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Performance analysis of coded MIMO OFDM in multipath fading channel

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Abstract: The average bit BER performance of coded MIMO OFDM with maximum likelihood detection in multipath Nakagami-Rice fading channels is analyzed. The upper bound of the BER probability is derived by means of the union bound of the pairwise error probabilities. Asymptotic results for the high SNR region enable to consider the effect of fading correlations and the diversity advantage. Simulations are performed to verify the correctness of the mathematical analysis, and the analysis is validated to bound the actual BER probability tightly in the high SNR region.

Keywords: MIMO, OFDM, maximum likelihood detection, multipath fading.

Classification: Wireless communication hardware

References

[1] M. Saito, S. Moriyama, and O. Yamada, "A digital modulation method for terrestrial digital TV broadcasting using trellis coded OFDM and its performance," *Proc. IEEE GLOBECOM'92*, pp. 1694–1698, Dec. 1992.

[2] IEEE standard for wireless LAN medium access control and physical layer specifications, IEEE802.11g, June 2003.

[3] IEEE standard for wireless LAN medium access control and physical layer specifications, IEEE802.11n, Sept. 2009.

[4] D. Divsalar and M. K. Simon, "The design of trellis coded MPSK for fading channels: performance criteria," *IEEE Trans. Commun.*, vol. 36, no. 9, pp. 1004–1012, Feb. 1988.

[5] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Feb. 1998.

[6] G. Jongren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Trans. Inf. Theory*, vol. 48, no. 3, pp. 611–627, March 2002.

[7] C. N. Chuah, D. N. C. Tse, J. M. Kahn, and R. A. Valenzuela, "Capacity scaling in MIMO wireless systems under correlated fading," *IEEE Trans. Inf. Theory*, vol. 48, no. 3, pp. 637–650, March 2002.

[8] M. K. Simon and M. S. Alouini, Digital communication over fading channels, Wiley-IEEE Press, Nov. 2004.

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1 Introduction

Orthogonal frequency division multiplex (OFDM) has been adopted in many practical communication systems, and advanced OFDM systems have been studied actively in response to the need for high-speed data communication. One of the advantages of OFDM is its robustness against multipath fading. OFDM can cope with multipath signals by simply coding over the frequency domain. Because of this advantage, coded OFDM has been recognized as a suitable method for high-speed data communication. In coded OFDM, generally coded data are interleaved to average the variance of the received powers caused by frequency selective fading. The interleaver rearranges the coded data in a non-contiguous way to prevent the burst error caused by attenuated subcarriers. When the size of the interleaver is sufficiently large, the order of time and frequency symbols is interchanged, and the effect of time selective fading may be averaged. In general, such an enlargement of the interleaver size requires a large amount of memory and increases latency. Currently, several systems utilizing OFDM are in practical use, such as integrated services digital broadcasting (ISDB) and IEEE802.11agn [1, 2, 3]. Most of systems interleave coded data only in the frequency domain or the space and frequency domains. If one uses the time-frequency interleaver with a sufficient interleave depth, the performance analysis is possible under the assumption of the statistical independence of received powers. However, if one uses an interleaver having insufficient interleave depth, the received powers of the signals on subcarriers are mutually correlated. The analysis result for time-frequency interleaving is not applicable to this case. Since the frequency or space-frequency interleaver is used in several practical systems, it is important to analyze the bit error rate (BER) performance for clarifying the fading effects on the system performance.

In this study, the BER performance of a coded multiple-input and multiple-output (MIMO) OFDM system in correlated multipath Nakagami-Rice fading channels is analyzed on the basis of the studies described in [4, 5, 6, 7]. A space frequency interleaver and a maximum likelihood (ML) detector are used for error correction. The upper bound of the BER probability is derived by means of the union bound of the pairwise error probabilities. By using the analysis results, the effects of fading correlation and the diversity advantage on the BER performance are studied. In the numerical example, simulations are performed to verify the correctness of the mathematical analysis. The analysis results are compared with the simulation results, and both sets of the results are confirmed to be in good agreement in the high SNR region.

