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Approximate 2-Degree-of-Freedom digital control for a boost DC-DC converter

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Abstract: If a duty ratio and a load resistance in a boost DC-DC converter are changed, the characteristics is varied greatly, that is, the boost DC-DC converter has the non-linear characteristics. In many applications of DC-DC converters, loads cannot be specified in advance, and they will be changed suddenly from no load to full load. The boost DC-DC converter used a conventional controller cannot suppress output voltage variation caused by large load change. In this paper an approximate 2-Degree-of-Freedom (2DOF) digital controller for suppressing the output voltage variation at sudden load change is proposed. This controller is actually implemented on a micro-processor and is connected to the boost DC-DC converter. Experimental results demonstrate that this type of digital controller is effective to suppress the variation.

Keywords: boost DC-DC converter, approximate 2-Degree-of-Freedom (2DOF), robust digital control, micro-processor

Classification: Electronic instrumentation and control

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1 Introduction

In many applications of DC-DC converters, loads cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. In a boost DC-DC converter, if a duty ratio, a load resistance and an input voltage are changed, the characteristics are varied greatly, that is, the boost DC-DC converter has the non-linear characteristics. Usually, a controller of the boost DC-DC converter is designed to an approximated linear controlled object at one operating point. Simple integral control, PID and root locus method etc. have been applied to control the boost DC-DC converter [1, 2, 3]. However, the boost DC-DC converter used these conventional controllers cannot suppress output voltage variation caused by large load change. Robust control method using an approximate 2-degree-of-freedom (2DOF) controller for improving start-up characteristics and load sudden change characteristics of DC-DC converters has been proposed [4]. However, it was applied to a buck DC-DC converter.

In this paper, the design method of the approximate 2DOF controller covering multi operating points with one controller is proposed. By this method, even if the dynamic characteristics of the controlled object are changed largely, the start-up response and the dynamic load response can be improved greatly. This controller is actually implemented on a microprocessor and is connected to the boost DC-DC converter. Experimental results demonstrate that the digital controller designed by the proposed method satisfies the given specifications and is useful practically.

2 Boost DC-DC converter

The boost DC-DC converter is shown in Fig. 1. In Fig. 1, v_i is an input AC voltage, C_{in} is a smoothing capacitor, Q_0 is a main switch, L_0 is a boost inductance and D_0 is a diode, C_0 is an output capacitor, R_L is an output load resistance, i_L is an inductor current, i_o is an output current, and v_o is an output voltage. Here C_{in} is 1 [μF], L_0 is 150 [μH] and C_0 is 940 [μF]. Using the averaging method, the static characteristics of the boost DC-DC converter and the state equation of the controlled object in some neighborhood of some operating point become as follows [5]:

$$V_s = \frac{1}{1 - \frac{1}{(1-\mu_s)^2}\frac{R_0}{R_L}} \frac{1}{1-\mu_s} V_i, \quad I_s = \frac{1}{R_L}\frac{V_s}{1-\mu_s} \tag{1}$$

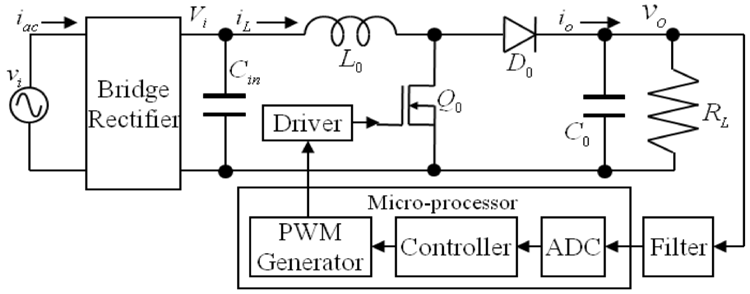

Fig. 1. Boost DC-DC converter

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$$\dot{x}(t) = A_c x(t) + B_c u(t)$$
$$y(t) = C_c x(t) \tag{2}$$

where

$$A_c = \begin{bmatrix} -\frac{1}{R_L C_0} & \frac{1-\mu_s}{C_0} \\ -\frac{1-\mu_s}{L_0} & -\frac{R_0}{L_0} \end{bmatrix}, B_c = \begin{bmatrix} -\frac{I_s}{C_0} \\ \frac{V_s}{L_0} \end{bmatrix}, x(t) = \begin{bmatrix} v_o(t) \\ i_L(t) \end{bmatrix}, \begin{array}{l} u(t) = \Delta\mu(t) \\ C_c = [1 \quad 0] \end{array}$$

Here the equivalent resistance of inductor R_0 is 1.8 [Ω] and the rectified input voltage V_i is 141 [$\mathrm{V_{DC}}$]. $\mu_s + \Delta\mu = \mu$, where μ is a duty ratio, μ_s is a value at each operating point and $\Delta\mu$ is a small perturbation from each operating point.

