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INVITED PAPER *Special Section on Deepening and Expanding of Information Network Science*

# Improving Performance of Heuristic Algorithms by Lebesgue Spectrum Filter

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**SUMMARY** The previous researches on the chaotic CDMA have theoretically derived the chaotic sequences having the minimum asynchronous cross-correlation. To minimize the asynchronous cross-correlation, autocorrelation of each sequence have to be $C(\tau) \approx C \times r^{\tau}$, $r = -2 + \sqrt{3}$, dumped oscillation with increase of the lag $\tau$. There are several methods to generate such sequences, using a chaotic map, using the Lebesgue spectrum filter (LSF) and so on. In this paper, such lowest cross-correlation found in the chaotic CDMA researches is applied to solution search algorithms for combinatorial optimization problems. In combinatorial optimization, effectiveness of the chaotic search has already been clarified. First, an importance of chaos and autocorrelation with dumped oscillation for combinatorial optimization is shown. Next, in order to realize ideal solution search, the LSF is applied to the Hopfield-Tank neural network, the 2-opt method and the 2-exchange method. Effectiveness of the LSF is clarified even for the large problems for the traveling salesman problems and the quadratic assignment problems.

***key words:*** *chaos, combinatorial optimization, lebesgue spectrum filter, CDMA, traveling salesman problema, quadratic assignment problems*

## 1. Introduction

In the previous researches applying chaotic sequences to the code division multiple access (CDMA), effectiveness of chaos has been clarified [1]–[3]. For realizing high performance in the CDMA, cross-correlation among the spreading sequences should be small. In Ref. [1], by analyzing chip-asynchronous CDMA, the Markov chain for realizing the lowest cross-correlation has been derived, and chaotic maps to generate such sequences with lowest cross-correlation have been proposed. A FIR filter for generating such optimal chaotic CDMA sequences has also been proposed, and the advantage of the sequences has been experimentally shown [3].

The theory of such sequences having lowest cross-correlation can be applied to various field. Minimization of the cross-correlation is important not only in wireless communications. As one of the applications of such a theory, solution search algorithms for combinatorial optimization problem can also be improved by low cross-correlation. In NP-hard combinatorial optimization problems, it is very important to develop effective heuristic solution search algorithms to find good near-optimum solutions, because it is impossible to get exact optimum solution in large-scale problems. In such solution search algorithms for large-scale problems, the search space is quite high dimensional and there exist a lot of local optima. Therefore, if we apply a neighboring solution search algorithm which only moves to better state, it easily stops at a local optimum. In order to improve the solution search performance, meta-heuristic algorithms have been applied to such solution search algorithms, such as the simulated annealing based on stochastic dynamics, the tabu search based on deterministic avoidance of searching same part of the solution space, and so on.

In a solution search in a high dimensional space, directions of the state change should be fully randomized in order to enable search of wide area of the solution space. Such a wide movement of the searching state can be realized by minimizing cross-correlation among the axes of moving directions. Furthermore, most of the heuristic solution search algorithms are asynchronous. Therefore, in order to improve the solution search dynamics, the theory of the chaotic CDMA, which minimizes the asynchronous cross-correlation, can be applied to the heuristic search algorithms to realize theoretically best meta-heuristics.

Effectiveness of the chaotic dynamics for combinatorial optimization has been also shown in a lot of previous researches [4]–[12]. In the earliest research on the chaotic optimization, chaotic dynamics could only be applied to very small toy problems. The latest chaotic approach that combines heuristic algorithms to chaotic dynamics has much higher performance [7] and is more effective than the tabu search in benchmark NP-hard problems [8], [9]. Theoretical analyses of effectiveness of chaotic search have also been conducted. One of important features of the effectiveness of chaos is that a specific autocorrelation of the chaotic dynamics has a positive effect on finding better solutions [12].

In this paper, we apply the theory of chaotic CDMA to combinatorial optimization algorithms. Ideal low cross-correlation to search wide area of the solution space is added to the algorithms. As typical NP-hard benchmark problems, traveling salesman problem (TSP), and quadratic assignment problem (QAP) are introduced. The performance of the proposal is analyzed by applying the ideal low cross-correlation to the Hopfield-Tank neural networks, the 2-opt method and 2-exchange method, In order to give lowest cross-correlation to these algorithms, we use the Lebesgue spectrum filter (LSF), which has been proposed in Ref. [3]. The performance of the proposed meta-heuristic algorithms are evaluated on the benchmark problems and the effectiveness of the lowest asynchronous cross-correlation derived for the chaotic CDMA is clarified also in combinatorial optimiza-

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tion.

