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Joint blind estimation of symbol timing offset and carrier frequency offset for OFDM systems with I/Q imbalance

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Abstract: In this paper, we present a blind algorithm for joint estimation of symbol timing offset (STO) and carrier frequency offset (CFO) for orthogonal frequency division multiplexing (OFDM) systems with I/Q imbalance. The proposed algorithm exploits the cyclostationarity of OFDM signals and relies on second-order statistics only. Estimation performance of the algorithm is evaluated by simulation and the results obtained in some typical scenarios are provided to show its effectiveness.

Keywords: blind estimation, cyclostationarity, symbol timing offset (STO), carrier frequency offset (CFO), orthogonal frequency division multiplexing (OFDM), I/Q imbalance

Classification: Science and engineering for electronics

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1 Introduction

Because of its high spectrum efficiency, orthogonal frequency division multiplexing (OFDM) has become one of the most popular technologies for wireless communications. However, OFDM is sensitive to synchronization error such as symbol timing offset (STO) and carrier frequency offset (CFO) [1]. In order to remove the negative effects, OFDM systems must have accurate STO and CFO estimation.

Estimation of STO and CFO for OFDM systems had been studied for long time and some representative results had been obtained in [2, 3]. Maximum likelihood STO and CFO estimation for OFDM systems in AWGN channel was presented in [2], while its performance degrades in multi-path dispersive channel. In [3], Bölcskei proposed a blind STO and CFO estimation algorithm, which is applicable for multi-path dispersive environments, based on cyclostationarity of OFDM signal.

Recently, direct-conversion receiver (DCR) has been employed in more and more systems because of its low costs, relaxed requirements of filters and easy implementation and integration [4]. However, DCR is vulnerable to impairments like DCO, I/Q imbalance, even-order distortion, flicker noise and carrier leakage [4], where I/Q imbalance is one of the most common ones. In [5], algorithms for STO and CFO estimation by making use of training sequences or pilots were presented, however it is well known that both of them will occupy bandwidth that could be used for payload transmissions.

In this paper, we present a blind algorithm for joint estimation of STO and CFO for OFDM systems with I/Q imbalance. The proposed algorithm exploits the cyclostationarity [6] of OFDM signals and relies on second-order statistics only. The rest of this paper is organized as follows. In section 2, we introduce the mathematical model for OFDM systems without I/Q imbalance and briefly introduce the algorithm presented in [3]. In section 3, the model with I/Q imbalance and the proposed estimation algorithm are presented. Simulation results and corresponding analysis are provided in section 4. In the last section, conclusions are drawn.

2 STO and CFO estimation in OFDM systems without I/Q imbalance

Consider an OFDM system with M symmetric sub-carriers for data transmission. The signal received at receiver can be presented as in (1),

$$r(t) = e^{j2\pi f_o t} e^{j\varphi} \sum_{l=-\infty}^{+\infty} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} H_{k,l} S_{k,l} e^{j2\pi k\Delta f(t-lT_t)} g(t-lT_t) + w(t) \tag{1}$$

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where CFO is denoted by f_o. φ is a random phase. $S_{k,l}$ and $H_{k,l}$ represent the symbol carried by the k^{th} sub-carrier of l^{th} OFDM symbol and corresponding frequency domain channel response, respectively. They are supposed to have second-order statistics that $E\{S_{k_1,l_1}S^*_{k_2,l_2}\} = \sigma_S^2\delta(k_1-k_2)\delta(l_1-l_2)$ and $E\{|H_{k,l}|^2\} = \sigma_H^2$, where the average symbol power and channel gain are denoted by σ_S^2 and σ_H^2, respectively. T_t is the time duration of an OFDM symbol together with its cyclic prefix (CP). It consists of two parts G and T corresponding to CP and OFDM symbol respectively, i.e. $T_t = G + T$. Δf is the frequency space between adjacent sub-carriers and satisfies that $\Delta f T = 1$. Since most OFDM systems employ rectangular pulse shaping, the shaping pulse $g(t)$ is assumed in this paper to have the form of (2).

