

# Iterative receiver design for general nonorthogonal unitary space-time constellations

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**Abstract:** A simple iterative receiver for general nonorthogonal unitary space-time constellations (NOUSTC) is proposed. The output of the conventional noncoherent detector is used to initialize the receiver. In each iteration, the decision from last iteration is used to transform the received signals to derive new signals that bear channel state information (CSI), which are then employed to estimate the CSI using a Wiener filter, following a coherent detector is implemented. In our scheme, for each time block we can use not only the past and the present CSI-bearing signals, but also the future ones, to estimate the CSI, thus resulting in a relatively refined channel estimate and ultimately a receiver with good performance. Simulation results verify the performance of the proposed receiver.

**Keywords:** channel estimation, unitary space-time modulation, iterative receiver, Wiener filter

**Classification:** Wireless communication hardware

## References

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## 1 Introduction

Unitary space-time modulation (USTM) [1] is very suitable for noncoherent detection using a quadratic receiver (QR) without channel state information (CSI) at the receiver. However, a coherent receivers (CR) with perfect CSI can bring about approximately 3 dB signal to noise ratio (SNR) over a QR. This motivates to the generalized quadratic receivers (GQR) [2] for USTM, which incorporate channel estimation without the help of additional training signals and bridge the performance gap between the QR and the CR. However, the scheme in [2] fits well for only some unitary space-time constellations (USTC) with specific structures. When it is applied to the general nonorthogonal unitary space-time constellations (NOUSTC) in [3], for each time block it has to employ decision-feedback method to get only the past and the present CSI-bearing signals, which are then used to estimate the CSI, leading to an inaccurate channel estimate and ultimately a receiver with poor performance.

In this letter, a simple iterative receiver for general NOUSTC is proposed. The output of the conventional noncoherent detector is utilized to initialize the receiver. In each iteration, the decision from last iteration is used to transform the received signals to derive the CSI-bearing signals which are then employed to estimate the CSI using a Wiener filter, following a coherent detector is implemented. In our scheme, we can use for each time block in each iteration not only the past and the present CSI-bearing signals, but also the future ones, to estimate the CSI, thus resulting in a relatively refined channel estimate and ultimately a receiver with good performance.

## 2 System model

Consider a communication link comprising  $M$  transmit antennas and  $N$  receive antennas that operates in a Rayleigh flat-fading environment. The channel gains are assumed to remain constant in one time block of  $T$  symbol intervals, and we assume  $T > M$ . Thus, at the  $k$ th time block, the received signals are given by [3]

$$\mathbf{x}(k) = \sqrt{\frac{\rho T}{M}} \mathbf{\Psi}(k) \mathbf{h}(k) + \mathbf{w}(k) \quad (1)$$

where  $\mathbf{x}(k) = [\mathbf{x}_1^T(k), \mathbf{x}_2^T(k), \dots, \mathbf{x}_N^T(k)]^T$  is the  $TN \times 1$  complex received signal vector, in which the superscript “T” denotes transpose and  $\mathbf{x}_n(k) = [x_n((k-1)T+1), x_n((k-1)T+2), \dots, x_n(kT)]^T$  is the received signal vector at receive antenna  $n$  and time block  $k$ . The quantity  $\rho$  is the mean SNR at each receive antenna.  $\mathbf{\Psi}(k) = \mathbf{I}_N \otimes \mathbf{\Phi}(k)$  is a  $TN \times MN$  matrix of complex transmitted signals in which the  $T \times M$  unitary matrix  $\mathbf{\Phi}(k)$ , chosen from the constellation  $\Omega = \{\mathbf{\Phi}_l | \mathbf{\Phi}_l^\dagger \mathbf{\Phi}_l = \mathbf{I}_M, l = 1, \dots, L\}$ , is transmitted from  $M$  transmit antennas at time block  $k$ . The  $MN \times 1$  complex channel gain vector is defined as  $\mathbf{h}(k) = [\mathbf{h}_1^T(k), \mathbf{h}_2^T(k), \dots, \mathbf{h}_N^T(k)]^T$ , in which  $\mathbf{h}_n(k) = [h_{1n}(k), h_{2n}(k), \dots, h_{Mn}(k)]^T$ . Each  $h_{mn}$  is the path gain from transmit antenna  $m$  to receive antenna  $n$  at time block  $k$ , and is  $\mathcal{CN}(0, 1)$