## 2. Chaotic Sequences Having Lowest Asynchronous Cross-Correlation

In Ref. [1], the lowest cross-correlation for chip-asynchronous DS/CDMA has been analyzed in detail. In discrete time systems, white noise sequences, whose auto-correlation is zero, have the lowest cross-correlation among the sequences. However, in continuous time asynchronous systems, the lowest cross-correlation among the sequences is achieved by those with negative autocorrelation. The asynchronous cross-correlation among the negative autocorrelation sequences becomes smaller than that of the white noise sequences. In this section, the autocorrelation of the sequences which have lowest asynchronous cross-correlation is derived, and a chaotic map to generate such sequences is shown [1].

### 2.1 Lowest Chip-Asynchronous Cross-Correlation

The chip-asynchronous cross-correlation is defined in Fig. 1. Two sequences, $\mathbf{X}$ and $\mathbf{Y}$, have two states, 1 or $-1$. Timing of changing the state is not synchronized. As defined in the analysis on the chip-asynchronous DS/CDMA [1], a chip is an the integer part of timing difference, which is expressed by $l$. The decimal part of timing difference is expressed by $\epsilon$, which is a positive number smaller than 1 ($0 < \epsilon < 1$).

The chip-asynchronous cross-correlation between $\mathbf{X}$ and $\mathbf{Y}$ can be formulated as the following equation,

$$I = (1-\epsilon)R_N^{E/O}(l;\mathbf{X},\mathbf{Y}) + \epsilon R_N^{E/O}(l+1;\mathbf{X},\mathbf{Y}), \qquad (1)$$

where, $R_N^{E/O}$ is even and odd cross-correlation functions, which can be expressed as follows,

$$R_N^{E}(l;\mathbf{X},\mathbf{Y}) = \sum_{n=0}^{N-l-1} X_n Y_{n+l} + \sum_{n=0}^{l-1} X_{n+N-l} Y_n, \qquad (2)$$

$$R_N^{O}(l;\mathbf{X},\mathbf{Y}) = \sum_{n=0}^{N-l-1} X_n Y_{n+l} - \sum_{n=0}^{l-1} X_{n+N-l} Y_n. \qquad (3)$$

$X_n$ is the value of $n$th chip of $\mathbf{X}$ and $N$ is the length of the sequences.

As the amplitude of the chip-asynchronous cross-correlation $I$, the expected value of $I^2/N$ is calculated in the followings. Since $\epsilon$ and $R_N^{E/O}(l;\mathbf{X},\mathbf{Y})$ are independent on each other, $E[I^2/N]$ can be expressed as follows,

$$\begin{aligned} E\left[\frac{I^2}{N}\right] &= \frac{1}{N}E\left[I^2\right] \\ &= \frac{1}{N}E\left[(1-\epsilon)^2\right]E\left[\left\{R_N^{E/O}(l;\mathbf{X},\mathbf{Y})\right\}^2\right] \\ &\quad + \frac{1}{N}E\left[\epsilon^2\right]E\left[\left\{R_N^{E/O}(l+1;\mathbf{X},\mathbf{Y})\right\}^2\right] \\ &\quad + \frac{1}{N}E\left[2\epsilon(1-\epsilon)\right] \\ &\quad \cdot E\left[R_N^{E/O}(l;\mathbf{X},\mathbf{Y})R_N^{E/O}(l+1;\mathbf{X},\mathbf{Y})\right] \end{aligned} \qquad (4)$$