$$g(t) = \begin{cases} 1 & \text{if } 0 \le t \le T_t \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Because most modern receivers have digital baseband processor, the received signal is sampled with interval denoted by T_s here. The sampled signal can be expressed as in (3).

$$r(n) = e^{j\frac{2\pi}{N}\varepsilon n}e^{j\varphi}\sum_{l=-\infty}^{+\infty}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2} H_{k,l}S_{k,l}e^{j\frac{2\pi}{N}k(n-lU)}g(n-lU)+w(n) \tag{3}$$

where the definitions of the samples are $r(n) = r(t)|_{t=nT_s-t_o}$, $g(n) = g(t)|_{t=nT_s-t_o}$ and $w(n) = w(t)|_{t=nT_s-t_o}$. ε is the CFO normalized to sub-carrier interval, i.e. $\varepsilon = f_oT$. Let $T_t = UT_s$ and $T = NT_s$ with assumption that both U and N are integers.

The algorithm proposed in [3] performs blind STO and CFO estimation by exploiting the cyclostationarity of received OFDM samples. In order to do so, the correlation function needs to be calculated first as in (4).

$$\begin{aligned} m_{2r}(n,m) &= E\{r(n)r^*(n-m)\} \\ &= e^{j\frac{2\pi}{N}\varepsilon m}\sigma_S^2\sigma_H^2 a(m)\sum_{l=-\infty}^{+\infty} g(n-lU)g^*(n-m-lU) \\ &\quad + \sigma_w^2\delta(m) \end{aligned} \tag{4}$$

where $a(m) = \sum_{\substack{k=-M/2\\k\neq 0}}^{M/2} e^{j\frac{2\pi}{N}km}$. Since it is evident that $m_{2r}(n+U,m) = m_{2r}(n,m)$, $r(n)$ is said to be second-order cyclostationary. And the discrete Fourier transform (DFT) of its correlation function can then be obtained in (5) as

$$\begin{aligned} M_{2r}(u,m) &= \frac{1}{U}\sum_{n=0}^{U-1} m_{2r}(n,m)e^{-j\frac{2\pi}{U}un} \\ &= \frac{1}{U}e^{j\frac{2\pi}{N}\varepsilon m}a(m)G_2(u,m)e^{-j\frac{2\pi}{U}u\tau} + \sigma_w^2\delta(u)\delta(m) \end{aligned} \tag{5}$$

where τ is the STO normalized to sampling period, i.e. $\tau = t_o/T_s$. According to Paseval's theorem, $G_2(u,m)$ is defined as

$$G_2(u,m) = \frac{1}{T_s}\int_{-1/2T_s}^{1/2T_s} G_c^*(f)G_c\left(f+\frac{u}{T_t}\right)e^{j2\pi f m T_s}df \tag{6}$$

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where $G_c(f) = \int_{-\infty}^{+\infty} g(t)e^{-j2\pi ft}$. For any specific system, $g(t)$ is determined and therefore $G_2(u,m)$ is known at receiver. So $G_2(u,m)$ together with $a(m)$ could be removed from $M_{2r}(u,m)$ as in (7).

$$\begin{aligned} Q_{2r}(u,m) &= G_2^{-1}(u,m)a^{-1}(m)M_{2r}(u,m) \\ &= \frac{1}{U}e^{j\frac{2\pi}{N}\varepsilon m}e^{-j\frac{2\pi}{U}u\tau} + G_2^{-1}(u,m)a^{-1}(m)\sigma_w^2\delta(u)\delta(m) \end{aligned} \tag{7}$$

As shown in (8), we can perform another transform on $Q_{2r}(u,m)$ with respect to m.

$$\begin{aligned} S_{2r}(u,f) &= \sum_{m=-\infty}^{+\infty} Q_{2r}(u,m)e^{-j2\pi fm} \\ &= \frac{1}{U}e^{-j\frac{2\pi}{U}u\tau}A\left(f-\frac{\varepsilon}{N}\right) + G_2^{-1}(u,0)a^{-1}(0)\sigma_w^2\delta(u) \end{aligned} \tag{8}$$

where $A(f) = \sum_{m=-\infty}^{+\infty} e^{-j2\pi fm}$. Since evidently, Estimation of ε and τ can be obtained by $\hat{\varepsilon} = N\cdot \underset{f}{argmax}\{|S_{2r}(u,f)|\}$ and $\hat{\tau} = -\frac{U}{2u\pi}arg\{S_{2r}(u,\frac{\hat{\varepsilon}}{N})\}$ [3].

