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Noisy FIR parameter estimation by combining of total least mean squares estimation and least mean squares estimation

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Abstract: The problem of FIR filtering with noisy input and output data can be solved by a total least squares (TLS) estimation. The performance of the TLS estimation is very sensitive to the ratio between the variances of the input and output noises. In this paper, we propose an iterative convex combination algorithm between TLS and least squares (LS). We combine two typical iterative algorithms, the total least mean square method (TLMS) and the least mean square method (LMS). TLMS is a typical iterative algorithm for TLS and LMS is a typical one for LS. This combined algorithm shows robustness against the noise variance ratio. Consequently, the practical workability of the TLS method with noisy data has been significantly broadened.
Keywords: noisy FIR system estimation, Total Least Squares, Least Squares, convex combination
Classification: Science and engineering for electronics

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1 Introduction

Noisy FIR (finite impulse response) filtering aims at estimating filter coefficients of FIR systems where all variables are observed in noise; not only the system output is corrupted by measurement noise, but also the measured input signal often may be corrupted by additive noise. The standard least-squares (LS) algorithm in noisy FIR filtering is well recognized as handling the case of corrupted output only. The case of corrupted input and output can be handled by a total least squares (TLS) method [1]. Strictly speaking, TLS is optimal for the case of noise with the same variance in both input and output. General noisy FIR filtering has different noise variance in input and output [1, 2]. Generalized TLS has been proposed to cope with general noisy FIR filtering [1, 2]. However, generalized TLS requires a priori information about the ratio of the variances of input and output noises. As knowing such information is not practical, we need a method without a priori information. Zheng proposed such an algorithm in [3]. This was a simple average of two different generalized TLS models. However, this algorithm still needs a rough boundary for the ratio of variances of the input and output noises.

In this paper, we propose a new method, requiring no a priori information about the variances ratio. This method adopts a kind of open boundary for the ratio by considering LS and TLS simultaneously. The proposed algorithm uses the convex combination described in [4]. In this work, we combine a result from LS with that from TLS for more improvement in the estimation accuracy than that from Zheng's algorithm. This new method alleviates the requirement for a priori knowledge of the variance ratio so that the practical workability of the TLS algorithm is significantly broadened.

2 The convex combination between TLS and LS

2.1 An adaptive least squares solution and adaptive total least squares solution

Given an unknown system with finite impulse response and assuming that both the input and output are corrupted by Gaussian white noise, the system can be estimated from a noisy observation of the input and output, as indicated in Fig. 1. The unknown system is described by

$$\mathbf{w} = [w_0, w_1, \cdots, w_{N-1}]^H \in \mathrm{C}^{N\times 1}, \tag{1}$$

where $\mathbf{w}$ may be time-varying or time-invariant. The output is given by

$$\tilde{d}(k) = \mathbf{x}^H(k)\mathbf{w} + n_{out}(n), \tag{2}$$

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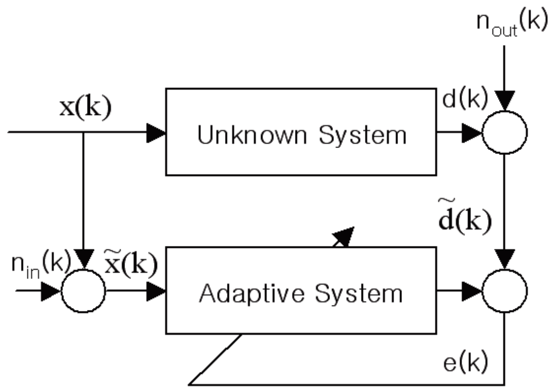

Fig. 1. The model of noisy FIR system

where the output noise $n_{out}(n)$ is a Gaussian white noise with variance σ_{out}^2 and is independent of the input signal. The noise free input vector is represented as

$$\mathbf{x}(k) = [x(k), x(k-1), \cdots, x(k-N+1)]^T. \tag{3}$$

The output error signal with time index k is

$$e_{LS}(k) = \tilde{d}(k) - \mathbf{w}_{LS}^T(k)\mathbf{x}(k) = \tilde{d}(k) - y_{LS}(k) \tag{4}$$

The LS solutions for the $\mathbf{w}_{LS}$ in Eq. (4) can be obtained by solving the optimization problem

$$\min E\{e_{LS}^2(k)\}. \tag{5}$$

In the LMS algorithm [5], Widrow has taken the squared-error, itself, as an estimation of $E\{e_{LS}^2(k)\}$. Then, at each iteration in the adaptive process, we have a simple gradient estimation algorithm.