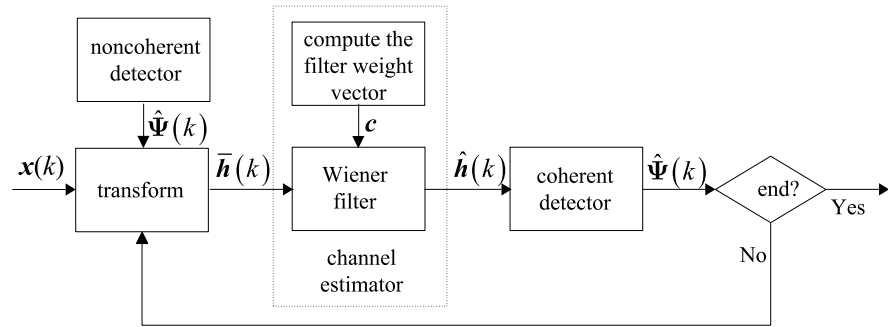


Fig. 1. The architecture of the proposed iterative receiver for NOUSTC.

distributed. The path gains over different branches are always independent, but those over the same branch are correlated from one time block to another with the correlation  $R_h(\tau) = E[h_{mn}(k + \tau)h_{mn}^*(k)]^T$ . Here,  $R_h(\tau)$  is assumed to be identical for all branches, i.e., it is independent of  $m$  and  $n$ .  $\mathbf{w}(k)$  is the  $TN \times 1$  complex noise vector defined similarly as  $\mathbf{x}(k)$ . We assume that all the entries of  $\mathbf{w}$  are independent and identically  $\mathcal{CN}(0, 1)$  distributed with respect to both time and space, and are independent of  $h$ .

### 3 Proposed iterative receiver for general NOUSTC

The architecture of the proposed receiver for NOUSTC is shown in Fig. 1 and its implementation steps will be given in the following.

#### 3.1 Noncoherent detector

If the channel  $h(k)$  is unknown to the receiver, and the signal matrices  $\{\Phi_l\}_{l=1}^L$  are transmitted with equal probabilities, the maximum likelihood receiver for USTM is a QR, given by [1]

$$\hat{\Psi}(k) = \arg \max_{\Psi_l \in \{\Psi_1, \dots, \Psi_L\}} \|\Psi_l^\dagger \mathbf{x}(k)\|_F^2 \quad (2)$$

where  $\|\cdot\|_F^2$  denotes the Frobenius norm, the superscript “†” denotes conjugate transpose and  $\Psi_l \triangleq \mathbf{I}_N \otimes \Phi_l$ . The output  $\hat{\Psi}(k)$  is used to initialize the receiver.

#### 3.2 Channel estimator

Different from [2], firstly we transform the received signals using the output  $\hat{\Psi}(k)$  of the QR for the first iteration, or using that of the last iteration for the  $k$ th ( $k \geq 1$ ) iteration to get the CSI-bearing signals for time blocks  $\{k + \tau\}_{\tau=-\tau_1}^{\tau_2}$  as

$$\bar{\mathbf{h}}(k + \tau) = \sqrt{\frac{M}{\rho T}} \hat{\Psi}^\dagger(k + \tau) \mathbf{x}(k + \tau) \quad (3)$$

$$= \tilde{\mathbf{h}}(k + \tau) + \mathbf{h}_e(k + \tau) \quad (4)$$

where we have defined the noisy channel components  $\tilde{\mathbf{h}}(k + \tau)$  for time blocks  $\{k + \tau\}_{\tau=-\tau_1}^{\tau_2}$  as

$$\tilde{\mathbf{h}}(k + \tau) = \mathbf{h}(k + \tau) + \sqrt{\frac{M}{\rho T}} \hat{\Psi}^\dagger(k + \tau) \mathbf{w}(k) \quad (5)$$

and the channel estimation error components  $\mathbf{h}_e(k + \tau)$  as

$$\mathbf{h}_e(k + \tau) = [\hat{\Psi}^\dagger(k + \tau) - \Psi^\dagger(k + \tau)] \Psi(k + \tau) \mathbf{h}(k + \tau) \quad (6)$$

