

# Spectral efficient cooperative transmission for half-duplex relay communications

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**Abstract:** A new transmission protocol using MIMO (Multiple Input Multiple Output) half-duplex relays is proposed to improve the capacity and the diversity gain of uplink relay communication systems. The proposed protocol employs two half-duplex DF (Decode-and-Forward) relays and Alamouti coded  $2 \times 2$  MIMO communication for links between each relay and the destination. A low complexity receiver using maximum ratio combining and Alamouti decoding is introduced to detect the signal of the proposed protocol. Performance is evaluated by theoretical analysis and computer simulations, and the results show that the proposed protocol outperforms conventional IEEE 802.16j systems in terms of ergodic capacity and outage probability.

**Keywords:** MIMO communication, half-duplex relay, alamouti code **Classification:** Science and engineering for electronics

#### References

- R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.
- [2] IEEE Draft Standard P802.16j/D3, "Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems. Multihop Relay Specification," 2008.
- [3] T. J. Oechtering and A. Sezgin, "A new cooperative transmission scheme using the space-time delay code," *Proc. ITG Workshop on Smart Antennas*, pp. 41–48, March 2004.
- [4] A. Ribeiro, X. Cai, and G. B. Ginanakis, "Opportunistic multipath for bandwidth-efficient cooperative networking," *Proc. IEEE ICASSP*, pp. 549–552, May 2004.
- [5] Y. Fan, C. Wang, J. S. Thompson, and H. V. Poor, "Recovering Multiplexing Loss through Successive Relaying Using Repetition Coding," *IEEE Trans. Wireless Commun.*, vol. 6, no. 22, pp. 4484–4493, Dec. 2007.





### 1 Introduction

The importance of relay communication has arisen especially for the uplink communication where a single antenna is preferred for the size and cost effectiveness of the transmitter. In the relay communication, diversity gain can be increased by utilizing relays as additional transmit antennas. However, the capacity is decreased by additional time resources that half-duplex relays require [1, 2]. There have been some researches to overcome this capacity loss of the half-duplex relay. A transmission scheme with two half-duplex AF (Amplify-and-Forward) relays that alternately forward messages to a destination was studied in [3] but it required high complexity maximum likelihood receiver. To reduce this complexity, the source and two relays used three different orthogonal codes in [4] but spectral efficiency was also reduced due to the spreading. The successive relaying studied in [5] guaranteed capacity improvement by means of a half-duplex relay equipped with two antennas but it required a feedback interference canceler, which is hard to be implemented. In summary, the previous researches have mitigated the capacity loss at the expense of bandwidth efficiency and receiver complexity.

In this letter, we propose a bandwidth efficient transmission protocol and a low complexity receiver alternating two half-duplex DF relays with MIMO communication. We show that the proposed protocol outperforms the IEEE 802.16j schemes in terms of the ergodic capacity and the outage probability.

# 2 System Model

We consider a relay system consisting of a source S, a destination D and two relays R1 and R2. It is assumed that the source is equipped with a single antenna because the uplink communication is considered. It is also assumed that the relays and the destination employ two antennas each so that they can enhance the reliability of the communication. The relays operate in a half-duplex way and assist the communication by decode-and-forwarding data received from the source. Note that the inter-relay inference between R1 and R2 is assumed to be canceled perfectly because it can be eliminated by the same receiver algorithm as described in Section 3.2.

## **3** Transmission Protocol and Receiver Design

## 3.1 Transmission Protocol

The transmission protocol of the proposed relay communication is described in Fig. 1 where  $(\cdot)^*$  is the complex conjugate operator. Two relays operate in two kinds of mode; a transmission mode and a reception mode. They change the mode every two time slots and transmit signals in turn. Let  $x_k$ denote a signal transmitted at a time slot k and N denote the number of signals in one frame. In the first two time slots, only R1 receives  $x_1$  and  $x_2$ from the source in the reception mode and R2 sleeps. In the next two time slots, R1 forwards  $x_1$  and  $x_2$  to the destination in the transmission mode and in the mean time R2 receives the new signals  $x_3$  and  $x_4$  from the source in the reception mode. In the same manner, the two relays  $R_1$  and  $R_2$  change their





