 0, whose spectrum mostly approximates the ideal sinc-function. It shows that the minimum standard deviation occurs at $\alpha = -0.1216$. Instead of using $\alpha$ value that has the best quality, for simplicity's sake, we choose $\alpha = -0.125$, which is close to $-0.1216$. The weighting coefficients of extended linear interpolation equation is

$$k_{EL_{-0.125}}(d) = \begin{cases} -0.875 \times |d| + 1, & 0 \leq |d| < 1 \\ -0.125 \times |d| + 0.125, & 1 \leq |d| < 2 \\ 0, & 2 \leq |d|\,. \end{cases} \tag{6}$$

The frequency response on the spectrum for $\alpha = -0.1216$ and $\alpha = -0.125$ are very close to that of bi-cubic interpolation except that the high-frequency response is a bit higher when $\alpha = -0.1216$ and $\alpha = -0.125$. Thus, the interpolation quality for $\alpha = -0.1216$ and $\alpha = -0.125$ is approximately equal to that of bi-cubic interpolation. From the hardware implementation point of view, $\alpha = -0.125$ is an optimal point to obtain good interpolation quality. The advantage of $\alpha = -0.125$ is that in hardware implementation the multiplication with distance $d$ can be accomplished by shifting $d$ to the right for 3 bits, instead of using a multiplier.

In a 1-dimensional interpolation, vertical and horizontal weighting coefficients have to be determined. The method of calculating vertical weighting coefficients is identical to the way of obtaining horizontal weighting coefficients. According to Fig. 1(b), eq. (6) can be modified to find all the vertical weighting coefficients.

$$\begin{aligned} w_i &= -0.125 \times d_0 \\ w_{i+1} &= ((-w_i) - d_0) + 1 \\ w_{i+2} &= d_0 + (-w_{i+3}) \\ w_{i+3} &= -(0.125 - (w_i)) \end{aligned} \tag{7}$$

where $w_i$, $w_{i+1}$, $w_{i+2}$, and $w_{i+3}$ are the vertical weighting coefficients of the corresponding source pixels $A_{i,j}$, $A_{i+1,j}$, $A_{i+2,j}$, and $A_{i+3,j}$; $d_0$ is the


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Table I. PSNRs of various scaling methods.

| Method / Test images | 2/3-> 3/2 (down/up) | | 3/2-> 2/3 (up/down) | |
|---|---|---|---|---|
| | Bi-cubic | Proposed | Bi-cubic | Proposed |
| Tank | 33.06 | 33.49 | 33.49 | 33.49 |
| Sailboat | 30.39 | 30.85 | 30.85 | 30.85 |
| Boat | 30.76 | 31.14 | 31.14 | 31.14 |
| Bridge | 26.91 | 27.30 | 27.30 | 27.30 |
| Goldhill | 31.75 | 32.03 | 32.03 | 32.03 |
| Lena | 33.74 | 34.16 | 34.16 | 34.16 |
| Peppers | 32.89 | 33.36 | 33.36 | 33.36 |
| Airplane | 33.07 | 33.41 | 33.41 | 33.41 |

distance between the source pixel $A_{i+1,j}$ and the virtual interpolated pixel $P_j$. According to eq. (7), the weighting coefficient generator, as depicted in Fig. 2, is designed for producing vertical or horizontal weighting coefficients. The complex operation is simplified in the generator, which includes only two adders and two subtractors.

## 4 Experimental results

The proposed method was coded in C++ and executed on a personal computer. To evaluate the performance of scaling, eight test images (512×512) used in the simulations are Tank, Sailboat, Boat, Bridge, Goldhill, Lena, Peppers, and Airplane. The performance of the proposed method with that of Bi-cubic is compared. The simulation results, shown in Table I, indicate that the proposed method presents a quality that is compatible to that of bi-cubic convolution interpolation. The kernel of the proposed method is built up of first-order polynomial interpolation; it presents a lower computation complexity of interpolation than Nuno-Maganda [7]. Nuno-Maganda [7] uses 12 multipliers and 12 adders to generate weighting coefficients and 20 multipliers and 12 adders for each convolution operation. With compatible image quality, the proposed method applies 4 adders to generate weighting coefficients and 8 multipliers and 6 adders for each convolution operation; our approach reduces about 70% of hardware cost.

## 5 Conclusions

This paper presented an efficient extended linear interpolation for digital image processing. Unlike bi-cubic convolution interpolations, whose weighting coefficients are generated with heavy computation, the kernel of the proposed method is built up of piecewise linear polynomials and approximates the ideal sinc-function in interval $[-2, 2]$. This approach reduces the computation efforts of interpolation. Thus, it can reduce the hardware complexity and is capable of obtaining compatible quality of bi-cubic convolution interpolation.


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