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Novel LMS algorithms based on status categorization

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Abstract: Least Mean Square (LMS) is an effective adaptive filtering algorithm with advantages of robustness and simplicity. In this paper, we propose two new algorithms, Categorized Variable Step Size LMS (CVSSLMS) and Combined CVSSLMS (CCVSSLMS), based on the categorization of filter status. The step sizes of the proposed algorithms are dynamically updated by optimization for each state. Experiment results show that the proposed algorithms outperform conventional LMS algorithms in both simplicity and robustness.

Keywords: LMS, adaptive filter, VSSLMS, MCLMS

Classification: Science and engineering for electronics

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1 Introduction

The Least Mean Square (LMS) [1, 2] is one of the most popular adaptive algorithms due to its simplicity and robustness. While the simplicity of an LMS algorithm is generally judged by the small number of additions and multiplications required, the robustness of an LMS algorithm is judged in two ways – first by how fast it converges and second by how small the mean square error (MSE) becomes.

These two factors determining the robustness of an LMS algorithm, convergence rate and MSE, generally depend on its step size. As widely known, an LMS algorithm with a large step size in general converges fast but the MSE becomes large. On the other hand, an LMS algorithm with a small step size tends to converge slowly but the MSE becomes small. To provide solutions to this tradeoff, there have been many research performed, including the algorithms using Variable Step Size LMS (VSSLMS) [3, 4, 5, 6] and those with Multiple Combined LMS (MCLMS) filters [7, 8]. In this letter we propose a Categorized Variable Step Size Least Mean Square (CVSSLMS) algorithm as a novel VSSLMS algorithm and a Combined CVSSLMS (CCVSSLMS) as an extension by combining with a fixed step size LMS filter.

2 The proposed algorithm

The main concern in designing an VSSLMS filter is how to effectively update the next step size, $\mu(n+1)$, by reflecting present state in order to make the adaptive filter, $\mathbf{w}(n)$, converge to the optimal filter, $\mathbf{w}_{opt}$. For example, Weepeng proposed an updating algorithm with the step size as a function of the gradient of square error and smoothing parameter [3]. In this paper, we categorized the adaptive filter $\mathbf{w}(n)$ into four states, each of which as a function of the change in gradient of present and previous square errors. After the categorization, different updating algorithm of the individual step size is applied in each category in order to reflect the present state of the filter.

2.1 Categorization of the filter coefficient

In the categorization, we considered two factors of the change in gradient, the sign and the amplitude. From the change in amplitude, we can judge whether the adaptive filter is converging or not. On the other hand, we can see from the amplitude change whether or not the adaptive filter has already passed the optimal filter. Since each of the two factors can have two

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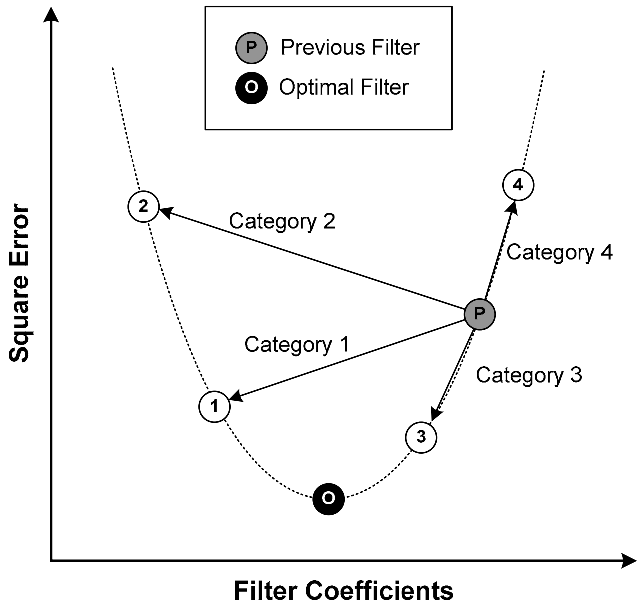

Fig. 1. Four categories in square error as a function of filter coefficients in a LMS Filter

values of changes (same or different for the sign; increase or decrease for the amplitude), there are four possible categories, which are shown in Fig. 1.

(1) Category 1 – a sign change and decreased amplitude
Because the amplitude of the gradient is decreased, the adaptive filter is converging to the optimal filter in this state. However, the present step size, $\mu(n)$, is considered large because the sign of the gradient is changed passing the optimal filter. The next step size, $\mu(n+1)$, in this category needs to be decreased.