3 STO and CFO estimation in OFDM systems with I/Q imbalance

With I/Q imbalance, the received samples can be presented as (9).

$$x(n) = \alpha r(n) + \beta^* r^*(n) \tag{9}$$

where α and β are parameters introduced by I/Q imbalance [7].

According to (9), the correlation function should be rewritten in (10) as

$$\begin{aligned} m_{2x}(n,m) &= E\{x(n)x^*(n-m)\} \\ &= |\alpha|^2 m_{2r}(n,m) + |\beta|^2 m_{2r}^*(n,m) \end{aligned} \tag{10}$$

And we also need to update the DFT as

$$M_{2x}(u,m) = |\alpha|^2 M_{2r}(u,m) + |\beta|^2 N_{2r}(u,m) \tag{11}$$

where

$$\begin{aligned} N_{2r}(u,m) &= \frac{1}{U}\sum_{n=0}^{U-1} m_{2r}^*(n,m)e^{-j\frac{2\pi}{U}un} \\ &= \frac{1}{U}e^{-j\frac{2\pi}{N}\varepsilon m}a(m)G_2(u,m)e^{-j\frac{2\pi}{U}u\tau} + \sigma_w^2\delta(u)\delta(m) \end{aligned} \tag{12}$$

The derivation in (12) employs the facts that $a^*(m) = a(m)$ and $g^*(n) = g(n)$. Consequently, we have the following results that

$$\begin{aligned} Q_{2x}(u,m) &= G_2^{-1}(u,m)a^{-1}(m)M_{2x}(u,m) \\ &= \frac{1}{U}[|\alpha|^2 e^{j\frac{2\pi}{N}\varepsilon m} + |\beta|^2 e^{-j\frac{2\pi}{N}\varepsilon m}]e^{-j\frac{2\pi}{U}u\tau} \\ &\quad + G_2^{-1}(u,m)a^{-1}(m)\sigma_w^2\delta(u)\delta(m) \end{aligned} \tag{13}$$

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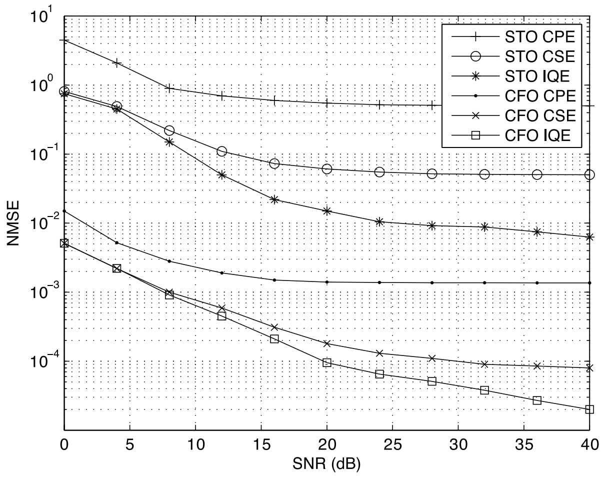

Fig. 1. NMSE of STO and CFO Estimation in Case A

$$\begin{aligned} S_{2x}(u,f) &= \sum_{m=-\infty}^{+\infty} Q_{2x}(u,m)e^{-j2\pi fm} \\ &= \frac{1}{U}e^{-j\frac{2\pi}{U}u\tau}\sum_{l=0}^{M-1}\left[|\alpha|^2 A\left(f-\frac{\varepsilon}{N}\right)+|\beta|^2 A\left(f+\frac{\varepsilon}{N}\right)\right] \\ &\quad + G_2^{-1}(u,0)a^{-1}(0)\sigma_w^2\delta(u) \end{aligned} \tag{14}$$