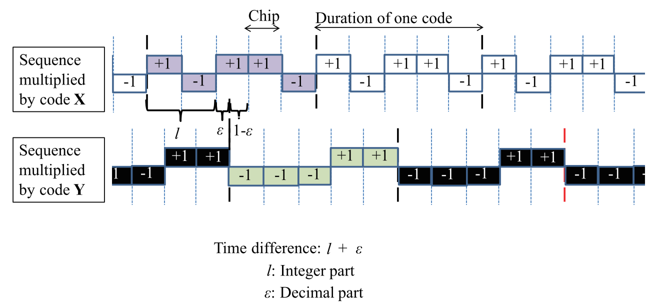


**Fig. 1** Chip-asynchronous cross-correlation.

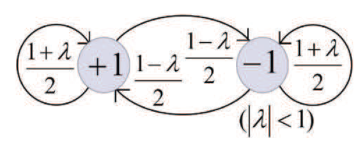

**Fig. 2** The Markov chain of two value sequences.

$\mathbf{X}$ and $\mathbf{Y}$ are assumed to be generated by the Markov chain shown in Fig. 2. Based on this assumption, $E[X_n X_{n+l}] = \lambda^l$ can be obtained. Because the values of $\epsilon$ has uniform distribution, $E[\epsilon] = \frac{1}{2}$ and $E\left[\epsilon^2\right] = \frac{1}{3}$.

When $N$ is large enough, the first term of Eq. (4) becomes as follows with the assumption that $X_m X_n$ and $Y_{m-l}Y_{n-l}$ are independent on each other,

$$\lim_{N\to\infty}\frac{1}{N}E\left[(1-\epsilon)^2\right]E\left[\left\{R_N^{E/O}(l;\mathbf{X},\mathbf{Y})\right\}^2\right] = \frac{1}{3}\left(\frac{1+\lambda^2}{1-\lambda^2}\right). \qquad (5)$$

The second term of Eq. (4) can be also calculated as follows,

$$\lim_{N\to\infty}\frac{1}{N}E\left[(1-\epsilon)^2\right]E\left[\left\{R_N^{E/O}(l+1;\mathbf{X},\mathbf{Y})\right\}^2\right] = \frac{1}{3}\left(\frac{1+\lambda^2}{1-\lambda^2}\right). \qquad (6)$$

The third term becomes as follows,

$$\begin{aligned} &\lim_{N\to\infty}\frac{1}{N}E\left[2\epsilon(1-\epsilon)\right] \\ &\cdot E\left[R_N^{E/O}(l;\mathbf{X},\mathbf{Y})R_N^{E/O}(l+1;\mathbf{X},\mathbf{Y})\right] \\ &= \frac{1}{3}\left(\frac{2\lambda}{1-\lambda^2}\right). \end{aligned} \qquad (7)$$

When $N$ is large enough, the amplitude of chip-asynchronous cross-correlation between $\mathbf{X}$ and $\mathbf{Y}$ in Eq. (4) can be obtained as follows by Eqs. (5), (6) and (7),

$$E\left[I^2/N\right] = \frac{2\left(1+\lambda+\lambda^2\right)}{3\left(1-\lambda^2\right)} \qquad (8)$$

This equation clarifies that the theoretically smallest chip-asynchronous cross-correlation can be generated by using

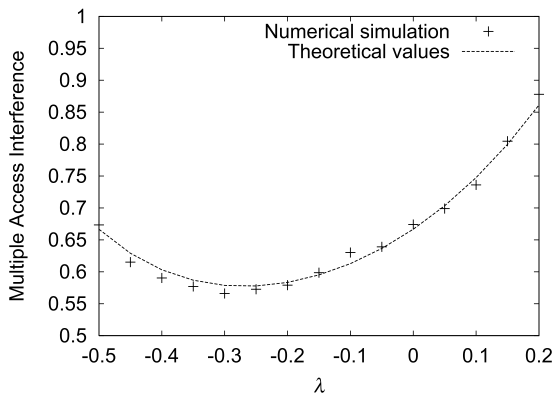


**Fig. 3** The amplitude of chip-asynchronous cross-correlation.

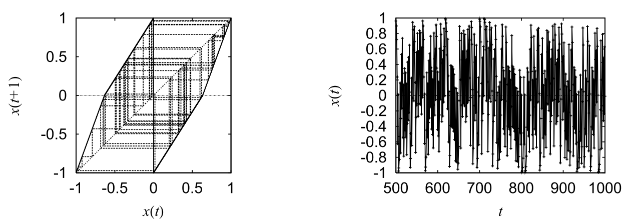


**Fig. 4** The Kalman map to generate the Markov sequence.

$\lambda = -2 + \sqrt{3}$. Figure 3 shows the dependency of $E\left[I^2/N\right]$ on $\lambda$. $E\left[I^2/N\right]$ becomes smallest when $\lambda = -2 + \sqrt{3}$. Therefore, it is clarified that the lowest asynchronous cross-correlation can be realized using the sequences whose auto-correlation is $C(\tau) \approx C \times r^\tau$, $r = -2 + \sqrt{3}$.