(2) Category 2 – a sign change and increased amplitude
In this category, the present step size, $\mu(n)$, is considered too large because it not only passes the optimal filter but also not even converges. Hence, the next step size, $\mu(n+1)$, needs to be decreased by more than for the category 1.

(3) Category 3 – no sign change and decreased amplitude
Although the adaptive filter in this category is converging to the optimal filter similarly to the category 1, it has not passed the optimal filter yet. To make it converge faster, the next step size, $\mu(n+1)$, needs to be increased.

(4) Category 4 – no sign change and increased amplitude
This category represents two cases. One is when the SNR of the mixed signal becomes very low. Because noise is considered as a pseudo-random signal, it seldom leads the square error of an adaptive filter to be large. Another is when the optimal filter is changed. In both cases, the step size, $\mu(n+1)$, needs to be increased by more than in the category 3 because the updated filter coefficients, $\mathbf{w}(n)$, are farther from

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the optimal filter coefficients, $\mathbf{w}_{opt}$, than the previous filter coefficients, $\mathbf{w}(n-1)$.

2.2 Categorized variable step size least mean square algorithm

As a generic VSSLMS filter, the CVSSLMS filter, $\mathbf{w}_{\mathrm{CVSS}}(n)$ is defined as Eq. (1).

$$\mathbf{w}_{\mathrm{CVSS}}(n+1) = \mathbf{w}_{\mathrm{CVSS}}(n) + \mu(n)e(n)\mathbf{x}(n), \tag{1}$$

where $\mu(n+1)$ is the step size, $e(n)$ is the error of the filter, and $\mathbf{x}(n)$ is the input signal. From the categorization, the update of the step size of the proposed CVSSLMS algorithm is modeled as Eq. (2).

$$\mu(n+1) = \begin{cases} \mu(n)/\left(1+\alpha_1\nabla(n-1)\nabla(n)\right) & for \text{ category } 1, \\ \mu(n)/\left(1+\alpha_2\nabla(n-1)\nabla(n)\right) & for \text{ category } 2, \\ \mu(n)\times\left(1+\alpha_1\nabla(n-1)\nabla(n)\right) & for \text{ category } 3, \\ \mu(n)\times\left(1+\alpha_2\nabla(n-1)\nabla(n)\right) & for \text{ category } 4. \end{cases} \tag{2}$$

Here, α_1 and α_2 are updating constants, and $\nabla(n)$ and $\nabla(n-1)$ are the gradients of present and previous square error, respectively. Each constant is used in updating the step size as a different form of a weighting function depending on its category. As mentioned in section 2.1, for example, the step size in category 2 needs to be decreased by a bigger amount than that in category 1. By making α_2 be larger than α_1, the next step size of category 2 becomes smaller than that of category 1. Similarly, different updating constants are used in category 3 and 4.

To avoid a divergence problem that seldom happens in category 3 and 4, the step size is limited as Eq. (3), according to Kwong [6]

$$\mu(n+1) = \min\left(\mu(n+1), \mu_{\max}\right), \tag{3}$$

where $\mu_{\max} \leq 2/3\mathrm{tr}(R)$, $R = E\left(\mathbf{x}(n)\mathbf{x}^T(n)\right)$, and $\mathrm{tr}(\cdot)$ is trace operation.

2.3 Combined categorized variable step size least mean square algorithm

In some LMS applications, the robustness of an algorithm is far more important than its simplicity. For this reason, MCLMS filters are used in many such fields in spite of its rather heavy calculations. For such applications, we propose a Combined Categorized Variable Step Size Least Mean Square (CCVSSLMS) algorithm as a combination of one VSSLMS filter and one Fixed Step Size LMS (FSSLMS) filter.

