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Reformulation of the ADI-BPM using a fundamental scheme

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Abstract: A fundamental scheme, developed for implicit finite-difference time-domain methods, is utilized to reformulate the beam-propagation method based on the alternating-direction implicit scheme (ADI-BPM). A derivative-free formulation is performed in the right-hand sides of equations to be solved, leading to quite efficient implementation of the algorithm. Numerical results reveal that the computation time and memory requirements are reduced to $\simeq$ 80%, providing the results equivalent to those of the conventional ADI-BPM.

Keywords: alternating-direction implicit (ADI) scheme, beam propagation method (BPM), numerical analysis, optical waveguide
Classification: Electromagnetic theory

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1 Introduction

During the last two decades, the beam-propagation method based on the alternating-direction implicit scheme (ADI-BPM) has been continuously employed for the analysis and design of three-dimensional (3-D) optical waveguides [1]. By virtue of the ADI splitting technique, problems of a broadly banded matrix are reduced to those of a tri-diagonal matrix that can be solved quite efficiently. The ADI-BPM has successfully been extended to semi-vectorial [2, 3] and full-vectorial versions [4, 5, 6, 7].

In this article, we reformulate the ADI-BPM with the help of a fundamental scheme [8] that has originally been developed for the efficient implementation of implicit finite-difference time-domain (FDTD) methods. We first present the formulation, in which derivative-free forms are obtained in the right-hand sides of resultant equations. This greatly facilitates the implementation of the algorithm, while maintaining the equivalence to the conventional ADI-BPM. To show the validity of the present method, termed FADI-BPM, we next analyze a waveguide sensor based on the surface plasmon resonance (SPR) [9]. Numerical results show that the computation time and memory requirements are reduced to $\simeq$ 80% of those of the conventional ADI-BPM.

2 Discussion

We here consider the formulation for the semi-vectorial ADI-BPM. The Fresnel equation to be solved is expressed as

$$\frac{\partial \phi}{\partial z} = \frac{1}{\sigma}\left[D_{xx} + D_{yy} + \nu\right]\phi \tag{1}$$

where

$$\begin{aligned} \sigma &= 2jkn_0 \\ \nu &= k^2(n^2 - n_0^2) \end{aligned}$$

in which k, n_0 and n are, respectively, the free-space wavenumber, the reference refractive index and the refractive index of the waveguide. In addition, the spatial derivatives and the field components are defined as

$$D_{xx} = n^2\frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\right), \; D_{yy} = \frac{\partial^2}{\partial y^2}, \; \phi = H_y$$

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for the TE mode and

$$D_{xx} = \frac{\partial^2}{\partial x^2},\ D_{yy} = n^2 \frac{\partial}{\partial y}\left(\frac{1}{n^2}\frac{\partial}{\partial y}\right),\ \phi = H_x$$

for the TM mode.

In the conventional ADI-BPM, (1) is split into two-step algorithm as

$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} = \left[1 + \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l} \tag{2a}$$

$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l+1} = \left[1 + \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} \tag{2b}$$

where $\phi^{l+1/2}$ is the intermediate fields.

To solve (2) more efficiently, we employ the fundamental scheme [8]. We first rewrite (2) as

$$v^l = \left[1 + \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l} \tag{3a}$$

$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} = v^l \tag{3b}$$

$$v^{l+1/2} = \left[1 + \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} \tag{3c}$$

$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l+1} = v^{l+1/2} \tag{3d}$$

where v's are the auxiliary variables. From (3d), the expression at one time step backward is given as

$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l} = v^{l-1/2}. \tag{4}$$

Then, (3a) can be rewritten with (4) as

$$\begin{aligned} v^l &= \left[1 + \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l} \\ &= 2\phi^l - \left[1 - \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l} \\ &= 2\phi^l - v^{l-1/2}. \end{aligned} \tag{5}$$

Using (3b), (3c) is similarly modified to

$$\begin{aligned} v^{l+1/2} &= \left[1 + \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} \\ &= 2\phi^{l+1/2} - \left[1 - \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} \\ &= 2\phi^{l+1/2} - v^l. \end{aligned} \tag{6}$$

With (5) and (6), (3) is finally reduced to the following efficient FADI-BPM equations to be solved:

$$v^l = 2\phi^l - v^{l-1/2} \tag{7a}$$

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$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{xx} + \frac{\nu}{2}\right)\right]\phi^{l+1/2} = v^l \tag{7b}$$

$$v^{l+1/2} = 2\phi^{l+1/2} - v^l \tag{7c}$$

$$\left[1 - \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^{l+1} = v^{l+1/2}. \tag{7d}$$

For the initial field ϕ^0, the algorithm requires the input initialization using (7d) as

$$v^{-1/2} = \left[1 - \frac{\Delta z}{2\sigma}\left(D_{yy} + \frac{\nu}{2}\right)\right]\phi^0. \tag{8}$$

It is worth mentioning that no spatial derivative appears in the right-hand sides of (7b) and (7d), in contrast to (2) of the conventional ADI-BPM. This leads to quite simple implementation of the algorithm, maintaining the equivalence to the conventional method. Incidentally, coupled equations should be solved for the fundamental FDTDs, so that several field components appear in the right-hand sides of implicit finite-difference equations to be solved [8]. In contrast, only a single component appears in the right-hand sides of (7b) and (7d). Therefore, particularly for the BPM formulation, we can take full advantage of the simplicity of the fundamental scheme.