$$y_{LS}(k) = \mathbf{w}_{LS}^T(k)\mathbf{x}(k) \tag{6}$$

$$\mathbf{w}_{LS}(k+1) = \mathbf{w}_{LS}(k) + \mu_{LS}(\tilde{d}(k) - y_{LS}(k))\mathbf{x}(k) \tag{7}$$

In addition to output noise, when a system has a noisy input vector given by

$$\tilde{\mathbf{x}}(k) = \mathbf{x}(k) + \mathbf{n}_{in}(k) \in C^{N\times 1}, \tag{8}$$

where the input noise vector $\mathbf{n}_{in}(k) = [n_{in}(k), n_{in}(k-1), \cdots, n_{in}(k-N+1)]^T$ and the element of the vector $n_{in}(k)$ is Gaussian white noise with variance σ_{in}^2, the total least squares estimation (TLS) provides an optimal solution [1]. Feng *et al.* proposed an iterative style solution for the TLS problem, the total least mean squares algorithm (TLMS) [6, 7]. This algorithm looks like the LMS algorithm in Eq. (6) and Eq. (7) with a complexity of $O(N)$ [6, 7]. The TLMS algorithm is derived by minimizing the following cost function,

$$\min E\{\tilde{\mathbf{w}}^H\overline{\mathbf{R}}\tilde{\mathbf{w}}\}, \quad \|\tilde{\mathbf{w}}\|_2 = \alpha \text{ and } \mathbf{w}_{TLS} = \tilde{\mathbf{w}}(1:N)/(-\tilde{\mathbf{w}}(N+1)), \tag{9}$$

where $\overline{\mathbf{x}}(n) = [\tilde{\mathbf{x}}^T(k), d(k)]^T \in C^{N+1\times 1}$ and $\bar{\mathbf{R}} = E\{\bar{\mathbf{x}}(k)\bar{\mathbf{x}}^H(k)\}$. The TLMS algorithm from [6] is

$$y_{TLMS}(k) = \mathbf{w}_{TLMS}^T(k)\tilde{\mathbf{x}}(k), \tag{10}$$

$$\tilde{\mathbf{w}}(k+1) = \tilde{\mathbf{w}}(k) + \mu_{TLMS}(\tilde{\mathbf{w}}(k) - \|\tilde{\mathbf{w}}(k)\|_2^2\, y_{TLMS}(k)\bar{\mathbf{x}}(k)), \tag{11}$$

$$\mathbf{w}_{TLMS}(k+1) = \tilde{\mathbf{w}}_{(1:N)}(k+1)/(-\tilde{\mathbf{w}}_{(N+1)}(k+1)). \tag{12}$$

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Although the TLS method is known to handle a noisy input and output system, strictly speaking, TLS gives an optimal solution when the ratio of the noise variances is one, $\gamma = \frac{\sigma_{out}^2}{\sigma_{in}^2} = 1$. In other cases, the estimation performance deteriorates [8, 9].

2.2 An adaptive convex combination of TLMS and LMS

In practical applications of TLS, there is uncertainty in the ratio between input noise variance and output noise variance, $\gamma = \frac{\sigma_{out}^2}{\sigma_{in}^2}$. Zheng proposed an average method between the TLS parameters from the two different ratios in [3]. However, this method still needs a boundary for the noise variance ratio. We propose an adaptive convex combination method between TLS and LS. The convex combination scheme has been used for LMS filters by Garcia *et al.* in [4]. We expect to improve the estimation performance in uncertain noise environments by applying a combination between TLS and LS. Generally, we know that TLS provides an optimal solution when $\gamma = \frac{\sigma_{out}^2}{\sigma_{in}^2} = 1$ and that LS gives an optimal solution when $\gamma = \frac{\sigma_{out}^2}{\sigma_{in}^2} = \infty$. Then, we expect this combination to widen the boundary of the noise variance ratio, thereby not needing a guess for the ratio of the input noise variance and the output noise variance.

The combination method uses a convex combination between the parameter of the TLS estimators and that of LS estimation.

$$\mathbf{w}_{comb}(k) = v(k)\mathbf{w}_{TLMS}(k) + (1 - v(k))\mathbf{w}_{LS}(k), \tag{13}$$

where the $v(k)$ is kept between the interval $(0, 1)$ by defining it as $v(k) = 1/(1 + e^{-a(k)})$. Using this expression, the error of the combined method can be calculated in the form of

$$e_{comb}(k) = v(k)e_{TLMS}(k) + (1 - v(k))e_{LS}(k), \tag{14}$$

where $e_{TLMS}(k) = \tilde{d}(k) - \mathbf{w}_{TLMS}^T(k)\tilde{\mathbf{x}}(k) = \tilde{d}(k) - y_{TLMS}(k)$ and $e_{LS}(k) = \tilde{d}(k) - \mathbf{w}_{LS}^T(k)\tilde{\mathbf{x}}(k) = \tilde{d}(k) - y_{LS}(k)$. $\tilde{d}(k)$ is a noisy desired output. The combination coefficient $v(k)$ is controlled by $a(k)$. This parameter is adapted to minimize the combined error using the LSM adaptation rule.