Following Martínez-Ramón's proposal of a convex combination of the weights of the two LMS filter [8], we designed a CCVSSLMS filter, $\mathbf{w}_{\mathrm{CVSS}}$, to be

$$\mathbf{w}_{\mathrm{CCVSS}}(n) = v(n)\mathbf{w}_{\mathrm{CVSS}}(n) + (1-v(n))\mathbf{w}_{\mathrm{FSS}}(n) \tag{4}$$

$$\text{whare} \begin{cases} v(n) = 1/1+e^{-a(n)} & \text{mixing coefficient,} \\ \mathbf{w}_{\mathrm{CVSS}}(n) & \text{CVSSLMS filter,} \\ \mathbf{w}_{\mathrm{FSS}}(n) & \text{FSSLMS filter.} \end{cases}$$

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Then the output and the error of the CCVSSLMS filter can be expressed as Eq. (5) and Eq. (6), respectively.

$$y_{\mathrm{CCVSS}}(n) = v(n) y_{\mathrm{CVSS}}(n) + (1 - v(n)) y_{\mathrm{FSS}}(n), \tag{5}$$

$$e_{\mathrm{CCVSS}}(n) = v(n) e_{\mathrm{CVSS}}(n) + (1 - v(n)) e_{\mathrm{FSS}}(n). \tag{6}$$

In order to minimize the overall error of the CCVSSLMS filter, $e_{\mathrm{CVSS}}(n)$, the combination parameter $a(n)$ is updated with Eq. (7).

$$\begin{aligned} a(n+1) &= a(n) - \frac{\mu_a}{2} \frac{d e^2_{\mathrm{CCVSS}}(n)}{da(n)} \\ &= a(n) - \mu_a e_{\mathrm{CCVSS}}(n) \left(e_{\mathrm{CVSS}}(n) - e_{\mathrm{FSS}}(n) \right) v(n) \left(1 - v(n) \right). \end{aligned} \tag{7}$$

In Eq. (7), μ_a is set to be a very high constant and $a(n)$ is limited in the range between $[-4, 4]$ as restricted by Arenas-García [7].

3 Experiments

We conducted an experiment to assess the performance of the proposed algorithm from the view points of robustness and simplicity. The two proposed algorithms – CVSSLMS algorithm and CCVSSLMS algorithm – are compared with the VSSLMS algorithm by Weepeng [3] and the MCLMS algorithm by Arenas-García [7].

3.1 Experiments environment

In the experiment, the desired signal, $d(n)$, is generated by a typical definition as given in Eq. (8).

$$d(n) = \mathbf{x}(n)^T \mathbf{w}_{opt} + t(n), \tag{8}$$

where the random sequence $\mathbf{x}(n)$ consists of a Bernoulli sequence with the value either of $+1$ or -1 with zero mean and unit variance, the noise signal $t(n)$ is a pseudorandom zero-mean unit-variance Gaussian process uncorrelated with $\mathbf{x}(n)$, and $\mathbf{w}_{opt}$ is an optimal filter of length ten. In order to verify the robustness of the proposed algorithms, we defined $\mathbf{w}_{opt}$ to have different coefficient vectors for different time intervals: $\mathbf{w}_{opt1}$ for the first 2000 samples ($n \leq 2000$) and $\mathbf{w}_{opt2}$ for $n > 2000$, which are defined by Eq. (9)

$$\mathbf{w}_{opti} = e^{-0.5(m-1)} r_i(m), \quad m = 1, 2, \ldots, 10 \text{ and } i = 1, 2, \tag{9}$$

where $r_i(m)$ is a pseudorandom zero-mean unit-variance Gaussian sequence generation function with a given i, and m is a set of indices for $\mathbf{w}_{opt1}$ and $\mathbf{w}_{opt2}$, hence $\mathbf{w}_{opt1}$ and $\mathbf{w}_{opt2}$ become different.

The constants for each comparing algorithm are chosen for an optimal performance. In VSSLMS algorithm by Weepeng [3], two constants are set as $\alpha = 0.99$ and $\rho = 4 \times 10^{-4}$. In MCLMS algorithm by Arenas-García [7], five constants are set as $\mu_1 = 0.0031$, $\mu_2 = 0.0121$, $\mu_3 = 0.0052$, $\mu_4 = 0.0017$, and $\mu_a = 10$. The proposed CVSSLMS algorithm uses $\mu_{\max} = 0.0444$, $\alpha_1 = 0.09$, and $\alpha_2 = 1.7$. In the proposed CCVSSLMS algorithm, a FSSLMS filter with $\mu = 0.0017$ is combined with CVSSLMS and $\mu_a = 10$.

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3.2 Experiment results – robustness

To assess the robustness of the proposed algorithm, the convergence rate and the Excessive Mean Square Error (EMSE) are obtained after the simulations. Fig. 2 and Fig. 3 show the results of the simulations.

