

# A new polarization estimation method based on spatial polarization characteristic of antenna

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**Abstract:** The spatial polarization characteristic of radar antenna is modeled in this paper. And based on it, a novel polarization estimation method of incoming wave signal is proposed. It is indicated that co-polarized component and cross-polarized component of antenna satisfy even-odd symmetry property, respectively. This conclusion is confirmed by electromagnetic computation and real test data. Our method does not need auxiliary orthogonal polarized antenna. The polarization can be estimated by using only received signal power. The accuracy of measurement method is guaranteed and the realization complexity is reduced. Simulation experiment proved the availability of the method.

**Keywords:** polarization estimation, spatial polarization, symmetry, signal power

**Classification:** Microwave and millimeter wave devices, circuits, and systems

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## 1 Introduction

According to antenna theory, there is a defined polarization direction of antenna with a given frequency and space point in the far-field area. When the antenna frequency is constant, the antenna polarization of radiation field vary with different observe direction, suggesting that the antenna polarization is a function of the spatial direction. The antenna polarization changing in different positions and oriented directions can be named as spatial polarization characteristic (SPC) of antenna [1, 2]. Polarization estimation is a key step in polarization information processing [3, 4], which can only be estimated by adopting multiple polarization antennas and multiple polarization channels. Single polarized radar is unable to measure polarization which directly restricted its processing ability in polarization area. A new polarization estimation method based on the amplitude symmetry and phase symmetry of SPC of single polarized antenna is proposed. The received signal power in the antenna scanning process is used for avoiding the impact of phase measurement error on the polarization estimation. Simulation experiment results verify the effectiveness of the algorithm. This method can enable single polarized radar to have some capability to process polarization information.

## 2 Symmetry of the antenna polarization characteristics

Supposing the main polarization of antenna is  $\hat{\theta}$  polarization, and the cross polarization is  $\hat{\phi}$  polarized. Then the incoming wave polarization in the polarization base can be expressed as [5]

$$\mathbf{E} = E_1\hat{\theta} + E_2\hat{\phi} \quad (1)$$

For most mechanical scanning radars, the antennas are symmetrical about the vertical normal section; that is, the antenna on the plane is mirror symmetrical about  $\phi = 0$ , and therefore the scattered wave polarization is also mirror symmetrical.

$$\hat{\phi}(\theta, -\phi) = -\text{Mirror} [\hat{\phi}(\theta, \phi)] \quad (2)$$

where Mirror refers to mirror-seeking. By using symmetry, it is easy to know that the polarization of scattered wave must satisfy

$$\begin{aligned} E_1(\theta, \phi) &= E_1(\theta, -\phi) \\ E_2(\theta, \phi) &= -E_2(\theta, -\phi) \end{aligned} \quad (3)$$

It is known that  $E_1$  and  $E_2$  are functions of  $(\theta, \phi)$ , and the amplitude and phase of main polarization component are symmetrical about  $\phi = 0$ ; that is,  $E_v(\phi) = E_v(-\phi)$  satisfies symmetry properties. The amplitude of cross-polarization component is symmetrical at  $\phi = 0$  plane; its phase has a jump of  $180^\circ$ , which meets the odd symmetry. For polarization ratio,  $\gamma(\phi) = \gamma(-\phi)$ .

The phase satisfies  $\varphi(-\phi) = \varphi(\phi) + \pi$  and  $\varphi \approx \begin{cases} 90^\circ & 0 < \phi < \phi_0 \\ -90^\circ & -\phi_0 < \phi < 0 \end{cases}$ .

### 3 Polarization estimation algorithm

In the horizontal and vertical polarization base, the polarization of incoming wave can be expressed as  $\begin{bmatrix} J_H \\ J_V \end{bmatrix}$ . Suppose the polarization state is constant during antenna scanning intervals, where  $\phi$  is scan angle. Then the received signal voltage can be expressed as

$$v(\phi) = J_v E_v(\phi) + J_H E_H(\phi) \quad (4)$$

(4) can be rewritten in the polarization ratio form as

$$v(\phi) = J_v E_v(\phi) + J_H E_H(\phi) = J_v E_v(\phi) \left( 1 + e^{j[\varphi_1 + \varphi_2(\phi)]} \tan \gamma_1 \tan \gamma_2(\phi) \right) \quad (5)$$

where  $\frac{J_H}{J_v} = \tan \gamma_1 e^{j\varphi_1}$ ,  $\frac{E_H(\phi)}{E_v(\phi)} = \tan \gamma_2(\phi) e^{j\varphi_2(\phi)}$ .  $\gamma_1$  is polarization angle of incoming signal, and  $\varphi_1$  is polarization phase difference. The return signal power is then obtained

$$|v(\phi)|^2 = |J_v|^2 |E_v(\phi)|^2 + |J_H|^2 |E_H(\phi)|^2 + 2|J_v| |E_v(\phi)| |J_H| |E_H(\phi)| \cos[\varphi_1 + \varphi_2(\phi)] \quad (6)$$