### 2.2 Generating the Lowest Cross-Correlated Sequences

In order to generate the chaotic code following the Markov chain in Fig. 2, the Kalman map can be used [1]. We can generate the sequence by the following equations,

$$x(t+1) = \begin{cases} \frac{2x-\lambda+1}{\lambda+1} & (-1 < x < \frac{\lambda-1}{2}), \\ \frac{2x-\lambda+1}{\lambda-1} & (\frac{\lambda-1}{2} \leq x < 0), \\ \frac{2x+\lambda-1}{\lambda-1} & (0 \leq x < \frac{\lambda-l}{2}), \\ \frac{2x+\lambda-1}{\lambda+1} & (\frac{\lambda-1}{2} \leq x < 1), \end{cases} \tag{9}$$

which is shown in Fig. 4.

In Ref. [3], the LSF has been applied to generate lowest cross-correlation, by giving negative autocorrelation to each sequence. The LSF is simple FIR filter,

$$\hat{f}(t) = \sum_{u=0}^{M} r^u f(t-u). \tag{10}$$

By setting $r = -2+\sqrt{3}$, we can give the autocorrelation minimizing chip-asynchronous cross-correlation to the sequence $f(t)$. The LSF has already applied to the spreading codes and have been shown effective to improve the performance of the DS/CDMA [3]. In this paper, the LSF will be applied to solution search algorithms to improve their performances.

## 3. Chaos Applied to Combinatorial Optimization

Effectiveness of the chaotic dynamics for combinatorial optimization has been shown by many researchers [4]–[12]. The first methods applying chaos to combinatorial optimization [4]–[6] was for the Hopfield-Tank neural networks [14]. Reference [4] transformed each neuron in the Hopfield-Tank neural networks to the form having chaotic dynamics, and showed the effectiveness of the chaotic dynamics for solution search. Reference [5] applied the chaotic neural network [15], [16] to Hopfield-Tank neural networks. The chaotic simulated annealing has also been proposed in Ref. [6]. The methods to use additive chaotic sequence to each neuron in the Hopfield-Tank neural networks have also been shown more effective than adding stochastic noise [11].

In order to apply such effective chaotic dynamics to large-scale problems, meta-heuristic algorithm using chaotic dynamics was proposed in Ref. [7]. Furthermore, very high performance chaotic meta-heuristic search [8], [9] was also proposed by realizing tabu search using the chaotic neural networks [15], [16]. This chaotic algorithm was implemented on circuits in order to run it in very high-speed [13].

## 4. Improving Hopfield-Tank Neural Network by Negative Autocorrelation

As benchmark combinatorial optimization problems to evaluate the performance of the algorithms, this paper introduces the TSPs and the QAPs. In this section, the Hopfield-Tank neural network [14] is used as a basic algorithm of the solution search. Because a search by the original Hopfield-Tank neural network stops at a local minimum and because its performance is poor, chaotic sequences are added to each neuron to avoid trapping at such undesirable states and much higher performance has been achieved.

The energy function of the basic neural network for the TSPs can be formulated as follows,

$$E_T = A[\{\sum_{i=1}^{N}(\sum_{k=1}^{N} x_{ik}(t)-1)^2\}+\{\sum_{k=1}^{N}(\sum_{i=1}^{N} x_{ik}(t)-1)^2\}] \\ +B\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} d_{ij}x_{ik}(t)\{x_{jk+1}(t)+x_{jk-1}(t)\}, \tag{11}$$

