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Dynamical mechanism for interrupted circuit with switching delay

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Abstract: The unavoidable nonidealities with switching delay in current-mode-controlled buck converters have been reported in the literature. Investigations are carried out on the dynamical mechanism and its experimental validation on an interrupted circuit with switching delay. Switching delay is seen to influence a region of a two-valued function on a discrete map, and to induce the coexistence of a periodic orbit.

Keywords: switching delay, interrupted electric circuit, return map, dynamical effect

Classification: Science and engineering for electronics

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1 Introduction

Nonlinear phenomena of power electronic systems have been extensively studied over the past few decades and have been applied in a variety of areas, such as control design of circuits and the improvement of a power supply EMC with chaos [1, 2, 3, 4, 5, 6]. On the other hand, the previous results are based on the assumption that any switching circuit theoretically works. In order to confirm the viability of the dynamical effects of missed switching, Banerjee *et al.* [7] showed the experimental results and sampled-data models under a simple assumption of a current-mode-controlled dc-dc converter with switching delay and high-frequency ripple. They found that the bifurcation behavior is quite different from that predicted under the assumption of ideal switching. This means that it is important topic to clarify the fundamental property of the missed switching in nonlinear dynamical systems because the switches in the real circuit do not often work theoretically. However, the accurate consideration about the dynamical effects of missed switching is insufficient because the bifurcation analysis of a two-dimensional Poincaré map is necessary to consider the qualitative property of the interrupted circuit with switching delay.

In this paper, we study the dynamical mechanism of the interrupted circuit with switching delay theoretically and experimentally in an attempt to clarify this point using two-dimensional Poincaré map. The test circuit is the simplest case of piecewise-smooth systems [8]. Therefore any current-mode DC-DC converters under switching delay should be based on the same dynamical mechanism.

2 Experimental setup

We propose a simple interrupted circuit shown in Fig. 1. Suppose that the orbit starts with v_k at $t = kT$ in switch A. Note that $\tau_{\rm d}$ shown in Fig. 2 (a) is the time delay of switching from switch A to switch B. When the orbit intersects the reference value $v_{\rm r}$ at time $kT + \tau_a - \tau_{\rm d}$, it changes to switch

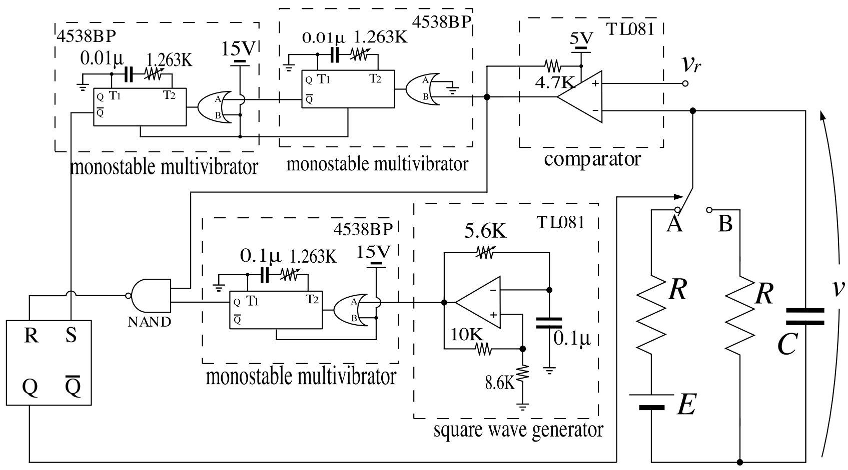

Fig. 1. Electrical scheme of the interrupted circuit with switching delay.

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B at time $kT + \tau_a$. Any clock pulse before the intersection is ignored. The orbit obeys switch B until the next clock pulse arrives. The time interval of system B is

