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Fast delay search algorithm for time-domain equalization in multicarrier systems

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Abstract: A fast delay search algorithm for time-domain equalizers (TEQs) in multicarrier communication systems is proposed. The algorithm simplifies the delay optimization process required in computing the TEQ tap weights, thus enabling TEQs to adapt quickly to channel variations. Simulation results show that the proposed delay search algorithm can reduce the TEQ's computational time significantly while maintaining comparable performances.

Keywords: time-domain equalizer, multicarrier, cyclic prefix
Classification: Microwave and millimeter wave devices, circuits, and systems

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1 Introduction

Multicarrier modulation such as orthogonal frequency division multiplexing (OFDM) modulation has been employed in wireline and wireless communication systems due to its robustness against frequency-selective fading. A cyclic prefix (CP) is appended at the beginning of each symbol block to mitigate intersymbol interference (ISI). However, when the length of the CP N_b is shorter than the length of the channel impulse response L, the received signal is corrupted by ISI, which then dramatically worsen the system performance. To avoid such performance degradation, a channel shortener called time-domain equalizer (TEQ) can be placed at the receiving end [1, 2, 3].

The most widely studied TEQs are designed based on criteria such as minimum mean-squared error (MMSE) [1], maximum shortening signal-to-noise ratio (MSSNR) [2] and minimum-ISI (Min-ISI) [3]. These TEQs, however, are non-adaptive in nature [4] and thus periodic training is required for channel estimation so as to compute the TEQ tap weights. If a TEQ is adapted slowly with respect to the environments, the computed TEQ tap weights will underperform in time-varying channels, leading to unsatisfactory performance. As such, a TEQ that can quickly adapt to channel variations is desired. In this letter, we propose a new delay search algorithm which can reduce the computational time of TEQs substantially, thus optimizing the performance of TEQs in multicarrier systems.

2 System description

The received OFDM signal is fed into a TEQ at the receiving end. As such, the equalized channel is the cascade of the channel and TEQ, where TEQ can be broken into two parts: TEQ design criteria and delay optimization process. The delay optimization process is required because the TEQ algorithm is delay dependent. By using TEQ design criteria such as MSSNR [2] and Min-ISI [3], different delay samples generally produce different sets of TEQ tap weights, and thus different equalized channels. As such, the delay optimization process is performed to identify the delay sample that yields the optimum TEQ tap weights.

3 Delay optimization in TEQs

We consider two existing TEQs, namely the MSSNR [2] and Min-ISI TEQs [3]. The goal of the MSSNR TEQ is to minimize the energy of the shortened impulse response outside a target window while keeping the energy inside it constant [2]. The MSSNR TEQ maximizes the shortening signal-to-noise ratio (SSNR) defined as

$$\mathrm{SSNR} = \frac{\boldsymbol{w}^T \boldsymbol{H}_{\mathrm{win}}^T \boldsymbol{H}_{\mathrm{win}} \boldsymbol{w}}{\boldsymbol{w}^T \boldsymbol{H}_{\mathrm{wall}}^T \boldsymbol{H}_{\mathrm{wall}} \boldsymbol{w}}, \tag{1}$$

where $\boldsymbol{w}^T \boldsymbol{H}_{\mathrm{win}}^T \boldsymbol{H}_{\mathrm{win}} \boldsymbol{w}$ and $\boldsymbol{w}^T \boldsymbol{H}_{\mathrm{wall}}^T \boldsymbol{H}_{\mathrm{wall}} \boldsymbol{w}$ represent the energy inside and outside the target window, respectively, while $\boldsymbol{w}$, $\boldsymbol{H}_{\mathrm{win}}$ and $\boldsymbol{H}_{\mathrm{wall}}$ denote the vector forms of the TEQ tap weights, the channel convolution matrix inside

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and outside the window, respectively [2]. The Min-ISI TEQ is a generalized version of the MSSNR TEQ which minimizes a weighted sum of the ISI power [3]. The Min-ISI TEQ maximizes the achievable bits per symbol, defined as

$$b = \sum_{i \in S} \log_2 \left(1 + \frac{\mathrm{SNR}_i}{\Gamma}\right) \text{ bits/symbol}, \tag{2}$$

where i, S, SNR_i, and Γ represent the subchannel index, the set of used subchannel indices, the signal-to-noise ratio (SNR) of the ith subchannel, and the SNR gap, respectively. The equalized channel of $L + N_w - 1$ samples entails a delay of d samples followed by a target window of $N_b + 1$ samples and the remaining tail bits of $L + N_w - 1 - d - (N_b + 1)$ samples, where N_w is the number of the TEQ tap weights. Delay optimization is repeated over D delay samples, where $D = (L + N_w - 1) - N_b$. Since the bulk of the TEQ's computational time is devoted to this repetitive process, a simplification of this process is the key to obtaining a more efficient TEQ algorithm.