where $N$ is the number of cities, $d_{ij}$ is the distance between the cities $i$ and $j$, $x_{ij}(t)$ is the output of the $(i,j)$th neuron at time $t$. When $x_{ij}(t) = 1$, the city $i$ is visited in the $j$th order. $A$ and $B$ are the weight for the constraint (formation of a closed tour) and the objective (minimization of total tour length). From Eq. (11), the connection weights between $(i,j)$th and $(k,l)$th neurons $w_{ijkl}$ and the threshold of neurons $\theta_{ij}$ can be obtained as follows,

$$w_{ijkl} = -A\{\delta_{ij}(1-\delta_{kl}) + \delta_{kl}(1-\delta_{ij})\} \\ - Bd_{ij}(\delta_{lk+1} + \delta_{l-k-1}), \tag{12}$$

$$\theta_{ij} = 2A, \tag{13}$$

where $\delta_{ij}$ is the Kronecker delta, $\delta_{ij} = 1$ when $i = j$, otherwise $\delta_{ij} = 0$.

For the QAPs whose objective function is

$$F(\boldsymbol{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} b_{p(i)p(j)}, \tag{14}$$

the energy function can be formulated as follows,

$$\begin{aligned} E_Q = A[\{\sum_{i=1}^{N}(\sum_{k=1}^{N} x_{ik}(t) - 1)^2\} + \{\sum_{k=1}^{N}(\sum_{i=1}^{N} x_{ik}(t) - 1)^2\}] \\ + B\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} a_{ij} b_{kl} x_{ik}(t) x_{jl}(t). \end{aligned} \tag{15}$$

When $x_{ij}(t) = 1$, the unit $i$ is assigned on the $j$th index. From Eq. (15), the connection weight and the threshold for the QAPs are obtained as follows,

$$w_{ijkl} = -A\{\delta_{ij}(1 - \delta_{kl}) + \delta_{kl}(1 - \delta_{ij})\} - Ba_{ij}b_{kl}, \tag{16}$$

$$\theta_{ij} = 2A. \tag{17}$$

## 4.1 Performance of the Neural Networks with Chaotic Noise

We use the following neuron update equation to apply additive noise to the neural network,

$$x_{ik}(t+1) = f[\sum_{j=1}^{N}\sum_{l=1}^{N} w_{ikjl} x_{jl}(t) + \theta_{ik} + \beta z_{ik}(t)], \tag{18}$$

where $z_{ij}(t)$ is a noise sequence adding to the $(i, j)$th neuron, $\beta$ is the amplitude of noise, and $f$ is the sigmoidal output function, $f(y) = 1/(1 + \exp(-y/\epsilon))$. The noise sequence introduced as $z_{ij}(t)$ is normalized to zero mean and unit variance.

Figures 5 and 6 show the results of the logistic map chaos, $z^c_{ij}(t+1) = az^c_{ij}(t)(1 - z^c_{ij}(t))$, and the white Gaussian noise used as the additive noise for the neural network. For the logistic map chaos, 3.82, 3.92, and 3.95 are used for the parameter $a$. The horizontal axis is the amplitude of the noise, $\beta$ in Eq. (18). The solvable performance on the ordinate is defined as the percentage of the optimum solution obtained in 1000 runs with different initial conditions. Achieving the optimum solution at each run is defined as hitting the optimum solution value at least once in a fixed iteration. Exact solution methods can find the exact solutions for each problem introduced in this paper, but those methods require larger amount of computation to obtain the solutions. In this paper, the solvable performance of each type of noise are evaluated using a small and fixed computational amount. The cutoff times of each run are fixed at 1024 iterations for TSP and at 4096 iterations for QAP, respectively. The parameters of the neural network are $A = 1, B = 1$, and $\epsilon = 0.3$