First, we compare CVSSLMS algorithm with Weepeng's VSSLMS algorithm from the viewpoint of the step size change. As shown in Fig. 2, it is clear that the CVSSLMS performs better than Weepeng's algorithm as it converges faster with dynamically-changed step size, which can be easily observed from the figure. Moreover, it also adapts better and faster after the change of the optimal filter at the 2000th sample. From the point of EMSE, it is also clear that the proposed algorithms provide far lower EMSE than Weepeng's as shown in Fig. 3.

In comparison with the MCLMS, Fig. 3 shows that both the proposed algorithms converge faster than MCLMS. After the change in the optimal filter, moreover, the proposed algorithms converge better with lower EMSE than the MCLMS. As a result, the proposed algorithms show more robust performance than conventional adaptive LMS algorithms.

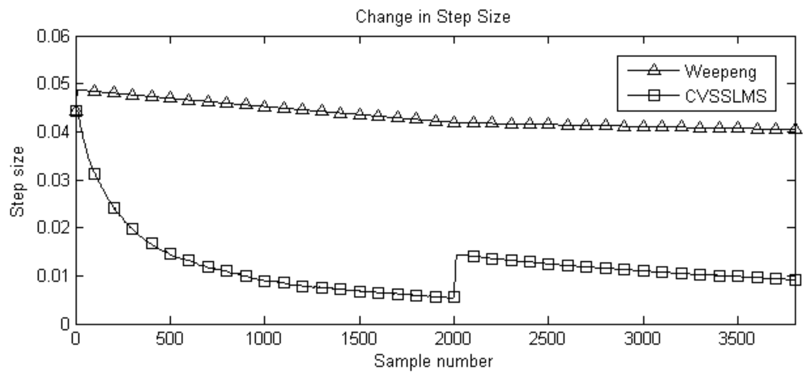

Fig. 2. Change in Step Size of VSSLMS algorithms

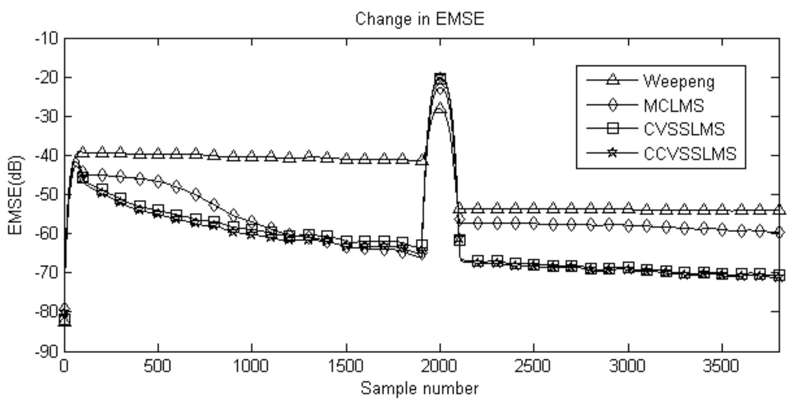

Fig. 3. Change in Excessive Mean Square Error

3.3 Experiment results – simplicity

To assess the simplicity of the proposed algorithms, we investigated the complexity of each algorithm by counting the number of multiplications. Table I

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Table I. Comparison in Complexity

Algorithm	Number of Multiplications	
	In general	In the simulation
Weepeng	7M+1	71
MCLMS	2LM + 5L + 1	101
CVSSLMS	5M + 2	52
CCVSSLMS	7M + 10	80

shows the number of multiplications used in general and in the simulation. Here, M means the LMS filter length, L means the number of the FSS filters used in MCLMS. Comparing the proposed algorithms with a single VSS filter, it is clear that the CVSSLMS algorithm needs less complexity than Weepeng's VSS algorithm. In comparing with multiple LMS filters, it is also clear that the CCVSSLMS algorithm requires lower complexity than the MCLMS.

4 Conclusions

In this letter, a Categorized Variable Step Size Least Mean Square (CVSSLMS) algorithm was proposed that updates its step size more effectively reflecting the gradient state. As an extended form, a Combined CVSSLMS (CCVSSLMS) algorithm is also proposed for the case that the robustness is highly important. To verify the performance of the proposed algorithms, nonstationary environment experiments were carried out, which confirmed that the proposed algorithms outperformed conventional LMS algorithms with lower complexity, lower EMSE, and faster convergence rate. This proves that the proposed algorithms – CVSSLMS and CCVSSLMS – are therefore great alternatives to existing LMS algorithms, with their simplicity of the logic and the efficiency and robustness in performance.

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