From Eqs. (1) and (2), the parameters in A_c and B_c depend on the duty ratio μ_s. Therefore, the state equation and the initial value of the converter at the operating point will be changed depending on the duty ratio and the operating point change. The load change and the duty ratio change of the controlled object are parameter change in Eq. (2). Such parameter change can be transformed to the equivalent disturbances q_u and q_y as shown in Fig. 2. Therefore, what is necessary is just to constitute the control system which pulse transfer functions from equivalent disturbances q_u and q_y to the output y become as small as possible in their amplitudes in order to robustize or suppress the influence of these parameter change. Here, the controller which satisfies the following specifications will be designed.

(1) The input voltage v_i is 100 [$\mathrm{V_{AC}}$], the output voltage v_o changes from 240 to 385 [$\mathrm{V_{DC}}$].

(2) The step responses are the almost same at resistive loads where 300 [Ω] $\leq R_L < 5$ [kΩ]. And the over-shoot is less than 10 [%] in the step response.

(3) The dynamic load response is smaller than 10 [$\mathrm{V_{DC}}$] against load change between 30 $\sim$ 500 [W].

Under these specifications, three operating points was determined as follows:

Point 1: The output voltage is 385 [$\mathrm{V_{DC}}$], the resistive load is 5 [kΩ].
Point 2: The output voltage is 385 [$\mathrm{V_{DC}}$], the resistive load is 300 [Ω]
Point 3: The output voltage is 240 [$\mathrm{V_{DC}}$], the resistive load is 300 [Ω]

The gain and phase characteristics of the boost DC-DC converter in the neighborhood of each operating point are different. The approximate 2DOF controller is designed to the controlled object in the neighborhood of one point selected from above.

3 Digital robust controller

The continuous system of eq. (2) is transformed into the discrete system, where sampling period $T_s = 10$ [μs]. Here, in order to compensate the delay time $L_d = 0.99T_s$ caused by the AD conversion time and the micro-processor operation time etc., one delay (state ξ_1) is introduced to the input of the controlled object. And, more one delay (state ξ_2) is also introduced to the

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input of the controlled object for the current feedback equivalent conversion. The state-space equation of the new controlled object with two delays is described as follows:

$$\begin{aligned} x_{dw}(k+1) &= A_{dw}x_{dw}(k) + B_{dw}v(k) \\ y(k) &= C_{dw}x_{dw}(k) \end{aligned} \tag{3}$$

where

$$A_d = \left[e^{A_cT}\right], B_d = \left[\int_0^T e^{A_c\tau}B_c d\tau\right], C_d = C_c, \xi_2(k-1) = \xi_1(k)$$

$$A_{dw} = \begin{bmatrix} A_w & B_w \\ 0 & 0 \end{bmatrix}, B_{dw} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{dw} = [C_c \quad 0 \quad 0]$$

$$A_w = \begin{bmatrix} e^{A_cT} & e^{A_c(T-L_d)}\int_0^{L_d} e^{A_c\tau}B_c d\tau \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_w = \begin{bmatrix} \int_0^{T-L_d} e^{A_c\tau}B_c d\tau \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ 1 \end{bmatrix}, x_{dw}(k) = \begin{bmatrix} x_d(k) \\ \xi_1(k) \\ \xi_2(k) \end{bmatrix} = \begin{bmatrix} v_o(k) \\ i_L(k) \\ u(k) \\ v(k-1) \end{bmatrix}$$

To the system of eq. (3), the model matching control system is constituted using the state feedback and the system is transformed to the one using only the output feedback (voltage feedback) as shown in Fig. 2 (a). The current feedback is transformed to the voltage and control input feedbacks without changing the pulse transfer function between $r - y$. The transfer function of the system of Fig. 2 (a) from r to y is described as follows:

$$W_{ry}(z) = \frac{(1+H_1)(1+H_2)(1+H_3)(z-n_1)(z-n_2)(1+H_4)}{(z+H_1)(z+H_2)(z+H_3)(1-n_1)(1-n_2)(z+H_4)} \tag{4}$$

Here n_1 and n_2 are the zeros of the discrete-time controlled object. If the relation of H_1, H_2 and H_3 are specified as $|H_1| >> |H_2|, |H_3|$, then W_{ry} can be approximated to the following first-order discrete model.