We can obtain the Wiener filter weight vector for the minimum mean-square error (MMSE) estimate of  $\mathbf{h}(k + \tau)$  based on  $\tilde{\mathbf{h}}(k + \tau)$  as if  $\tilde{\mathbf{h}}(k + \tau)$  were available. We define  $\tilde{h}_j(k + \tau)$  as the  $j$ th element of  $\tilde{\mathbf{h}}(k + \tau)$ , and let  $\bar{\mathbf{h}}_j(k) = [\tilde{h}_j(k - \tau_1), \tilde{h}_j(k - \tau_1 + 1), \dots, \tilde{h}_j(k + \tau_2)]^T$ . Then the MMSE estimate of  $h_j(k)$  based on  $\bar{\mathbf{h}}_j(k)$  is

$$\hat{h}_j(k) = \mathbf{c}^\dagger \bar{\mathbf{h}}_j(k) \quad (7)$$

where  $\bar{\mathbf{h}}_j(k)$  is defined similarly as  $\tilde{\mathbf{h}}_j(k)$ , and  $\mathbf{c}$  is the Wiener filter weight vector which can be derived from the Wiener-Hopf equation [4] as

$$\mathbf{c} = \mathbf{R}_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}^{-1} \mathbf{r}_{\tilde{\mathbf{h}}h} \quad (8)$$

where  $\mathbf{R}_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}} = \mathbb{E}[\tilde{\mathbf{h}}_j(k) \tilde{\mathbf{h}}_j^\dagger(k)]$  is the autocorrelation matrix of  $\tilde{\mathbf{h}}_j(k)$  and  $\mathbf{r}_{\tilde{\mathbf{h}}h} = \mathbb{E}[\tilde{\mathbf{h}}_j(k) h_j^*(k)]$  is the cross correlation vector, in which the superscript “\*” denotes conjugate of a scalar quantity. The  $(s, t)$ th entry of  $\mathbf{R}_{\tilde{\mathbf{h}}\tilde{\mathbf{h}}}$  is given by

$$r_{st} = \begin{cases} 1 + M/\rho T, & s = t \\ R_h(s - t), & s \neq t \end{cases} \quad (9)$$

and  $\mathbf{r}_{\tilde{\mathbf{h}}h}$  is given by

$$\mathbf{r}_{\tilde{\mathbf{h}}h} = [R_h(-\tau_1), R_h(-\tau_1 + 1), \dots, R_h(\tau_2)]^T \quad (10)$$

From Eq. (8)-Eq. (10), we can see that the Wiener filter weight vector  $\mathbf{c}$  is independent of the channel branch number  $j$  and the time-block number  $k$ . Thus, it can be computed in advance for each iteration, and Eq. (7) becomes

$$\hat{\mathbf{h}}(k) = \bar{\mathbf{H}}(k) \mathbf{c} \quad (11)$$

where  $\bar{\mathbf{H}}(k) = [\bar{\mathbf{h}}(k - \tau_1), \bar{\mathbf{h}}(k - \tau_1 + 1), \dots, \bar{\mathbf{h}}(k + \tau_2)]$ .

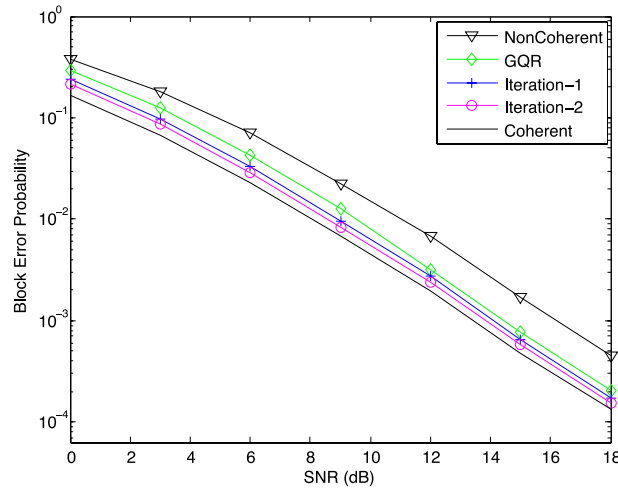
### 3.3 Coherent detector

After estimate the CSI from Eq. (11), coherent detection can be implemented by

$$\hat{\Psi}(k) = \arg \min_{\Psi_l \in \{\Psi_1, \dots, \Psi_L\}} \left\| \mathbf{x}(k) - \sqrt{\frac{\rho T}{M}} \Psi_l \hat{\mathbf{h}}(k) \right\|^2 \quad (12)$$

The output of the CR is then fed back to Eq. (3) and thus another iteration begins. The iterations cease after a specified number of iterations.

