ELectronics EXpress

IEICE Electronics Express, Vol.6, No.7, 424–429

Equivalent circuit simulation of light propagation in optical fiber

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Abstract: On the basis of the formal resemblance between the nonlinear Schrödinger equation that governs the evolution of an optical signal in a single-mode fiber and the telegrapher's equations that describe the voltage and current on an electrical transmission line, the nonlinear Schrödinger equation is modeled in an electrical equivalent circuit. Through the equivalent circuit simulations for the propagation of an optical fundamental soliton, the validity of the proposed model, that is, the correctness of the handling of the dispersion, the nonlinearity, and the attenuation in the model, is verified.

Keywords: optical fiber, nonlinear Schrödinger equation, telegrapher's equation, equivalent circuit, optical soliton, circuit simulator

Classification: Optical fiber

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1 Introduction

To analyze the transmission properties of optical signals in fibers, the split-step Fourier method [1] and the finite-difference time-domain method [2] are

DOI: 10.1587/elex.6.424
Received February 06, 2009
Accepted March 01, 2009
Published April 10, 2009

ELectronics EXpress
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commonly used. These methods require complicated and/or sophisticated programming techniques and special calculation tools and are therefore awkward to designers of ICs for fiber-optic communications, who must be able to grasp signal transmission properties in optical fibers. The ability to analyze optical signal transmission properties with ordinary electrical circuit simulators would be of benefit to IC designers, because it would make it possible to analyze transmission properties of optical fibers and design ICs in parallel on a single circuit simulator. In this paper, we propose an equivalent circuit simulation method for light propagation in an optical fiber. The proposed method is founded upon the formal resemblance between the nonlinear Schrödinger (NLS) equation that governs light propagation in an optical fiber and the telegrapher's equations that describe voltage and current on an electrical transmission line. Owing to the formal resemblance, the NLS equation can be modeled in an electrical equivalent circuit.

2 Basic equations and equivalent circuit

Under the slowly-varying-envelope approximation, the electric field amplitude $A(z,T)$ in a single-mode optical fiber at transmission distance z and time T (in a moving frame with group velocity) obeys the NLS equation [1]

$$j\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + j\frac{\alpha}{2}A + \gamma\left|A\right|^2 A = 0, \tag{1}$$

where α is the attenuation and β_2 is the dispersion coefficient. γ is the nonlinear parameter defined as $\gamma \equiv (2\pi n_2)/(\lambda S_{eff})$, where n_2 is the nonlinear refractive index, λ is the optical wavelength, and S_{eff} is the effective core area of the fiber. The amplitude A in Eq. (1) is assumed to be normalized such that $|A|^2$ represents the optical power. By setting $(z,T) \to (\tau,\xi)$ and defining parameters Vr, Vi, Ir, and Ii as

$$\begin{cases} A = \mathrm{Re}[A] + j\mathrm{Im}[A] \equiv Vr + j\cdot Vi \\ j\dfrac{\beta_2}{2}\dfrac{\partial A}{\partial \xi} = -\dfrac{\beta_2}{2}\dfrac{\partial}{\partial \xi}\mathrm{Im}[A] + j\dfrac{\beta_2}{2}\dfrac{\partial}{\partial \xi}\mathrm{Re}[A] \equiv Ir + j\cdot Ii \end{cases}, \tag{2}$$

Eq. (1) is rewritten as the following pairs of differential equations:

$$\begin{cases} -\dfrac{\partial Vr}{\partial \xi} = -\dfrac{2}{\beta_2}Ii \\ -\dfrac{\partial Ir}{\partial \xi} = \dfrac{\partial Vr}{\partial \tau} + \dfrac{\alpha}{2}Vr + \gamma\left(Vr^2 + Vi^2\right)Vi \end{cases}, \tag{3}$$

$$\begin{cases} -\dfrac{\partial Vi}{\partial \xi} = +\dfrac{2}{\beta_2}Ir \\ -\dfrac{\partial Ii}{\partial \xi} = \dfrac{\partial Vi}{\partial \tau} + \dfrac{\alpha}{2}Vi - \gamma\left(Vr^2 + Vi^2\right)Vr \end{cases}. \tag{4}$$

If we view the parameters τ and ξ as "time" and "space" variables, respectively, and the parameters (Vr, Vi) and (Ir, Ii) as "voltages" and "currents," respectively, Eqs. (3) and (4) are none other than two sets of telegrapher's equations for two different electrical signal lines. Therefore, Eqs. (3) and (4)