4 New fast delay search algorithm

Both the MSSNR and Min-ISI TEQs are based on computation of eigenvalues and eigenvectors, thus conventional optimization techniques such as the genetic algorithm cannot be used for delay search. We propose a new delay search algorithm which is comprised of two stages: rough search and fine search. The function of the rough search stage is to locate the most probable delay region which embraces the optimum TEQ tap weights, while the function of the fine search stage is to identify the exact optimal delay that corresponds to the optimum TEQ tap weights. The proposed algorithm begins with a rough search over every interval of n delay samples. The delay obtained through the rough search is denoted as d_r, which is then used as the starting point in the fine search stage. The rough search is formulated as

$$d_r = \arg \max_{d_j \in A} (b_j), \tag{3}$$

where $A = \{n, 2n, 3n, \ldots, pn\}$, $pn \leq D$ and $p \in \mathbb{Z}$. The notations j, A and b_j represent the rough delay sample index, the set of rough delay sample indices and the number of bits per symbol of the jth rough delay sample, respectively. The fine search runs in the neighborhood of d_r and the optimum delay obtained in this stage is denoted as

$$d_f = \arg \max_{d_k \in B} (b_k), \tag{4}$$

where B ranges from $d_r - (n-1)$ to $d_r + (n-1)$. The notations k, B and b_k represent the fine delay sample index, the set of fine delay sample indices and the number of bits per symbol of the kth fine delay sample, respectively. Compared with conventional way that searches over D delay samples, the proposed delay search algorithm greatly simplifies the delay optimization process and thus reducing the TEQ's computational time.

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5 Results and discussion

The performances of the MSSNR and Min-ISI TEQs using conventional delay search are compared with that of using the proposed delay search algorithm. We simulated the eight standard carrier-serving area (CSA) channel models widely adopted for digital subscriber loops using the Linemod software [5]. The fast Fourier transform (FFT) size and CP length are 512 and 32, respectively [6]; N_w is set to 25. The input signal power of 23 dBm is distributed equally over the subcarriers. The channel noise is modeled as −140 dBm additive white Gaussian noise (AWGN) plus the near-end-cross-talk (NEXT) noise.

In conventional delay search, the delay optimization process is performed over $D = 50$ delay samples because TEQs were observed to perform poorly for $D > 50$ [3]. Table I shows the total number of run for different n values using the proposed delay search algorithm. In fact, different n values result in different number of run for the rough and fine search, thus the total number of run for the entire delay optimization process. For example, the proposed delay search algorithm with $n = 5$ requires 10 and 9 runs of the rough and fine searches, respectively, giving a total of 19 runs only. As compared with conventional delay search (total number of run = 50), the use of the proposed delay search algorithm with $n = 5$ leads to a 62% reduction in the computational time. Apparently, the TEQ tap weights can be computed quickly by using the proposed delay search algorithm.

On the other hand, it is important to examine the system performance of

Table I. Comparisons of different n values in terms of number of run using the proposed delay search algorithm on CSA loop 1 and bit rates achieved in percentage of that using conventional delay search, $N_w = 25$, $N_b = 32$.

Delay interval, n	Number of run for delay optimization using the proposed delay search algorithm				Bit rate achieved using the proposed delay search algorithm (in % of that achieved using conventional delay search)	
	Rough search	Fine search	Total	Reduction in computational time w.r.t. conventional delay search (%)	MSSNR TEQ (%)	Min-ISI TEQ (%)
2	25	3	28	44	100	100
3	16	5	21	58	100	100
4	12	7	19	62	100	100
5	10	9	19	62	100	100
6	8	11	19	62	100	100
7	7	13	20	60	100	100
8	6	15	21	58	100	100
9	5	17	22	56	100	100
10	5	19	24	52	100	100

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the proposed delay search algorithm with respect to conventional delay search algorithm. Table II shows the average bit rate results obtained from 1000 separate simulations for the MSSNR and Min-ISI TEQs using conventional and proposed delay search algorithms on 8 CSA loops. It is evident that for both the MSSNR and Min-ISI TEQs, the bit rates achieved by the proposed

Table II. Average bit rates for the MSSNR and Min-ISI TEQs using conventional and proposed delay search algorithms on 8 CSA loops, $n = 5$, $N_w = 25$, $N_b = 32$.