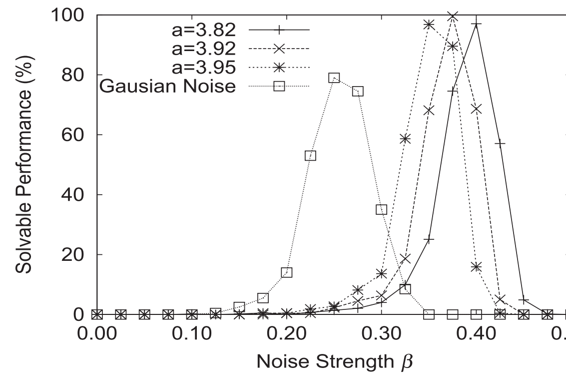


**Fig. 5** Solvable performance of chaotic noise and white Gaussian noise on the TSP.

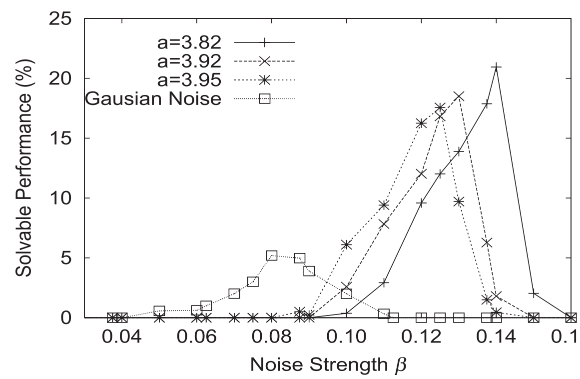


**Fig. 6** Solvable performance of chaotic noise and white Gaussian noise on the QAP.

for the TSP and $A = 0.35, B = 0.2$, and $\epsilon = 0.075$ for the QAP, respectively. The problems introduced in this paper are a 20-city TSP in [12] and a QAP with 12 nodes, nug12 in QAPLIB [17].

The results in both Figs. 5 and 6 show that the chaotic noise performs much better than the stochastic noise, on a comparison of the peaks of the solvable performances as the noise amplitude changes. The noise amplitude values for the best performance are different among the noise sequences. This is not because of a difference in the variances of the original sequences because each sequence is normalized before being added as $z_{ik}(t)$.

## 4.2 Improving Performance of Neural Networks by LSF

The autocorrelation coefficients of chaotic sequences used in the results of Figs. 5 and 6 are shown in Fig. 7. The figure shows that the autocorrelation of the effective chaotic sequences has a negative value at lag 1 and dumped oscillation. In the previous section, it is shown that minimum asynchronous cross-correlation can be realized using the sequences having autocorrelation, $C(\tau) \approx C \times r^\tau, r < 0$. In this subsection, such autocorrelation $C(\tau) \approx C \times r^\tau$ is applied to the Hopfield-Tank neural network and the effectiveness of the negative autocorrelation in solvable performance is evaluated.

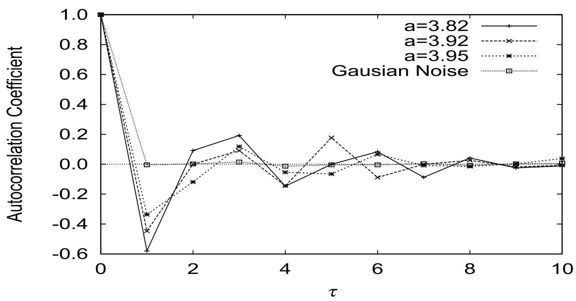


**Fig. 7** Autocorrelation coefficients of chaotic noise that has high solvable performance.

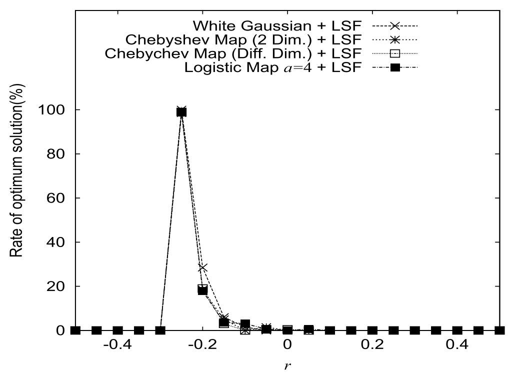


**Fig. 8** Solvable performance on the 20-city TSP with changing parameter $r$ in autocorrelation $C(\tau) \approx C \times r^{\tau}$.