2 System model

Figure 1 shows the block diagram of a coded MIMO OFDM system with a space frequency interleaver. At the transmitter, the channel encoder encodes information to codes $\mathbf{x}$ of length NN_T. The serial-to-parallel (SP) converter converts these codes into the parallel sequence. The space frequency interleaver interchanges the order of the code elements in the space and frequency

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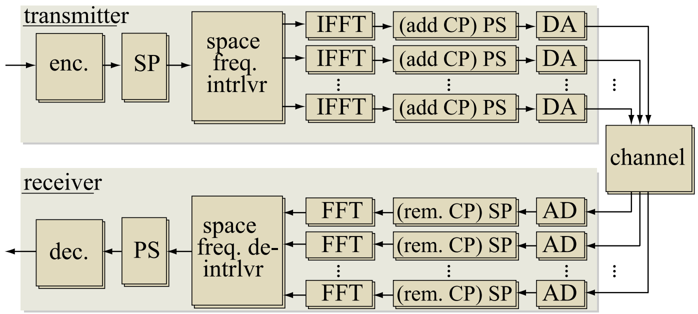

Fig. 1. Block diagram of coded MIMO OFDM system.

domains to remove the effect of frequency selective fading. Then, the outputs of the interleaver are separated to the N_T branches, and are fed into the inverse fast Fourier transformers (IFFTs). In each IFFT, the code elements are modulated and summed up, and N_T OFDM signals are generated. Then, the parallel-to-serial converters (PSs) convert the outputs of IFFTs into serial sequences, and simultaneously, the signals are extended cyclically by adding cyclic prefixes (CPs). The digital-to-analog converters (DAs) convert these signals into continuous signals. After the transmit operation, the signals are transmitted from N_T antennas.

The signals affected by correlated multipath fading are received by N_R antennas. After the receiver operation, the signals are sampled by analog-to-digital converters (ADs) at the same frequency in the transmitter. The sampled sequences are paralleled by the SP converters, and simultaneously, the CPs are removed. The output received signals are equivalently expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{F}_{N_T}\mathbf{\Pi}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{NN_R \times NN_T}$ is the channel transfer matrix, and $\mathbf{\Pi} \in (0,1)^{NN_T \times NN_T}$ is the space frequency interleave matrix. $\mathbf{F}_n$ is an extension of the DFT matrix written as $\mathbf{F}_n = (\mathbf{I}_n \otimes \mathbf{F})$, where $\mathbf{F} \in \mathbb{C}^{N \times N}$ is the unitary DFT matrix, $\mathbf{I}_n$ is the n by n identity matrix, and $\otimes$ stands for the Kronecker product. The noise term $\mathbf{n} \in \mathbb{C}^{NN_R \times 1}$ is assumed to be uncorrelated complex additive circularly symmetric white Gaussian noise, that meets the relation, $E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma_n^2 \mathbf{I}_{NN_R}$, where $E[\cdot]$ stands for the mean value. After the removal of the CPs, the signals are fed into the fast Fourier transformers (FFTs). In the FFTs, the signals are separated and demodulated. The outputs of FFTs are fed into the space frequency de-interleaver wherein the data are reordered in a contiguous way. The PS converter converts the output of the de-interleaver into a serial sequence. Then, the decoding is performed by comparing the squared Euclidean distance between the code words.

3 Analysis

The union bound of the BER probability of the coded MIMO OFDM is written as

$$P_b \leq \sum_{\mathbf{x}, \hat{\mathbf{x}} \in \mathcal{X}} \frac{h\left(\mathbf{x}, \hat{\mathbf{x}}\right)}{N\left(\mathbf{x}\right)} p\left(\mathbf{x}\right) P\left(\mathbf{x}, \hat{\mathbf{x}}\right), \tag{2}$$

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where $h(\mathbf{x}, \hat{\mathbf{x}})$ is the Hamming distance between the information assigned to the codes $\mathbf{x}$ and $\hat{\mathbf{x}}$, $N(\mathbf{x})$ is the amount of information assigned to the code $\mathbf{x}$, $p(\mathbf{x})$ is the *a priori* probability of $\mathbf{x}$, $P(\mathbf{x}, \hat{\mathbf{x}})$ is the pairwise error probability, namely the decoder detects an erroneous codes $\hat{\mathbf{x}}$ in the transmission of $\mathbf{x}$, and $\mathcal{X}$ denotes the set of all code patterns. It is assumed that the channel is statistically static and that the receiver has accurate channel state information. Since the Q-function is bounded by $Q(x) \leq \frac{1}{2}\exp(-\frac{x^2}{2})$, for $x \geq 0$, the pairwise error probability between $\mathbf{x}$ and $\hat{\mathbf{x}}$ for ML detection conditioned on the channel transfer matrix is bounded by

$$P(\mathbf{x}, \hat{\mathbf{x}}) \leq \frac{1}{2}\exp\left(-\left|\left|\mathbf{H}\mathbf{F}_{N_T}\mathbf{\Pi}(\mathbf{x}-\hat{\mathbf{x}})\right|\right|^2/4\sigma_n^2\right), \tag{3}$$

where $||\cdot||^2$ stands for the squared Euclidean norm of the vector. Under the assumption of a sufficient length of CP and non-inter-block interference (IBI), Eq. (3) is rewritten as [6, 7]