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Path	Τ1	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	Τ <sub>5</sub>	T <sub>6</sub>	 $T_{N+1}$	T <sub>N+2</sub>
S→D	<b>x</b> <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x4	x <sub>5</sub>	x <sub>6</sub>	 -	-
S→R1	<b>x</b> <sub>1</sub>	x <sub>2</sub>	-	-	x5	x <sub>6</sub>	 -	-
S→R2	-	-	x <sub>3</sub>	x4	-	-	 -	-
R1→D	-	-	$\mathbf{x}_1$	-x*2	-	-	x <sub>N-1</sub>	-x* <sub>N</sub>
			x <sub>2</sub>	x*1			 $\mathbf{x}_{\mathbf{N}}$	x* <sub>N-1</sub>
R2→D	-	-		-	x <sub>3</sub>	-x*4	-	-
					$\mathbf{x}_4$	x*3		

Fig. 1. The transmission protocol of the proposed relay communication.

modes every two time slots so that the source can transmit a new signal continuously. Therefore, the capacity loss of the half-duplex relay can be neglected for a large frame length N. We also consider Alamouti coded  $2 \times 2$  MIMO communication between the relays and the destination to simplify the receiver design as well as to improve the diversity gain.

## 3.2 Receiver Design

In the following, we introduce the receiver algorithm. Let  $\mathbf{y}_k$  and  $\mathbf{x}_k$  denote a received signal vector and a transmit signal vector at a time slot k. In addition, let  $\mathbf{H}_k$  and  $\mathbf{n}_k$  denote a channel matrix and a complex additive white Gaussian noise vector  $\sim \mathcal{CN}(0, N_0)$  at the time slot k, respectively. For the odd number k,  $\mathbf{y}_k$  and  $\mathbf{y}_{k+1}$  can be expressed as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k, \quad \mathbf{y}_{k+1} = \mathbf{H}_{k+1} \mathbf{x}_{k+1} + \mathbf{n}_{k+1}$$
(1)

where  $\mathbf{y}_k = [y_{1,k} \ y_{2,k}]^T$  and  $\mathbf{n}_k = [n_{1,k} \ n_{2,k}]^T$  are vectors with two elements for each receive antenna,  $\mathbf{x}_k = [x_{k-2} \ x_{k-1} \ x_k]^T$ ,  $\mathbf{x}_{k+1} = [-x_{k-1}^* \ x_{k-2}^* \ x_{k+1}]^T$ and  $(\cdot)^T$  is the transpose operator. The channel matrix  $\mathbf{H}_k$  can be expressed as  $\mathbf{H}_k = \begin{bmatrix} h_{R_1D_1,k} \ h_{R_2D_1,k} \ h_{SD_1,k} \\ h_{R_1D_2,k} \ h_{R_2D_2,k} \ h_{SD_2,k} \end{bmatrix}$  where  $h_{R_iD_j,k}$  denotes a random complex-valued channel gain with a unit variance between the  $i^{th}$  antenna of the relay and the  $j^{th}$  antenna of the destination at the time slot k. In the same manner,  $h_{SD_j,k}$  represents a random complex-valued channel gain with a unit variance between the source and the  $j^{th}$  antenna of the destination at the time slot k. From (1), we define modified received signals  $y'_k$ and  $y'_{k+1}$  as

$$y'_{k} = h'_{1,k}x_{k-2} + h'_{2,k}x_{k-1} + n'_{k}, \ y'_{k+1} = h'^{*}_{2,k+1}x_{k-2} - h'^{*}_{1,k+1}x_{k-1} + n'_{k+1}$$
(2)

where  $h'_{1,k} = h_{SD_2,k}h_{R_1D_1,k} - h_{SD_1,k}h_{R_1D_2,k}$  and  $h'_{2,k} = h_{SD_2,k}h_{R_2D_1,k} - h_{SD_1,k}h_{R_2D_2,k}$  and  $n'_k = h_{SD_2,k}n_{1,k} - h_{SD_1,k}n_{2,k}$ . Since we assume a quasi static channel condition in the time domain, the channels can be approximated to  $h'_{1,k} \approx h'_{1,k+1}$ ,  $h'_{2,k} \approx h'_{2,k+1}$  and then  $y'_k$  and  $y'_{k+1}$  can be transformed to

$$\begin{bmatrix} y'_k \\ y'_{k+1} \end{bmatrix} = \begin{bmatrix} h'_{1,k} & h'_{2,k} \\ h'^*_{2,k} & -h'^*_{1,k} \end{bmatrix} \begin{bmatrix} x_{k-2} \\ x_{k-1} \end{bmatrix} + \begin{bmatrix} n'_k \\ n'_{k+1} \end{bmatrix}.$$
 (3)