DOI: 10.1587/elex.6.424
Received February 06, 2009
Accepted March 01, 2009
Published April 10, 2009

IEICE Electronics Express, Vol.6, No.7, 424–429

can be modeled in electrical circuits, the circuitry of which is formally the same as that of the well-known distributed-element equivalent circuit for an electrical transmission line [3]. The equivalent circuits of a small section $\Delta\xi$ for Eqs. (3) and (4) are shown in Figs. 1 (a) and (b), respectively. The equivalent circuits in Fig. 1 are primal and artless in their form and can not be used in ordinary circuit simulators as they are, since the voltage and current sources cross-refer to the current and voltage on the other signal line. To obtain a practical circuit that can be implemented in circuit simulators, we have only to unite the two primal circuits into a single circuit. An example of the unification is shown in Fig. 2 where CCVS, VCCS, and VCVS denote the current-controlled voltage source, voltage-controlled current source, and voltage-controlled voltage source, respectively. The dual-input single-output VCCSs and VCVSs output an electrical signal, the value of which is proportional to the product of the two input voltages. The circuit shown in Fig. 2

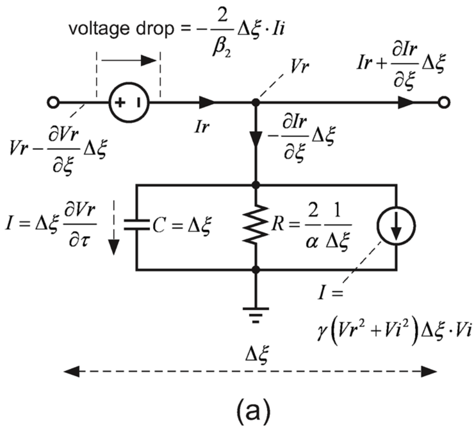

Fig. 1. Primal equivalent circuits of a small section $\Delta\xi$ for the NLS equation. (a) Circuit representation of Eq. (3). (b) Circuit representation of Eq. (4).

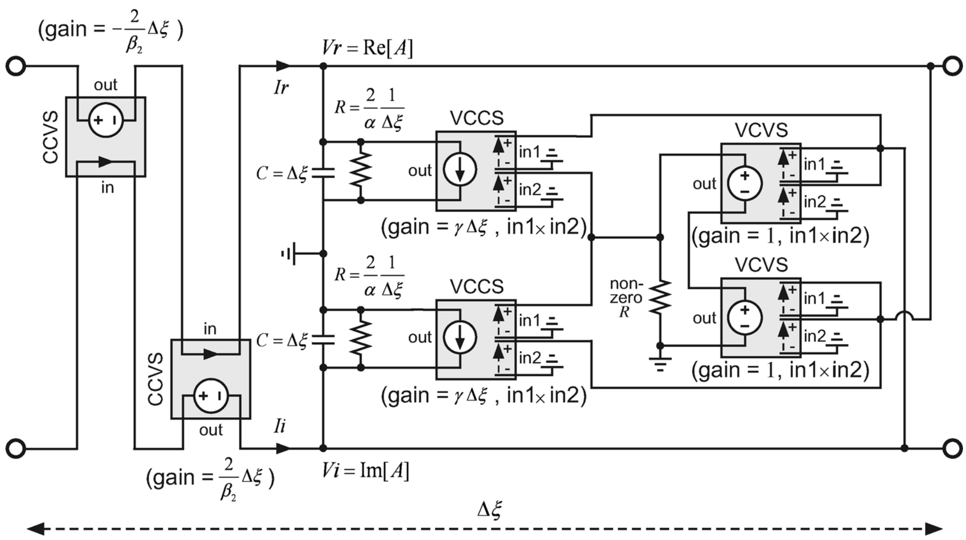

Fig. 2. A practical equivalent circuit of a small section $\Delta\xi$ for the NLS equation.

DOI: 10.1587/elex.6.424
Received February 06, 2009
Accepted March 01, 2009
Published April 10, 2009

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has two (upper and lower) main signal lines, each of which corresponds to the signal lines in Figs. 1 (a) and (b), respectively. The upper and lower CCVSs in Fig. 2 provide the same function as the voltage sources in Figs. 1 (a) and (b), respectively. The upper and lower VCCSs in Fig. 2 correspond to the current sources in Figs. 1 (a) and (b), respectively. All the elements used in the circuit in Fig. 2 are available in ordinary circuit simulators.