$$\tau_b = T\left[1 - \left(\frac{\tau_a}{T} \bmod 1\right)\right]. \quad (1)$$

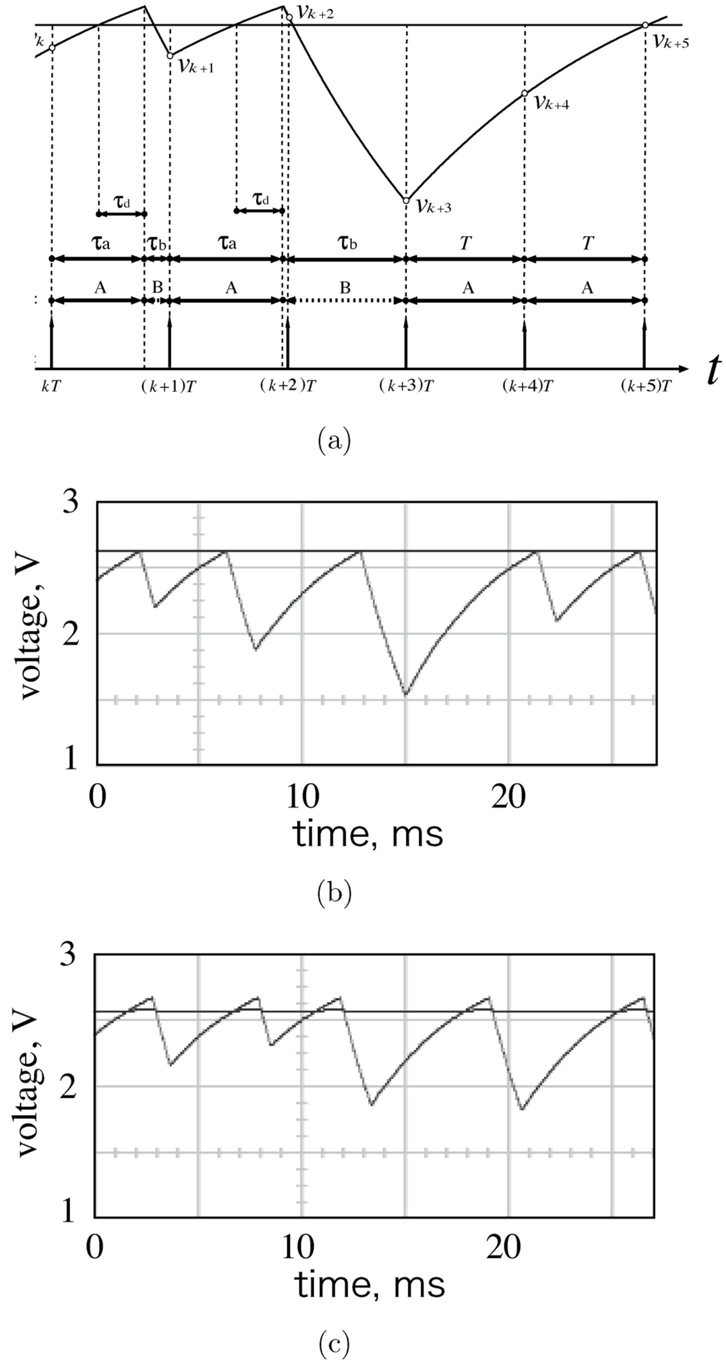

Fig. 2. Behavior of capacitance voltage v. (a) Illustrated example. (b) Measured waveform in the ideal switching case at $\tau_d = 0.0$ ms. (c) Measured waveform in the nonideal switching case at $\tau_d = 0.8$ ms.

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After that, the orbit obeys system A again. We choose the values for parameters in Fig. 1 as

$$E = 3.0 \text{ V}, \ R = 40 \text{ k}\Omega, \ C = 0.1\,\mu\text{F}, \ T = 2.4 \text{ ms}. \tag{2}$$

Figure 2 (b)(c) show measured waveforms in the ideal and nonideal switching cases.

3 Poincaré map

We derive the Poincaré map for analyzing the qualitative properties of the interrupted circuit with switching delay. The borders of the Poincaré map are given by two initial conditions,

$$D_1 = v_{\mathrm{r}}, \ D_2 = (v_{\mathrm{r}} - E)e^{\frac{1}{RC}(T-\tau_{\mathrm{d}})} + E, \tag{3}$$

where $D_2 < D_1$. We define the following three mappings. In $v_k \leq D_2$, the orbit starting from the initial value v_k with $t = kT$ intersects the reference value v_{r} beyond time $t = (k+1)T$. In $D_1 \leq v_k$ and $D_2 \leq v_{k-1}$, the orbit obeys the dynamics with switch B. In $D_2 < v_k < D_1$ or $(D_1 \leq v_k$ and $v_{k-1} < D_2)$, time τ_a at which orbit obeys switch A is obtained as

$$\tau_a = RC \log \frac{v_k - E}{v_{\mathrm{r}} - E} + \tau_{\mathrm{d}}. \tag{4}$$