From (3), the estimated transmit signals  $\hat{x}_{R,k-2}$  and  $\hat{x}_{R,k-1}$  from the relay can be obtained by using Almouti decoding process. We can derive  $\hat{x}_{R,k}$  and  $\hat{x}_{R,k+1}$  in the same way that we used for  $\hat{x}_{R,k-2}$  and  $\hat{x}_{R,k-1}$ . In the following, we obtain the estimated transmit signals  $\hat{x}_{S,k}$  and  $\hat{x}_{S,k+1}$  from the sources. First, we define  $\mathbf{r}_k$  and  $\mathbf{r}_{k+1}$  as

$$\mathbf{r}_k = \mathbf{y}_k - \mathbf{H}_{R,k} \hat{\mathbf{x}}_{R,k} = \mathbf{h}_{SD,k} \ x_k + \mathbf{n}_k \tag{4}$$

$$\mathbf{r}_{k+1} = \mathbf{y}_{k+1} - \mathbf{H}_{R,k+1} \hat{\mathbf{x}}_{R,k}^* = \mathbf{h}_{SD,k+1} \ x_{k+1} + \mathbf{n}_{k+1}$$
(5)

where 
$$\mathbf{r}_{k} = [r_{1,k} r_{2,k}]^{T}$$
,  $\mathbf{h}_{SD,k} = [h_{SD_{1},k} h_{SD_{2},k}]^{T}$ ,  $\mathbf{H}_{R,k} = \begin{bmatrix} h_{R_{1}D_{1},k} & h_{R_{2}D_{1},k} \\ h_{R_{1}D_{2},k} & h_{R_{2}D_{2},k} \end{bmatrix}$ 

and  $\hat{\mathbf{x}}_{R,k} = [\hat{x}_{R,k-2} \ \hat{x}_{R,k-1}]^T$ . From  $\mathbf{r}_k$  and  $\mathbf{r}_{k+1}$ , we can get  $\hat{x}_{S,k}$  and  $\hat{x}_{S,k+1}$ . Finally, the estimated transmit signal  $\hat{x}_k$  can be obtained by using maximum ratio combining (MRC) of  $\hat{x}_{R,k}$  and  $\hat{x}_{S,k}$ , and  $\hat{x}_{k+1}$  can be derived in the same manner. Finally, the transmitted signals  $x_k$  and  $x_{k+1}$  are decoded from  $\hat{x}_k$  and  $\hat{x}_{k+1}$ , respectively.

#### 4 Performance Analysis

It is assumed that the capacity is dominated by the source-destination link and the relay-destination link because the source-relay link is in a good channel condition. It is also assumed that the destination has the perfect knowledge of all the channels from the source and the relays. The effective channel matrix  $\mathbf{H}_e$  can be written as

$$\mathbf{H}_{e} = \begin{bmatrix} h_{SD_{1}} & h_{SD_{2}} & 0 & 0 & h'_{1} & h'_{2} \\ 0 & 0 & h_{SD_{1}} & h_{SD_{2}} & h'_{2} & -h'_{1}^{*} \end{bmatrix}^{T}.$$
 (6)

Using  $\mathbf{H}_e$ , the ergodic capacity  $C_p$  of the proposed protocol can be expressed as

$$C_p = \mathcal{E}_{\mathbf{H}_e} \left[ \frac{1}{2} \log_2 \det \left( \mathbf{I}_2 + \gamma \mathbf{H}_e^H \mathbf{H}_e \right) \right]$$
(7)

where  $\mathbf{E}_{\mathbf{H}_e}[\cdot]$  denotes the expectation operator with respect to  $\mathbf{H}_e$ ,  $[\cdot]^H$ is the Hermitian transpose operator,  $\mathbf{I}_2$  is the 2 × 2 identity matrix and  $\gamma$ represents the signal power per transmit antenna to noise power ratio (SNR). To make the analysis simple, it is assumed that  $\gamma = \gamma_{SD_j} = \gamma_{R_iD_j}$  where  $\gamma_{SD_j}$  and  $\gamma_{R_iD_j}$  are the SNRs of the source-destination link and the relaydestination link, respectively and *i* and *j* represent the indexes of the transmit antenna and the receive antenna, respectively. In the following, we derive the outage probability  $P_p^{out}$  of the proposed protocol to analyze its diversity performance.  $P_p^{out}$  can be modeled as