To perform the equivalent circuit simulation of light propagation in an optical fiber, we have only to plurally cascade the unit circuit shown in Fig. 2 and carry out a transient circuit simulation with initial voltage values distributed beforehand on the nodes of the cascaded circuit. The temporal distribution $|A(z,T)|^2$ at a transmission distance z is then obtained as the distribution of $Vr^2 + Vi^2$ on the nodes of the cascaded circuit. Note that the "length" $\Delta\xi$ of the unit circuit corresponds to the span on the time (T) axis, since the exchange of the time and space variables, $(z,T) \leftrightarrow (\tau,\xi)$, is assumed in the derivation of the equivalent circuit. For this reason, the passing of time in the transient circuit simulation corresponds to the extension of transmission distance for an optical signal.

3 Equivalent circuit simulation of optical soliton transmission

To verify the correctness of the equivalent circuit model, we performed circuit simulations of the transmission of an optical fundamental soliton. The fundamental soliton features the conservation of its pulse shape with respect to time T over long transmission distances, because of the balance between a pulse broadening effect due to the dispersion and a pulse sharpening effect due to the nonlinearity. Therefore, correct simulation of the evolution of an optical fundamental soliton with the equivalent circuit model would be evidence of the correct handling of the dispersion and the nonlinearity in the model. The correctness of the attenuation handling will also be examined.

For standard silica fibers, the dispersion coefficient $\beta_2 \equiv -(\lambda^2 D)/(2\pi c)$, where D is the dispersion parameter and c is the speed of light in vacuum, is negative at $\lambda = 1.55\,[\mu\text{m}]$ (the anomalous dispersion regime). When $\beta_2 < 0$ and $\alpha = 0$, the fundamental soliton as an analytic solution of Eq. (1) is expressed as [1, 4, 5, 6]

$$\begin{cases} A(z,T) = \sqrt{P_0}\exp\left(j\dfrac{|\beta_2|}{2T_0^2}z\right)\operatorname{sech}\left(\dfrac{T}{T_0}\right), \quad 1 = \dfrac{T_0^2/|\beta_2|}{1/(\gamma P_0)} \\ T_0 \equiv \dfrac{t_0}{2\cosh^{-1}\sqrt{2}} \end{cases}, \tag{5}$$

where P_0 and t_0 are the peak power and the full width at half maximum (FWHM) of the pulse $|A|^2$, respectively. Note that the dispersion length L_D and the nonlinear length L_{NL} defined by

$$L_D \equiv T_0^2\Big/|\beta_2|, \quad L_{NL} \equiv 1/(\gamma P_0) \tag{6}$$

are equal for the fundamental soliton [See the second equation in (5)].

For the equivalent circuit simulation of the fundamental soliton transmission, we set up a cascaded circuit composed of twenty-thousand unit

ELectronics EXpress

DOI: 10.1587/elex.6.424
Received February 06, 2009
Accepted March 01, 2009
Published April 10, 2009

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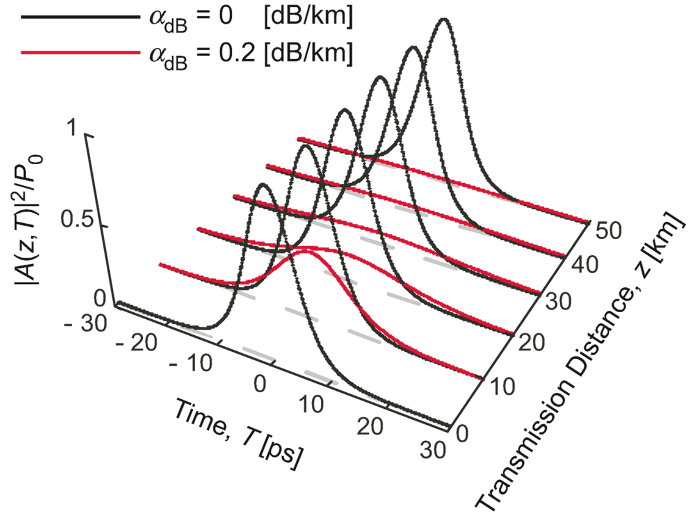
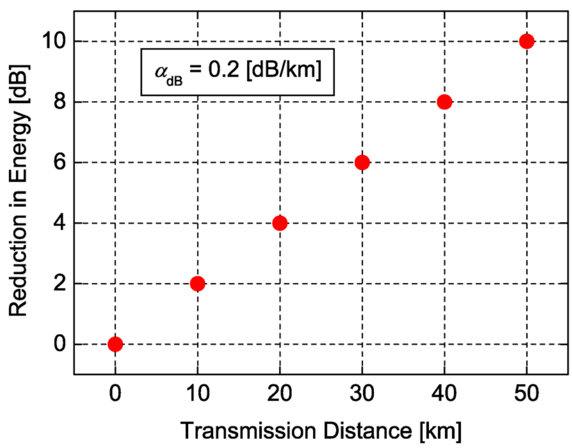
circuits with $\Delta\xi = 0.1$ [ps] and assigned the cascaded circuit to a time domain of $-1000\,[\text{ps}] \leq T \leq 1000\,[\text{ps}]$. Fiber parameters used in the simulation are $\lambda = 1.55\,[\mu\text{m}]$, $D = 18\,[\text{ps/nm/km}]$, $S_{eff} = 50\,[\mu\text{m}^2]$, and $n_2 = 3.18 \times 10^{-20}\,[\text{m}^2/\text{W}]$ [1, 4]. As an initial input soliton, $A(z = 0, T)$ given by Eq. (5) with $t_0 = 10$ [ps] was assumed and corresponding initial voltages were distributed on the Vr nodes of the cascaded circuit. The value of P_0 is uniquely determined to be 0.2767 [W] by the second equation in (5).