As a result, the voltage at time $t = (k+1)T$ of each region is governed by the following equation.

$$v_{k+1} = F(v_k) = \begin{cases} (v_k - E)e^{-\frac{1}{RC}T} + E, & v_k \leq D_2 \\ \dfrac{v_k - E}{v_{\mathrm{r}} - E}(v_{\mathrm{r}} - E + Ee^{\tau_{\mathrm{d}}})\, e^{-\frac{1}{RC}T}, & \\ \qquad (D_2 < v_k < D_1) \quad \text{or} \quad (D_1 \leq v_k \text{ and } v_{k-1} < D_2) & \\ v_k e^{-\frac{1}{RC}T}, \quad D_1 \leq v_k \text{ and } D_2 \leq v_{k-1} & \end{cases} \tag{5}$$

Note that the value of v_{k+1} depends on v_k and v_{k-1}. This means that the behavior of Fig. 1 is described by the two-dimensional Poincaré map. However, v_{k-1} can be used only to show the condition of Eq. (5).

4 Dynamical effect of switching delay

Figure 3 (a) shows an example of the experimental Poincaré map with $\tau_{\mathrm{d}} = 0.0$. The coexisting attractor is never seen under the ideal switching case because the Poincaré map is described by a single-valued function and the characteristic multiplier of the Poincaré map is constant. Figure 3 (b) shows an example of the Poincaré map with $\tau_{\mathrm{d}} = 0.8$ ms. Note that a region of a two-valued function can be observed in $D_1 \leq v_k$. This result illustrates the possibility of the coexistence of the periodic or the chaotic attractor. In addition, the invariant interval $F(I) \subseteq I$ can be analytically obtained as

$$I = \left[v_{\mathrm{r}}e^{-T}, (v_{\mathrm{r}} - E)e^{-\frac{1}{RC}\tau_{\mathrm{d}}} + E\right]. \tag{6}$$

Therefore when the unavoidable nonlinearities are caused by switching delay, the periodic orbit can observe a large voltage range compared with the ideal interrupted circuit.

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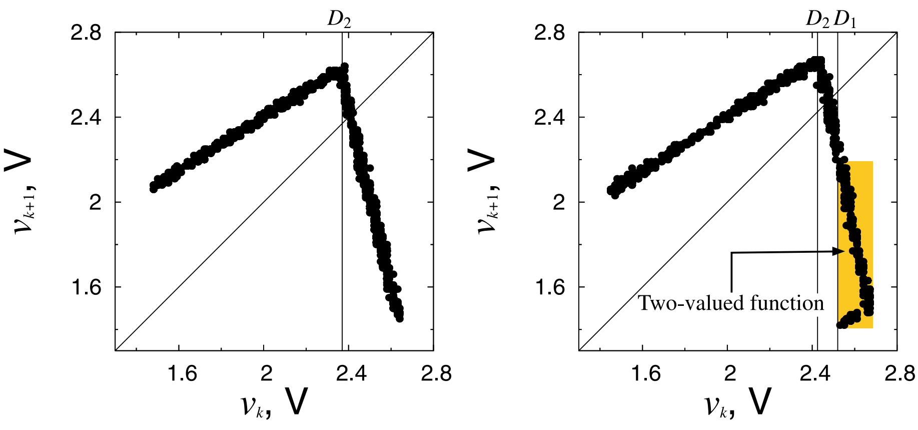

(a) (b)

Fig. 3. Measured Poincaré maps in the ideal and nonideal switching cases. (a) Ideal switching ($\tau_\mathrm{d} = 0.0\,\mathrm{ms}$). (b) Switching delay ($\tau_\mathrm{d} = 0.8\,\mathrm{ms}$).

5 Conclusion

New theoretical and experimental results concerning the dynamical effects of an interrupted circuit with switching delay have been obtained. The measured Poincaré map has provided strong support of our results. This study revealed that the behavior of the interrupted circuit with switching delay is quite different from that predicted under the assumption of ideal switching because the Poincaré map is described by a multivalued function.

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