$$P_p^{out} = \int_0^m p_u du \approx 1 - e^{-m} \left( 1 + m + \frac{1}{2}m^2 + \frac{1}{6}m^3 \right) \tag{8}$$

where  $u = \sum_{i=1}^{2} (|h_{I,SD_i}|^2 + |h_{Q,SD_i}|^2 + |h'_{I,i}|^2 + |h'_{Q,i}|^2)$ ,  $p_u$  is the probability density function of  $u, m = \frac{2^Q - 1}{\gamma}$  and Q is the threshold value of the required capacity.  $h_{I,SD_i}$  and  $h_{Q,SD_i}$  denote the real part and the imaginary part of  $h_{SD_i}$ , respectively, and  $h'_{I,i}$  and  $h'_{Q,i}$  are the real part and the imaginary



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part of  $h'_i$ . Assuming that u follows *Chi-square distribution*, its probability density function can be approximated to  $p_u \approx \frac{1}{6} u^3 e^{-u}$ . Using the power series expression of  $e^{-m}$ ,  $P_p^{out}$  can be simplified to

$$P_p^{out} \approx \frac{1}{12}m^4 - \frac{1}{36}m^6 \propto \frac{(2^H - 1)^4}{12}(\frac{1}{\gamma})^4.$$
 (9)

This result shows that the outage probability is proportional to  $(1/\gamma)^4$  in high SNR regions, which demonstrates that the fourth-order diversity can be achieved by the proposed protocol.

### **5** Simulation results

Independently distributed Rayleigh fading channels were assumed for the source-destination link and the relay-destination link whilst the source-relay link was assumed to be error free. The performances of IEEE 802.16j hybrid type and non-hybrid type systems [2] are analyzed and compared to those of the proposed protocol in SNR conditions of  $\gamma_{R_iD_j} = \gamma_{SD_j}$ ,  $\gamma_{R_iD_j} = \gamma_{SD_j} + 1 \, \text{dB}$  and  $\gamma_{R_iD_j} = \gamma_{SD_j} + 3 \, \text{dB}$ . The hybrid type utilizes the source-destination link whilst the non-hybrid type does not. For a fair comparison, all the systems exploit two antennas at the destination and the same aggregate transmit power. Fig. 2 shows the ergodic capacities as a function of SNR in which the proposed protocol has maximum 75 percentage improvement in high SNR regions compared to the IEEE 802.16j hybrid type. Fig. 3 depicts the outage probability when  $Q = 1.5 \, \text{bps/Hz}$ . The analytical result is also plotted and compared to the numerical result. It is observed that the proposed protocol achieves similar diversity order to its competitors and outperforms them by maximum 2 dB at the outage probability  $= 1 \times 10^{-3}$ .

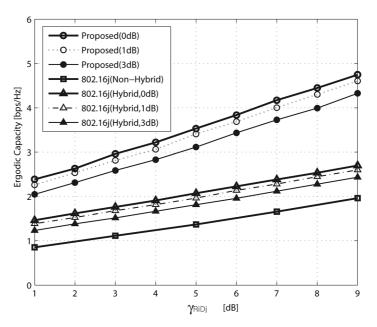
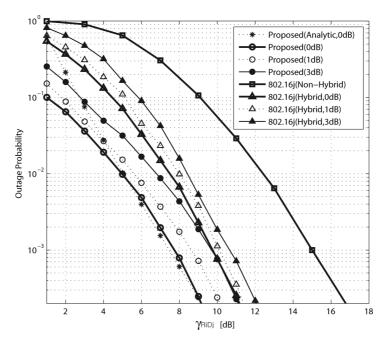


Fig. 2. The ergodic capacities of the proposed protocol and the conventional protocols.







**Fig. 3.** The outage probabilities of the proposed protocol and the conventional protocols.

# 6 Conclusions

In this letter, a new transmission protocol with MIMO half-duplex DF relays and a low complexity receiver was proposed to improve the capacity and the diversity gain of the uplink relay communication systems. The performance was evaluated in terms of the ergordic capacity and the outage probability, and the results showed that the proposed protocol outperformed the conventional IEEE 802.16j systems in Rayleigh fading environments.

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