$$P(\mathbf{x}, \hat{\mathbf{x}}) \leq \frac{1}{2}\exp\left(-\left|\left|\Phi(\mathbf{x}, \hat{\mathbf{x}})\mathbf{h}\right|\right|^2/4\sigma_n^2\right), \tag{4}$$

where $\mathbf{h} \in \mathbb{C}^{NN_TN_R\times 1}$ is the circularly symmetric complex Gaussian random vector with unit variance and mean $\mathbf{m}$, and $\Phi(\mathbf{x}, \hat{\mathbf{x}})$ is written as

$$\Phi(\mathbf{x}, \hat{\mathbf{x}}) = (\mathbf{I}_{N_R} \otimes [\mathbf{D}_1(\mathbf{x}, \hat{\mathbf{x}})\,\mathbf{D}_2(\mathbf{x}, \hat{\mathbf{x}})\cdots\mathbf{D}_{N_T}(\mathbf{x}, \hat{\mathbf{x}})])\,\mathbf{F}_{N_TN_R}\mathbf{C}^{\frac{1}{2}}. \tag{5}$$

In Eq. (5), $\mathbf{D}_n(\mathbf{x}, \hat{\mathbf{x}})$ is the diagonal matrix with the entry of the vector difference of the code vectors $\mathbf{x}$ and $\hat{\mathbf{x}}$ transmitted from the n-th antenna, and $\mathbf{C} \in \mathbb{C}^{NN_TN_R\times NN_TN_R}$ is the covariance matrix of the vector with random entries of multipath components. The mean vector $\mathbf{m}$ is approximated by the pseudo-inverse of the square root of the covariance matrix as $\mathbf{m} = \mathbf{C}^{+\frac{1}{2}}\mathbf{m}_a$. $\mathbf{m}_a$ is the mean vector of the channel responses, and $(\cdot)^+$ stands for the pseudo-inverse of the matrix. By averaging, pairwise error probability for the given mean vector is rewritten as [4, 5]

$$P(\mathbf{x}, \hat{\mathbf{x}}) \leq \frac{1}{2}\prod_{n=1}^{NN_TN_R}\frac{1}{1+\lambda_n/4\sigma_n^2}\exp\left(-\frac{K_n\lambda_n/4\sigma_n^2}{1+\lambda_n/4\sigma_n^2}\right). \tag{6}$$

λ_n in Eq. (6) is the n-th element of the eigenvalues of $\Phi(\mathbf{x}, \hat{\mathbf{x}})^H\,\Phi(\mathbf{x}, \hat{\mathbf{x}})$, and K_n is the n-th element of K-factors given by $K_n = |\mathbf{v}_n\mathbf{m}|^2$. $\mathbf{v}_n$ denotes the n-th entry of the eigenvectors of $\Phi(\mathbf{x}, \hat{\mathbf{x}})^H\,\Phi(\mathbf{x}, \hat{\mathbf{x}})$ yielding an orthonormal basis of $\mathbb{C}^{NN_TN_R\times NN_TN_R}$. The average BER probability conditioned on the fading mean vector is derived by substituting Eq. (6) into Eq. (2). The rank of $\Phi(\mathbf{x}, \hat{\mathbf{x}})^H\,\Phi(\mathbf{x}, \hat{\mathbf{x}})$ largely depends on the delay profile in addition to the code structure. Since some of the fading components have a low mean power, the rank of $\Phi(\mathbf{x}, \hat{\mathbf{x}})^H\,\Phi(\mathbf{x}, \hat{\mathbf{x}})$ does not become a definite factor for determining the system performance. Concerning the code structure, in time selective fading, the product of the Euclidean distance yields the performance measure [4, 5]. In coded OFDM, the received powers of the subcarriers are mutually correlated, and therefore, the discussions in time selective fading can not be directly applied. However, it is possible to reach a similar consideration,

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namely, the error probability largely depends on the product of the Euclidean distance, and as increasing of the code length, the improvement due to the diversity is dominated by the number of effective multipath components.

4 Numerical example

Computer simulations are performed to verify the tightness of the upper bound of the mathematical analysis under the following conditions: MIMO OFDM with $N = 16$ subcarriers is considered; a set of Golay complementary codes of length 16 is employed; each code occurs with equal probability; the CP length is 1/4 symbol duration; signals are transmitted by $N_T = 2$ antennas and received by $N_R = 2$ antennas; and the CP length is sufficiently large, and hence, IBI does not occur.