$$W_{ry}(z) \approx W_m(z) = \frac{1+H_1}{z+H_1} \tag{5}$$

In Fig. 2 (a), the parameters are as follows:

$$\begin{aligned} f_1 &= -f_1 + \frac{f_2}{a_{12}}\left(a_{11} - f_4 + \frac{f_2 b_{11}}{a_{12}}\right), f_2 = -\frac{f_2}{a_{12}} \\ ff_3 &= -f_3 + \frac{f_2 a_{13}}{a_{12}}, \quad ff_4 = -f_4 + \frac{f_2 b_{11}}{a_{12}} \end{aligned} \tag{6}$$

Where, $f_i, i = 1, \ldots, 4$ are the state feedback gains obtained from constituting the model matching system using the current feedback. The system added the inverse system and the filter to the system in Fig. 2 (a) is constituted as shown in Fig. 2 (b). In Fig. 2 (b), the transfer function $K(z)$ is as follows:

$$K(z) = \frac{k_z}{z - 1 + k_z} \tag{7}$$

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$W_{Qy}(z)$ is the transfer function from the equivalent disturbance $Q = [q_u \; q_y]^{\mathrm{T}}$ to y. The transfer functions of the system of Fig. 2 (b) from r to y and Q to y are given by the following equations. Here, if $W_s(z) \approx 1$, then the equations are approximated to the most right-hand side.

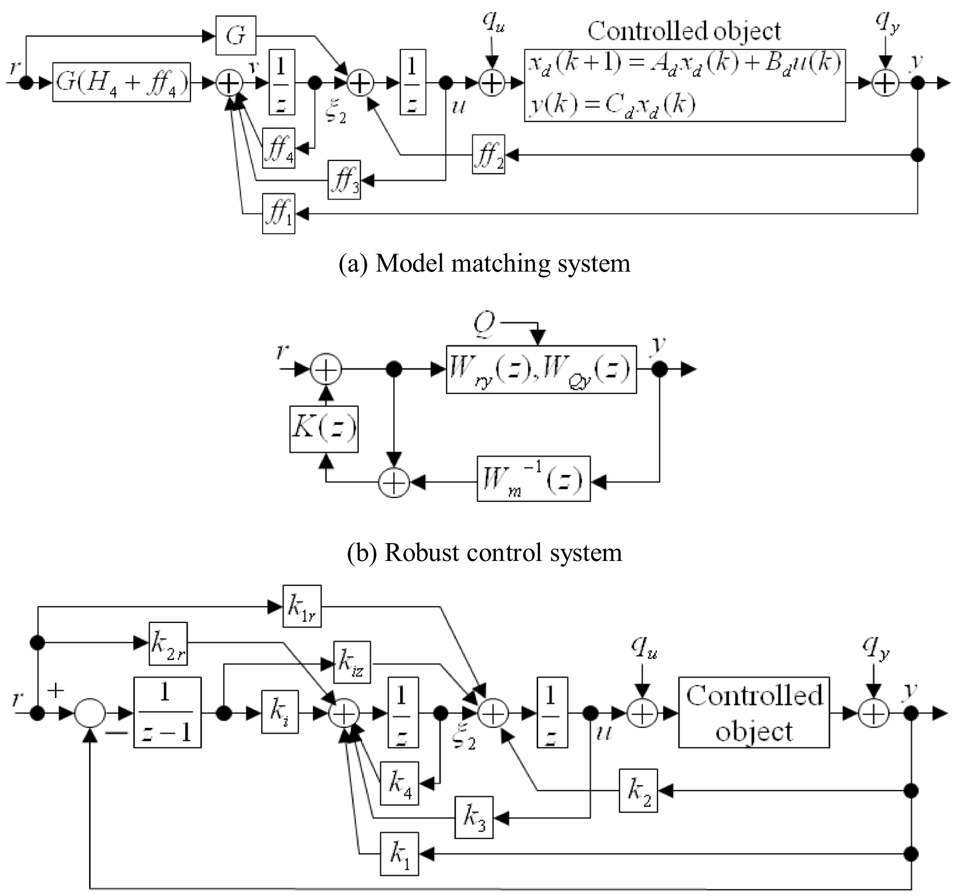
$$y = \frac{1+H_1}{z+H_1}\frac{z-1+k_z}{z-1+k_z W_s(z)} W_s(z) r \approx \frac{1+H_1}{z+H_1} r \tag{8}$$

$$y = \frac{z-1+k_z}{z-1+k_z}\frac{z-1+k_z}{z-1+k_z W_s(z)} W_{Qy}(z) Q \approx \frac{z-1}{z-1+k_z} W_{Qy}(z) Q \tag{9}$$