By the symmetry of the antenna polarization characteristics,  $\varphi_2(-\phi) = \varphi_2(\phi) + \pi$ ,  $\gamma_2(\phi) = \gamma_2(-\phi)$ . Thus the odd and even components of signal power can be expressed as

$$v_{\text{even}} = \frac{|v(\phi)|^2 + |v(-\phi)|^2}{2} = |J_v|^2 |E_v(\phi)|^2 + |J_H|^2 |E_H(\phi)|^2 \quad (7)$$

$$v_{\text{odd}} = \frac{|v(\phi)|^2 - |v(-\phi)|^2}{2} = 2 \cos[\varphi_1 + \varphi_2(\phi)] |J_H| |E_H(\phi)| |J_v| |E_v(\phi)| \quad (8)$$

Based on (7), the polarization amplitude estimation  $\begin{bmatrix} |J_H|^2 \\ |J_V|^2 \end{bmatrix}$  can be realized by the least square estimation method, and the observation equation for the received signal power  $|v(\phi)|$  can be established.

The magnitude characteristic of the polarization vector is expressed as

$$\mathbf{J} = \begin{bmatrix} |J_V|^2 & |J_H|^2 \end{bmatrix} \quad (9)$$

Let the observed times be  $N$ , and let the measurement error be  $\mathbf{C}$ . Then the observation equation is

$$\mathbf{V}_{\text{even}} = \mathbf{H} \cdot \mathbf{J} + \mathbf{C} \quad (10)$$

$$\text{Where } \mathbf{H} = \begin{bmatrix} |E_V(\phi_1)|^2 & |E_H(\phi_1)|^2 \\ |E_V(\phi_2)|^2 & |E_H(\phi_2)|^2 \\ \vdots & \vdots \\ |E_V(\phi_N)|^2 & |E_H(\phi_N)|^2 \end{bmatrix}.$$

According to parametric estimation theory, the least squares estimation of  $\mathbf{J}$  is

$$\hat{\mathbf{J}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{V}_{\text{even}} \quad (11)$$

The estimated polarization magnitude is

$$\begin{cases} |\hat{J}_v| = \sqrt{\hat{\mathbf{J}}(1)} \\ |\hat{J}_h| = \sqrt{\hat{\mathbf{J}}(2)} \end{cases} \quad (12)$$

Similarly, the maximum likelihood estimation of polarization phase difference can be obtained by only using power information of return signals. The two estimated values are

$$\begin{cases} \hat{\varphi}_1 = \frac{1}{N} \sum_{n=1}^N \left[ a \cos \left( \frac{v_{odd,n}}{2 |\hat{J}_h| |\hat{J}_v| |E_v(\phi_n)| |E_h(\phi_n)|} \right) - \varphi_2(\phi) \right] \\ \hat{\varphi}_1 = \frac{1}{N} \sum_{n=1}^N \left[ -a \cos \left( \frac{v_{odd,n}}{2 |\hat{J}_h| |\hat{J}_v| |E_v(\phi_n)| |E_h(\phi_n)|} \right) - \varphi_2(\phi) \right] \end{cases} \quad (13)$$

In order to reduce the phase measurement ambiguity, the observation equation can be established and the polarization phase difference can be estimated by solving the least square method. Thus formula (8) can be rewritten as

$$\frac{v_{odd}}{2 |J_h| |E_h(\phi)| |J_v| |E_v(\phi)|} = \cos [\varphi_1 + \varphi_2(\phi)] \quad (14)$$

Therefore, the inverse cosine of above equation is given by

$$\varphi_1 = a \cos \left[ \frac{v_{odd}}{2 |J_h| |E_h(\phi)| |J_v| |E_v(\phi)|} \right] - \varphi_2(\phi) \quad (15)$$

As  $|E_h(\phi)|, |E_v(\phi)|, \varphi_2(\phi)$  is a known quantity, the observation equation is as follows:

$$\mathbf{Z} = \begin{bmatrix} a \cos \left[ \frac{v_{odd}}{2 |J_h| |E_h(1)| |J_v| |E_v(1)|} \right] - \varphi_2(1) \\ a \cos \left[ \frac{v_{odd}}{2 |J_h| |E_h(2)| |J_v| |E_v(2)|} \right] - \varphi_2(2) \\ \vdots \\ a \cos \left[ \frac{v_{odd}}{2 |J_h| |E_h(N)| |J_v| |E_v(N)|} \right] - \varphi_2(N) \end{bmatrix}^{N \times 1} \quad (16)$$

$$\mathbf{H} = [1 \quad 1 \dots 1]^{1 \times N} \quad (17)$$

Then we have the phase term

$$\hat{\varphi}_1 = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{Z} \quad (18)$$

By Eq. (11), there is an additive noise  $\mathbf{n}$  in the polarization estimation process. The variance is  $\sigma^2$  which can be considered to be complex Gaussian white noise, and the estimation error  $\tilde{\mathbf{h}} = \hat{\mathbf{h}} - \mathbf{h}$  can be expressed as

$$|\tilde{\mathbf{h}}| = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{n} \quad (19)$$