From Eq. (3) it can be seen that we can obtain for each time block  $k$  not only the past and the present CSI-bearing signals  $\{\bar{\mathbf{h}}(k + \tau)\}_{\tau=-\tau_1}^0$ , but also the future ones  $\{\bar{\mathbf{h}}(k + \tau)\}_{\tau=1}^{\tau_2}$ , which are all used to estimate the CSI, thus resulting in a relatively refined channel estimate, which enables the proposed receiver to perform better than the GQR for general NOUSTC.



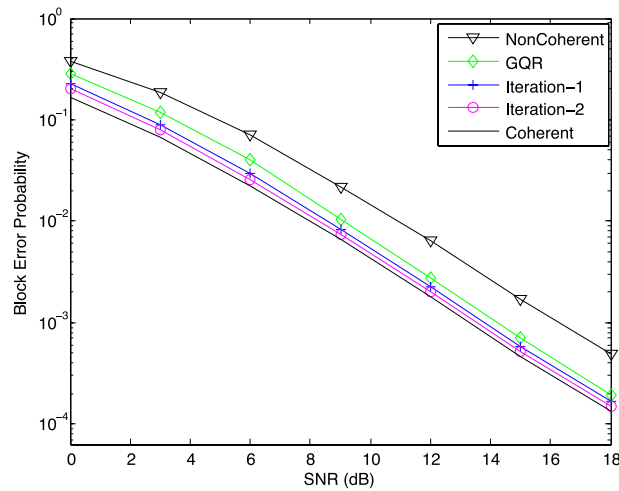
**Fig. 2.** Block error probability of the proposed receiver with  $\tau_1 = \tau_2 = 5$ , compared with that of the GQR with  $\tau_1 = 10, \tau_2 = 0$ , the conventional noncoherent detector, and the ideal coherent detector.

#### 4 Simulation results

During simulation, we assume that the fading process on each link is correlated according to Jakes model [5] with autocorrelation function  $R_h(\Delta t) = J_0(2\pi f_d T_s \Delta t)$ , where  $J_0(\cdot)$  is the zeroth-order Bessel function of the first kind,  $f_d$  is the maximum Doppler frequency,  $T_s$  is the symbol interval, and  $\Delta t$  is an integer denoting the distance between two samples of the channel. The normalized fade rate is set to be  $f_d T_s = 0.0025$ . Without loss of generality, only  $M = 2$  and  $N = 1$  are assumed.

The block error probability of the proposed receiver for the NOUSTC of size  $L = 17$  with  $M = 2$  in Table II of [3] as a function of SNR is shown in Fig. 2, compared with that of the GQR in [2], the conventional noncoherent detector, and the ideal coherent detector with perfect CSI. Iteration-1 and Iteration-2 in the legend of the figure denote the iteration runs once and twice, respectively. We have assumed that  $\tau_1 = \tau_2 = 5$  for the proposed receiver and  $\tau_1 = 10, \tau_2 = 0$  for the GQR. In other words, the channel memory span is set to be the same for them when estimating the channel. It can be seen from Fig. 2 that both of the proposed receiver and the GQR perform better than the conventional noncoherent detector, moreover, the proposed receiver shows its obvious advantage over the GQR even the iteration runs only once.

Fig. 3 shows the simulated results of the same NOUSTC as that in Fig. 2 but with  $\tau_1 = \tau_2 = 8$  for the proposed receiver and  $\tau_1 = 16, \tau_2 = 0$  for the GQR. We can see that the proposed receiver provides notable performance improvements over the GQR. Specifically, at an block error probability of  $10^{-3}$  in Fig. 3, the performance of the proposed receiver after 2 iterations is roughly 0.5 dB better than the GQR. It is also obvious from Fig. 3 that the performance curve of the proposed receiver after 2 iterations is very close to the lower bound given by the ideal ML coherent detector. Moreover, through simulation we find that additional iteration can not bring evident perfor-



**Fig. 3.** Block error probability of the proposed receiver with  $\tau_1 = \tau_2 = 8$ , compared with that of the GQR with  $\tau_1 = 16, \tau_2 = 0$ , the conventional noncoherent detector, and the ideal coherent detector.

mance gain. In other words, the algorithm converges on the whole after 2 iterations.

## 5 Conclusion

An iterative receiver for general NOUSTC was proposed. In each iteration MMSE channel estimation and coherent detection are successively performed. For each time block not only the past and the present CSI-bearing signals, but also the future ones, are utilized to estimate the CSI in the proposed scheme, thus resulting in a relatively refined channel estimate and ultimately a receiver with good performance. Simulation results verify the performance of the new receiver.

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