Where

$$W_s(z) = \frac{(1+H_2)}{(z+H_2)}\frac{(1+H_3)}{(z+H_3)}\frac{(z-n_1)}{(1-n_1)}\frac{(z-n_2)}{(1-n_2)} \approx 1$$

From eqs. (8), (9), it turns out that the characteristics from r to y can be specified with H_1 and the characteristics from q_u and q_y to y can be specified independently with k_z. That is the system in Fig. 2 (b) is an approximate 2DOF system, and its sensitivity against the disturbances becomes lower with the increase of k_z. If the equivalent conversion of the controller in Fig. 2 (b), the approximate 2DOF digital integral type control system will be obtained as shown in Fig. 2 (c). In Fig. 2 (c), the parameters of the controller are as follows:



(a) Model matching system

(b) Robust control system

(c) Approximate 2DOF digital robust integral type controller

Fig. 2. Model matching and robust control system configuration

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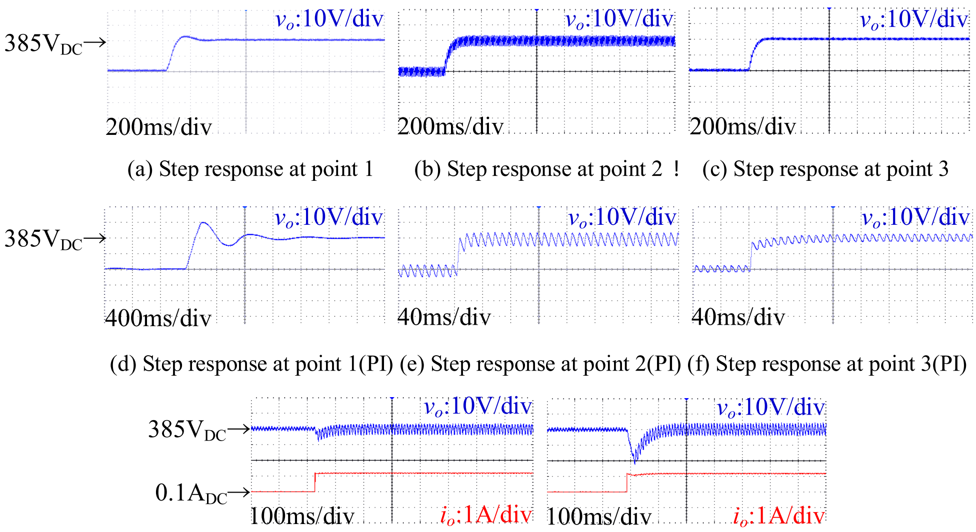
$$k_1 = f\!f_1 - \frac{G(H_4 + f\!f_4)k_z}{1+H_2}, k_2 = f\!f_2 - \frac{Gk_z}{1+H_2}, k_3 = f\!f_3, k_4 = f\!f_4$$
$$k_i = G(H_4 + f\!f_4)k_z, k_{iz} = Gk_z, k_{1r} = G, k_{2r} = G(H_4 + f\!f_4) \tag{10}$$

4 Experimental results

The controller is designed to the state equation (2) in the neighborhood of the operating point 1. In this experiment, the micro-processor SH7216 by Renesas Electronics Corp. is used. The experimental results of the step responses and the load sudden change are shown in Fig. 3 (a) to (c) and (g), respectively. Even if the operating point changes, the step responses are almost same. The output voltage variation in the sudden load change is under 10 [V_{DC}]. The experimental results used usual PI controller is shown in Fig. 3 (d) to (f) and (h), respectively. The step responses no satisfy the specification and are different greatly at each operating point. The output voltage variation in the sudden load change is over 20 [V_{DC}] and the recovery time is over 100 [ms]. From these results, the control systems used PI controller cannot satisfy specifications. As a result, it turns out that the proposed method is effective practically.



(a) Step response at point 1 (b) Step response at point 2 (c) Step response at point 3

(d) Step response at point 1(PI) (e) Step response at point 2(PI) (f) Step response at point 3(PI)

(g) Dynamic load responses(2DOF) (h) Dynamic load responses(PI)

Fig. 3. Experimental results of output voltage responses

5 Conclusion

In this paper, the concept of controller that for the boost DC-DC converter to attain good robustness was given. The proposed digital controller was implemented on the micro-processor. The DC-DC converter built-in this micro-processor was manufactured. It was shown from experiments that the proposed approximate 2DOF digital controller can suppress the variation of step responses in each operating point and the output voltage variation in sudden load change. This fact demonstrates the usefulness and practicality of the proposed method.

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