Furthermore,  $\tilde{\mathbf{h}}$  follows zero mean complex Gaussian distribution  $\tilde{\mathbf{h}} \sim N(0, R_{\tilde{\mathbf{h}}})$ , and its covariance matrix  $R_{\tilde{\mathbf{h}}}$  is

$$R_{\tilde{\mathbf{h}}} = \sigma^2 (\mathbf{H}^T \mathbf{H})^{-1} \quad (20)$$

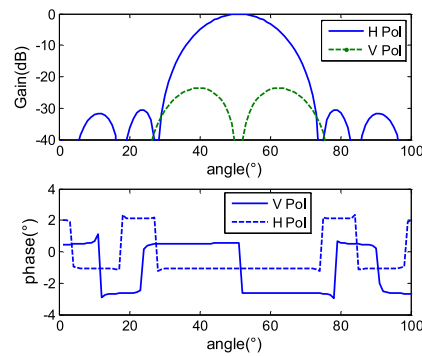
Equation set (20) is solvable when the  $\mathbf{H}^T \mathbf{H}$  is reversible

$$\mathbf{H}^T \mathbf{H} = \begin{bmatrix} \sum_{n=1}^{n=N} |E_V(\phi_n)|^4 & \sum_{n=1}^{n=N} |E_V(\phi_n)|^2 |E_H(\phi_n)|^2 \\ \sum_{n=1}^{n=N} |E_H(\phi_n)|^2 |E_V(\phi_n)|^2 & \sum_{n=1}^{n=N} |E_H(\phi_n)|^4 \end{bmatrix} \quad (21)$$

Due to the antenna spatial polarization characteristics, the two vectors of the matrix are linearly independent, thus proving that the above equation has solutions. So this algorithm is feasible.

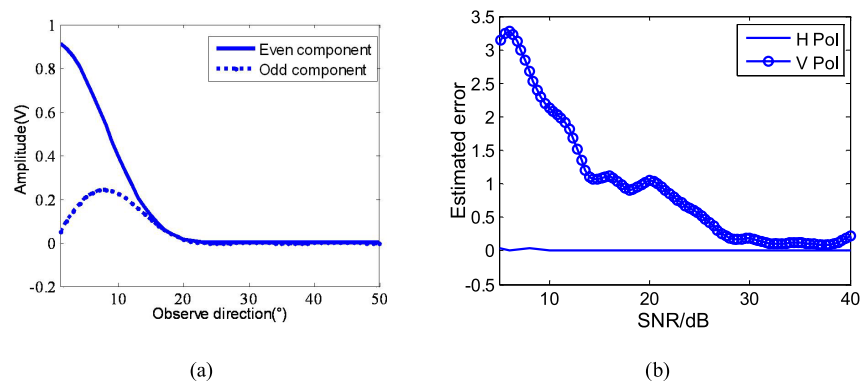
#### 4 Simulation experiment

Simulation experiment is carried to validate the proposed method. The amplitude and phase pattern of antenna main polarization component and cross-polarization component with respect to different directions are shown in Fig. 1, where azimuth interval is  $0.1^\circ$ , and the spatial sampling number is 101.



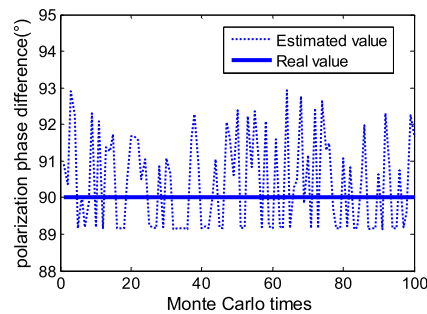
**Fig. 1.** Full polarization amplitude and phase pattern

Looking at Fig. 1, we observe that the main polarization component meets even symmetry properties and the cross polarization component meet odd symmetry property. The polarization parameter of the signal is  $\left[ \cos \frac{\pi}{4} \right]$ ,



**Fig. 2.** (a) Even and odd component of received signal power, (b) Polarization amplitude estimation performance curve

$\sin \frac{\pi}{4} \exp(j \frac{\pi}{2}) \Big]^T$ . By using the proposed method in Section 3, the odd component and even component of return signal power are obtained as shown in Fig. 2 (a). The polarization estimation performance as well as the estimation precision with respect to SNR is given in Fig. 2 (b). It is obvious that the horizon polarization component estimated is more precise than vertical polarization. The vertical polarization can obtain a good estimate in the high SNR condition. The statistics result of polarization phase difference estimation is shown in Fig. 3. The Monte Carlo simulation is carried out 100 times. The estimation error is close to  $\pm 3^\circ$ .



**Fig. 3.** Polarization phase difference estimation

## 5 Conclusion

Based on the antenna inherent symmetry characteristics, a novel polarization estimation method is given in the paper. The polarizations states can be estimated by using single channel instead of two orthogonal polarization channels. The amplitude and phase pattern data of antenna main and cross polarization components are taken as a known quantity, and then the polarization is estimated by processing the received signal power. Our method does not need conventional orthogonal polarization channel isolation thus reducing the system complexity, while guaranteeing the measurement accuracy. Theoretical deduction and experimental result verify the effectiveness of our method.

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