Figure 2 shows a relation of the average BER probability versus E_b/N_0 in uncorrelated multipath fading channels as a parameter of the number of arrival signals N_L. In the simulation, the average fading power is assumed to follow an exponentially decaying power delay profile with equispaced delays having a decay factor of $\delta = 0.2$ [8]. The amplitudes of multipath signals are distributed according to the Nakagami-Rice distribution with K-factors of 3.0 dB for the first path and zero for the others. The signals transmitted from multiple antennas are affected by the channel independently. From the figure, good agreement between the simulation and analysis results is observed in the high E_b/N_0 region. The BER probability improves with an increase in the number of multipath components. For more than 4 paths, the amount of performance improvement remains small because of the small mean power of the delay components. If the target BER probability becomes small, the effect of paths with small mean power appears gradually.

Figure 3 shows the average BER probability versus E_b/N_0 in correlated multipath fading channels. In the simulation, the total number of arriving signals is fixed to $N_L = 16$, and the rank of the covariance matrix r is changed.

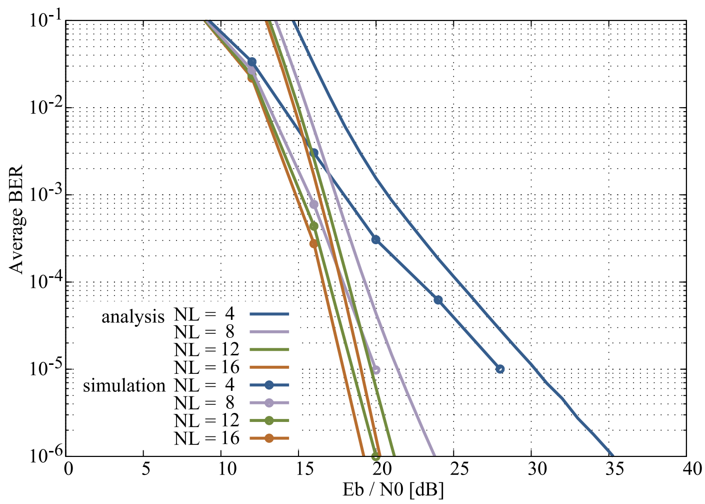

Fig. 2. Average BER probability in uncorrelated multipath Nakagami-Rice fading channel.

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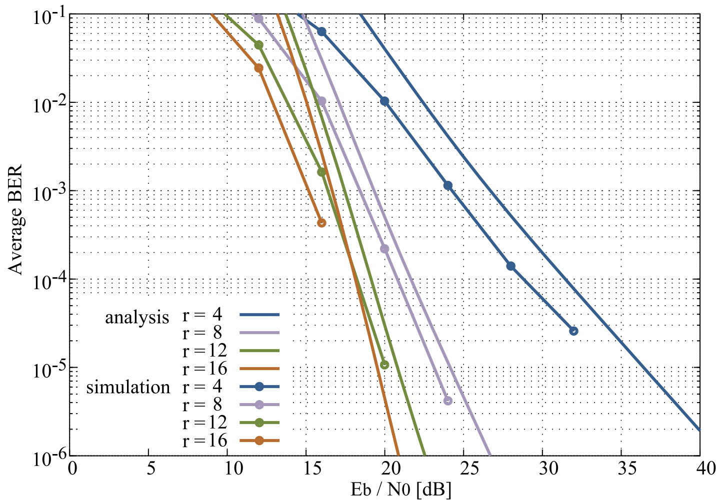

Fig. 3. Average BER probability in correlated multipath Rayleigh fading channel.

All the paths are affected by the Rayleigh fading. Similar to the case shown in Fig. 2, the average fading power is assumed to follow an exponentially decaying power profile model having a decay factor of $\delta = 0.2$. From the figure, it is seen that the analysis results are consistent with the simulation results in the high E_b/N_0 region. The BER probability improves as the rank of the covariance matrix increases. For $r \geq 8$, a small amount of improvement is observed. Similar to the case shown in Fig. 2, the reason for this little improvement is the small mean power of the delay signals. These results confirms that the diversity advantage depends on the number of effective multipath components.

5 Conclusion

The average BER performance of coded MIMO OFDM using a space frequency interleaver with ML detection in correlated multipath Nakagami-Rice fading channels has been studied. The upper bound of the BER probability was derived by means of the union bound of the pairwise error probabilities. The analysis contributes to measuring the system performance in a static channel in the case of an insufficient interleave depth and to better understanding of the effect of fading correlation and the diversity advantage on the BER performance. Even when the code length is longer than interleaving size, this analysis is applicable with a simple extension. The results show that the rank of the matrix does not become a definite measure of the system performance and that the effective multipath components affect the system performance. In a practical system, some of the signals may arrive at a receiver with small time difference. In such a case, the effect of these signals approaches flat fading, so that to gain a diversity advantage requires the components having a certain degree of time difference depending on the transmission bandwidth. The simulations were performed to verify the correctness of the mathematical analysis, and the analysis was validated to bound the actual BER probability tightly in the high SNR region.

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