Loop No.	Average bit rate (MSSNR TEQ)		Average bit rate (Min-ISI TEQ)		Average bit rate achieved using the proposed delay search algorithm (in % of that achieved using conventional delay search)	
	Conventional delay search (Mb/s)	Proposed delay search (Mb/s)	Conventional delay search (Mb/s)	Proposed delay search (Mb/s)	MSSNR TEQ (%)	Min-ISI TEQ (%)
1	6.2572	6.2572	8.6034	8.6034	100.00	100.00
2	7.0918	7.0918	9.8319	9.8319	100.00	100.00
3	6.8959	6.8959	8.2672	8.2620	100.00	99.94
4	6.0171	6.0171	8.2353	8.2353	100.00	100.00
5	6.9330	6.9330	8.7074	8.6934	100.00	99.84
6	7.2181	7.2181	8.0113	8.0113	100.00	100.00
7	5.6110	5.6110	7.9967	7.9967	100.00	100.00
8	5.7460	5.7460	7.0479	7.0479	100.00	100.00

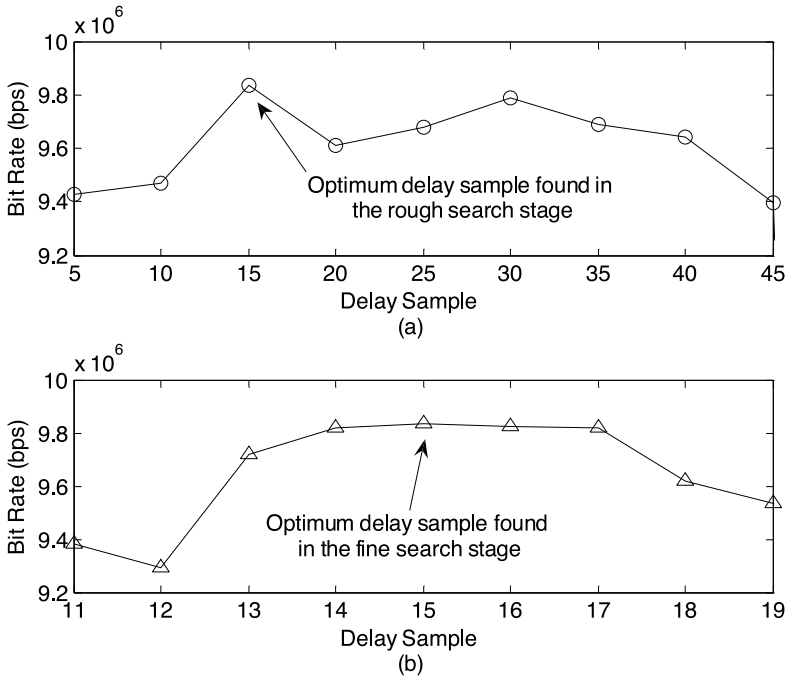

Fig. 1. Achievable bit rates using the proposed delay search algorithm for the Min-ISI TEQ during (a) rough search, (b) fine search, $n = 5$, $N_w = 25$ and $N_b = 32$ on CSA loop 2.

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delay search algorithm are over 99% of that achieved by using conventional delay search on all CSA loops. Indeed, this suggests that the proposed delay search algorithm can reduce the computational time significantly without degrading the achievable bit rate. It is worth mentioning that due to the concern about the reliability of the proposed algorithm, the results are the average values obtained from 1000 separate simulations.

Figures 1 (a) and (b) show the achievable bit rates for the Min-ISI TEQ using the proposed delay search algorithm with $n = 5$ for the rough search and fine search on CSA loop 2, respectively. The rough search goes through delay samples at $d = 5, 10, \ldots, 50$, then $d_r = 15$ is chosen as the starting point for the fine search. Then the fine search goes through delay samples at $d = 11, 12, \ldots, 19$. As shown in Table I, the optimum n for the proposed delay search algorithm are 4, 5 and 6 since the highest computational time reduction can be obtained without degrading the achievable bit rate.

6 Conclusion

In this letter, a fast delay search algorithm for the Min-ISI and MSSNR TEQs was proposed. The proposed algorithm simplifies the delay optimization process required in computing the TEQ tap weights, thus enabling TEQs to adapt quickly with respect to the environments. Simulation results indicated that the proposed method greatly reduces the computational time of TEQs without compromising the system performance.

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