To investigate the usefulness of the FADI-BPM, we analyze the waveguide-based SPR sensor shown in Fig. 1 [9]. The configuration parameters are $d = d_{\mathrm{w}} = 2.0\,\mu\mathrm{m}$, $t_{\mathrm{m}} = 0.045\,\mu\mathrm{m}$ and $t_{\mathrm{ad}} = 0.02\,\mu\mathrm{m}$. The width of the metal, adsorbed and analyte layers is $w = 10\,\mu\mathrm{m}$. The refractive indexes are $n_{\mathrm{co}} = n_{\mathrm{ad}} = 1.47$ and $n_{\mathrm{sub}} = 1.46$. The dispersion of the metal (Au) is taken into account using the critical points model, which is denoted as the CP3 model in [10]. Water is used as the analyte ($n_{\mathrm{a}} = 1.332$), which is sufficiently thick ($1\,\mu\mathrm{m}$) to yield a converged solution. The structure is discretized with $\Delta x = 0.05\,\mu\mathrm{m}$, $\Delta y = 0.005\,\mu\mathrm{m}$ and $\Delta z = 0.1\,\mu\mathrm{m}$.

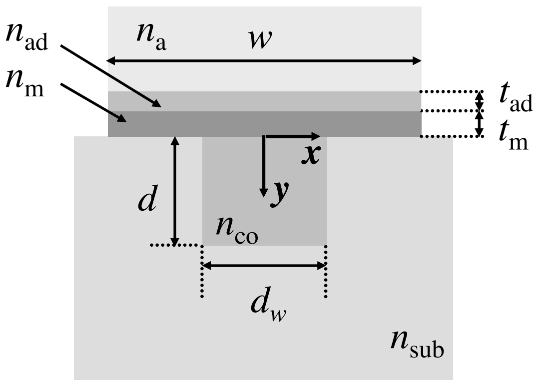

Fig. 1. Configuration of a waveguide-based SPR sensor.

The initial input field is the eigenmode (H_x fundamental mode) of the waveguide that corresponds to the structure without the metal, adsorbed and analyte layers in Fig. 1. We calculate the output guided-mode power as a function of wavelength. Fig. 2 shows the results, in which the power obtained from the conventional ADI-BPM is compared with that from the

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FADI-BPM. It is found that these two results are in perfect agreement, validating the equivalence of the two methods.

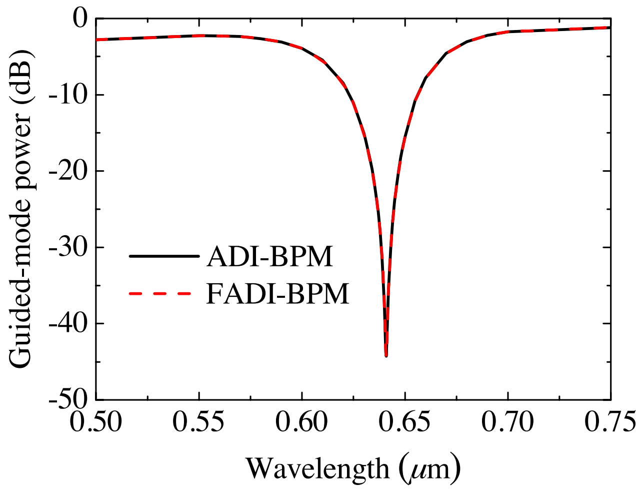

Fig. 2. Output guided-mode power versus wavelength. The sensor section is 200 μm in length. The resonance wavelength is around $\lambda = 0.64\,\mu$m.

Finally, we check the efficiency of the FADI-BPM. The CPU time and memory requirements are depicted in Fig. 3, in which a workstation with a Xeon X5650 processor (2.66 GHz) is used. The computation requirements are found to be successfully reduced to about 80% of the conventional ADI-BPM, demonstrating the efficiency improvement of the fundamental scheme.

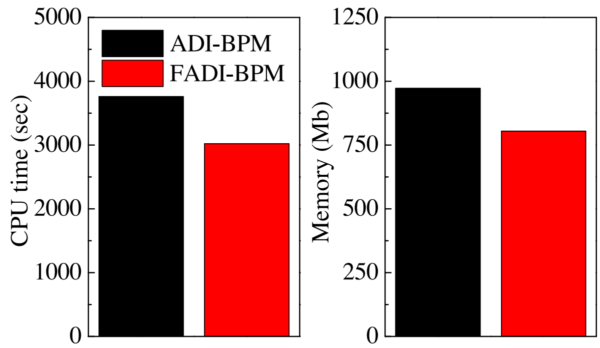

Fig. 3. CPU time and memory requirements.

3 Conclusion

A fundamental scheme has been applied for the first time to the formulation of the ADI-BPM. Efficiency improvement is investigated through the analysis of the waveguide-based SPR sensor. Although only the z-invariant structure is studied here, the present method is applicable to z-variant structures, which will be discussed elsewhere. In addition, the efficient formulation with the fundamental scheme can be applied to the full-vectorial ADI-BPM.

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Therefore, most of the existing ADI-BPM algorithms may be improved with the help of the fundamental scheme.

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