Thanks to the fact that $\alpha > \beta$, estimation of ε and τ can be obtained by (15) and (16).

$$\hat{\varepsilon} = sgn[S_{2x}(u,f) - S_{2x}(u,-f)]N\, \underset{f}{argmax}[|S_{2x}(u,f)| + |S_{2x}(u,-f)|] \tag{15}$$

where $sgn(.)$ indicates the sign function.

$$\hat{\tau} = -\frac{U}{2u\pi}arg\left[S_{2x}\left(u,\frac{\hat{\varepsilon}}{N}\right)+S_{2x}\left(u,-\frac{\hat{\varepsilon}}{N}\right)\right] \tag{16}$$

For implementation considerations, although we do not have access to ensemble cyclic quantities, we could estimate them by time domain averaging as shown in [6].

4 Simulations results and analysis

In this section, performance of proposed algorithm is compared with the ones in [2] and [3] by simulation. Common conditions of the simulations are selected as $B = 20\,MHz$, $N = 64$, $M = 52$, $N_{CP} = 16$, with QPSK modulation. The frequency-selective fading channel has 15 paths, with exponential power delay profile such that $|h_p|^2 = e^{-p/5}$ for $p = 0, \ldots, 15$. Two cases of STO, CFO and I/Q imbalance are considered in the simulations as Case A: $\tau = 0.1$, $\varepsilon = 0.1$, amplitude imbalance $(AI) = 1\,dB$, phase imbalance $(PI) = 10deg.$; Case B: $\tau = 0.3$, $\varepsilon = 0.3$, $AI = 2\,dB$, $PI = 10deg.$.

The curves with different markers in the figures represent the NMSE of STO and CFO estimation, defined as $E\{|\hat{\tau}-\tau|^2\}$ and $E\{|\hat{\varepsilon}-\varepsilon|^2\}$, achieved by the 3 different algorithms, respectively. SNR is difined as $1/\sigma_w^2$. 'CPE'

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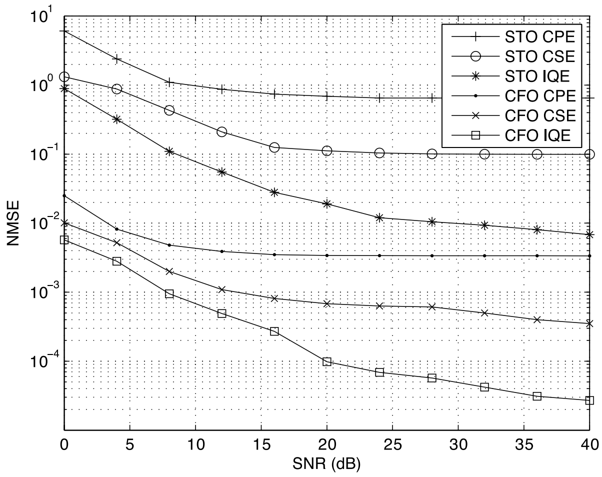

Fig. 2. NMSE of STO and CFO Estimation in Case B

represents the algorithm in [2], ‘CSE’ represents the algorithm in [3] and ‘IQE’ represents the algorithm proposed in this paper.

In Fig. 1, NMSE achieved by the three algorithms in case A is given to show the STO and CFO estimation performance in OFDM systems with relative small I/Q imbalance. CPE has larger estimation error than the other two due to the effect of multi-path dispersive channel. CSE and IQE have similar performance in low SNR area. In high SNR area, IQE outperform CSE because of its use of new metric which takes I/Q imbalance into accounts.

In Fig. 2, NMSE achieved by the algorithms in case B is given to show the STO and CFO estimation performance in OFDM systems with relative large I/Q imbalance. The performance comparison results of three algorithms are similar to those of Fig. 1. The main difference is that the performance difference between IQE and CSE become larger because of the relative large I/Q imbalance.

5 Conclusions

In this paper, we present a blind algorithm for joint estimation of STO and CFO for OFDM systems with I/Q imbalance. The proposed algorithm exploits the cyclostationarity of OFDM signals and relies on second-order statistics only. Simulation results show the proposed algorithm has superior performance to existing ones.

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