$$\begin{array}{l} a(k+1) = a(k) - \frac{\mu_a}{2}\frac{\partial e_{comb}^2(k)}{\partial a(k)} \\ = a(k) + \mu e_{comb}(k)(y_{TLMS}(k) - y_{LS}(k))v(k)(1 - v(k)) \end{array}, \tag{15}$$

because $\frac{\partial e_{comb}^2(k)}{\partial a(k)} = 2e_{comb}\frac{\partial v(k)}{\partial a(k)}(e_{TLMS}(k) - e_{LS}(k))$ and $e_{TLMS}(k) - e_{LS}(k) = y_{LS}(k) - y_{TLMS}(k)$.

3 Simulations and results

We performed computer simulations to obtain an experimental evaluation of the proposed algorithm. The example considered in this simulation is that of identifying the unknown FIR system with the following coefficients

$$\mathbf{w}_{\text{true}} = [-0.3, -0.9, 0.8, -0.7, 0.6]^T. \tag{16}$$

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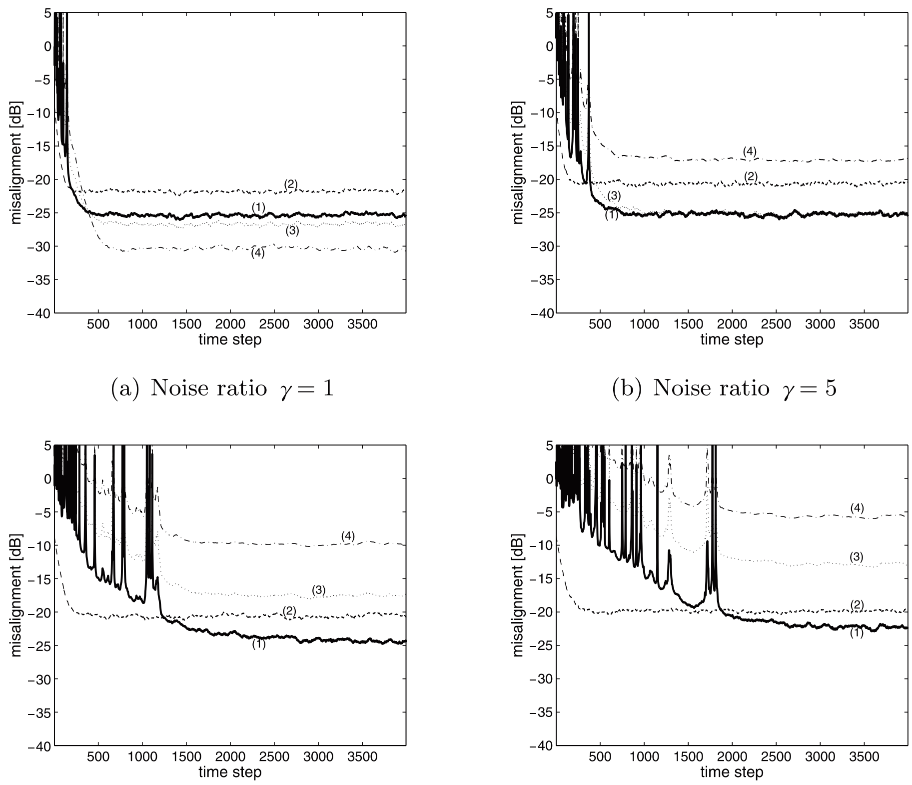
The true input signal $x(t)$ is white Gaussian noise with zero mean and unit variance. A study of this example was conducted in [8, 9].

The overall estimation is measured by the misalignment defined as

$$\rho = 10 \log_{10} \frac{|\mathbf{w}_{\text{true}} - \mathbf{w}_{\text{estim}}|^2}{|\mathbf{w}_{\text{true}}|^2}. \tag{17}$$