Fig. 3 shows the transient circuit simulation results obtained by using HSPICE. The black lines in Fig. 3 (a) indicate the evolution of the optical fundamental soliton for $\alpha_{\text{dB}} = 0\,[\text{dB/km}]$ ($\alpha_{\text{dB}} = (10/\log_e 10)\alpha$, where α is expressed in unit of km^{-1}). For $\alpha_{\text{dB}} = 0$, the resistors in Fig. 2, which are responsible for α, were omitted. As can be seen in Fig. 3 (a), the soliton



(a)



(b)

Fig. 3. Equivalent circuit simulation results for the optical fundamental soliton transmission. (a) Evolution of the fundamental soliton. (b) Energy reduction calculated with Eq. (7).

DOI: 10.1587/elex.6.424
Received February 06, 2009
Accepted March 01, 2009
Published April 10, 2009

maintains its envelope shape over a transmission distance of 50 km when $\alpha_{\mathrm{dB}} = 0$. This implies that the handling of the dispersion and the nonlinearity in the proposed equivalent circuit model is correct. It should be noted that the dispersion length and the nonlinear length are $L_D = L_{NL} \approx 1.4\,\mathrm{km} \ll 50\,\mathrm{km}$ for the parameter values used in the simulation, indicating that the transient circuit simulation up to a 50-km transmission distance sufficiently includes the influence of the dispersion and the nonlinearity.

The red lines in Fig. 3 (a) indicate the evolution of the optical fundamental soliton for $\alpha_{\mathrm{dB}} = 0.2\,[\mathrm{dB/km}]$. When the attenuation exists, the soliton gradually decays, losing its energy as the transmission distance increases. To verify the correctness of the attenuation handling in the simulation, the energy reduction calculated as

$$\frac{\displaystyle\int_{-1000\,\mathrm{ps}}^{1000\,\mathrm{ps}} \left| A(z,T)_{\alpha_{\mathrm{dB}}=0\,\mathrm{dB/km}} \right|^2 dT}{\displaystyle\int_{-1000\,\mathrm{ps}}^{1000\,\mathrm{ps}} \left| A(z,T)_{\alpha_{\mathrm{dB}}=0.2\,\mathrm{dB/km}} \right|^2 dT} \tag{7}$$

is plotted in Fig. 3 (b) as a function of transmission distance. From the figure, we can confirm that the circuit simulation results shown in Fig. 3 (a) correctly reflect the value of the attenuation constant.

4 Conclusion

On the basis of the formal resemblance to the telegrapher's equations, an equivalent circuit model for the NLS equation that governs the light propagation in a single-mode fiber is proposed. Through equivalent circuit simulations for the optical fundamental soliton transmission, the validity of the model has been confirmed. The object of the circuit simulations was limitted to a single pulse signal of the fundamental soliton; however, needless to say, other signals like a pseudo-random-bit-sequence (PRBS) signal are also treatable with the model. The proposed model and the simulation method utilizing it will be especially useful to designers of ICs for optical communications systems, since the method requires only a circuit simulator as a calculation tool.

Acknowledgments

The authors thank Dr. T. Hirono for his valuable and helpful advice.

DOI: 10.1587/elex.6.424
Received February 06, 2009
Accepted March 01, 2009
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