The experiment initially starts with input noise $n_{in}(t)$ and output noise $n_{out}(t)$, having variances of $\sigma_{in}^2 = 0.1$ and $\sigma_{out}^2 = 0.1$. These noise variances make this experiment a typical TLS problem. We set the input signal power to as much as necessary so that the signal-to-noise ratio (SNR) at the input becomes $\text{SNR}_{in} = 5\,\text{dB}$. Fig. 2 shows the convergence curves of 4 different algorithms. Fig. 2 illustrates the results for 4 different noise ratios, $\gamma = [1, 5, 10, 15]$, $\gamma = \frac{\sigma_{out}^2}{\sigma_{in}^2}$. Fig. 3 also illustrates misalignment, ρ, with respect to the noise ratio, γ, from the proposed algorithm, the TLMS algorithm in [6], Zheng's algorithm in [3] and the LMS algorithm. The curves are the results from 5 different noise ratios, $\gamma = [1, 5, 10, 15, 20]$, with a fixed σ_{in}^2. In the typical TLS case, where $\gamma = 1$, the TLMS is much better than the other algorithms. As the output noise gets greater, the proposed algorithm shows its superiority. Zheng's algorithm in [3], which uses an elementary combination method, also degrades when $\gamma > 5$. The mean impulse response estimates of four different methods are shown in Table I, which also demon-



(a) Noise ratio $\gamma = 1$ (b) Noise ratio $\gamma = 5$

(c) Noise ratio $\gamma = 10$ (d) Noise ratio $\gamma = 15$

Fig. 2. The misalignment convergence curves comparison ((1): the proposed method, (2): LMS, dash, (3): Zheng's method [3], (4): TLMS [6])

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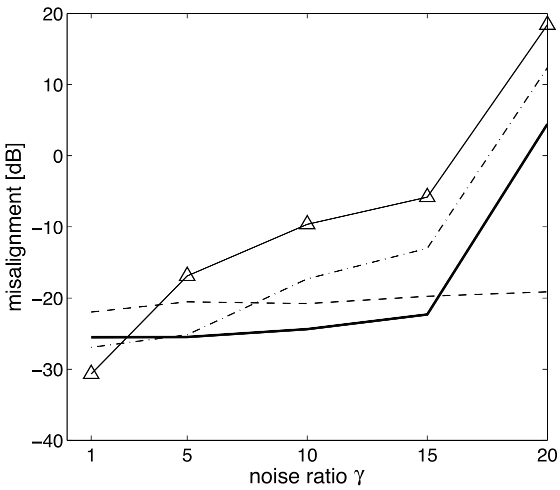

Fig. 3. The estimation error against the ratio between input and output noise variances (solid line: the proposed method, dashed line: LMS, dash and dotted line: Zheng's method [3], solid line with triangles: TLMS [6])

Table I. Mean impulse response estimates of different methods

Filter Coefficient		w_1	w_2	w_3	w_4	w_5	$\rho_{av}^{2)}$ [dB]
Actual value		-0.3	-0.9	0.8	-0.7	0.6	
$\gamma^{1)} = 1$	Proposed algorithm	-0.258	-0.771	0.680	-0.591	0.508	-25.5
	LMS	-0.232	-0.691	0.607	-0.527	0.457	-22.0
	TLMS [6]	-0.299	-0.903	0.798	-0.701	0.594	-30.7
	Zheng's [3]	-0.266	-0.797	0.703	-0.614	0.525	-27. 0
$\gamma = 5$	Proposed algorithm	-0.277	-0.880	0.759	-0.680	0.590	-25.4
	LMS	-0.207	-0.685	0.586	-0.527	0.461	-20.5
	TLMS [6]	-0.424	-1.284	1.126	-1.006	0.855	-16.9
	Zheng's [3]	-0.316	-0.984	0.856	-0.767	0.658	-25.2
$\gamma = 10$	Proposed algorithm	-0.323	-0.967	0.843	-0.746	0.613	-24.4
	LMS	-0.244	-0.700	0.611	-0.541	0.438	-20.8
	TLMS [6]	-0.578	-1.848	1.614	-1.429	1.190	-9.6
	Zheng's [3]	-0.411	-1.274	1.113	-0.985	0.814	-17.3
$\gamma = 15$	Proposed algorithm	-0.312	-1.018	0.903	-0.786	0.674	-22.3
	LMS	-0.203	-0.685	0.608	-0.523	0.457	-19.8
	TLMS [6]	-0.748	-2.350	2.080	-1.840	1.541	-5.8
	Zheng's [3]	-0.475	-1.517	1.344	-1.181	0.999	-13.0

1) γ means the ratio between output noise variance and input noise variance.

2) ρ_{av} mean the average misalignment.

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strates the proposed algorithm significantly broadens the practical workable region of the TLS algorithm.

4 Conclusion

We reported on our proposed algorithm to the problem of noisy FIR filtering by means of a combination involving TLS and LMS. The focus was set on the sensitivity with little tolerance in the TLS method with respect to the ratio between the variances of the input and output noises. The proposed combination algorithm has greatly widened the range of the noise variances ratio. Therefore, the practical workability of the TLS method with noisy data has been significantly broadened.

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