In order to give such autocorrelation, we apply the LSF to the Hopfield-Tank neural networks. The modified update equation with the LSF is defined as follows,

$$y_{ik}(t+1) = \sum_{j=1}^{N}\sum_{l=1}^{N} w_{ikjl}x_{jl}(t) + \theta_{ik} + \beta z_{ik}(t) \qquad (19)$$

$$\hat{y}_{ij}(t+1) = \sum_{u=0}^{M} r^{u} y_{ij}(t+1-u) \qquad (20)$$

$$x_{ik}(t+1) = 1/(1+\exp(-\hat{y}_{ik}(t+1)/\epsilon)). \qquad (21)$$

In this formulation, the internal state is defined as $y(t)$ and the LSF is applied to it. The LSF can be transformed to the following form suitable for numerical computation,

$$\hat{y}_{ik}(t+1) = r\hat{y}_{ik}(t) + y_{ik}(t+1). \qquad (22)$$

Figures 8 and 9 show the results of the TSP and the QAP solved with stochastic and chaotic noise whose autocorrelation is $C(\tau) \approx C \times r^{\tau}$ as $r$ is varied. The solvable performance is evaluated as the percentage of obtaining optimum solutions in 1000 runs with different random initial conditions. In this simulation, the several types of white noise is used for $z_{ik}(t)$ in order to tune the autocorrelation only by the parameter $r$.

Figures 8 and 9 clearly show that negative $r$ induces higher performance. The results demonstrate that a negative autocorrelation with oscillation ($r < 0$) has a higher performance than white noise ($r = 0$) and positive autocorrelation

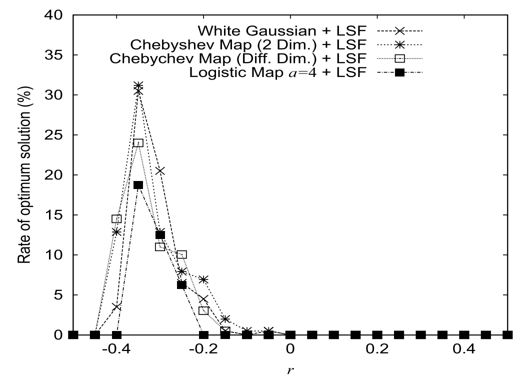


**Fig. 9** Solvable performance on the 12-node QAP with changing parameter $r$ in autocorrelation $C(\tau) \approx C \times r^{\tau}$.

noise ($r < 0$). In comparing the best results with Figs. 5 and 6, we can see that stochastic noise with a negative autocorrelation has almost the same performance as chaotic noise. As shown in Fig. 7, chaotic noise also has a negative autocorrelation with damped oscillation.

## 5. Improving Heuristic Search by LSF

In the previous section, performance improvement by the LSF and the negative autocorrelation has been analyzed on the Hopfield-Tank neural network algorithm. However, the Hopfield-Tank neural network is not applicable to large-scale problems, because it requires $N \times N$ neurons for the problem size $N$ and the number of the mutual connections between the neurons becomes the order of $N^4$. In this section, effectiveness of the LSF and negative autocorrelation is also analyzed on heuristic algorithms, which can be easily applied to much larger combinatorial optimization problems.

### 5.1 Improving 2-Opt for TSP by LSF

The 2-opt method is a simple algorithm applicable to large-scale TSPs. Various meta-heuristic algorithms have been realized based on the 2-opt. As shown in Fig. 10, the 2-opt flips two pairs of links with keeping a closed tour. It is possible to improve the solution by the flipping when the tour length can be shorten. However, such a simple search only decreasing the tour length stops at an undesirable local minimum. In the conventional meta-heuristics, stochastic random fluctuations or tabu searches have been applied to this simple algorithm to improve the solutions.

In the previous works, effectiveness of chaos on the meta-heuristics has also been shown. In Ref. [7], chaotic dynamics has been applied to the 2-opt and its effectiveness has been clarified. Reference [9] has proposed chaotic tabu search, which has been realized by using refractory effects of the chaotic neural networks. Effectiveness of the chaotic tabu search applied to the 2-opt was shown in very large-scale TSPs.

In the general 2-opt method, each flip will be applied one by one. Therefore, in the high-dimensional searching

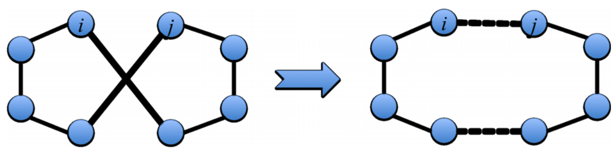


**Fig. 10** 2-opt method for TSP.

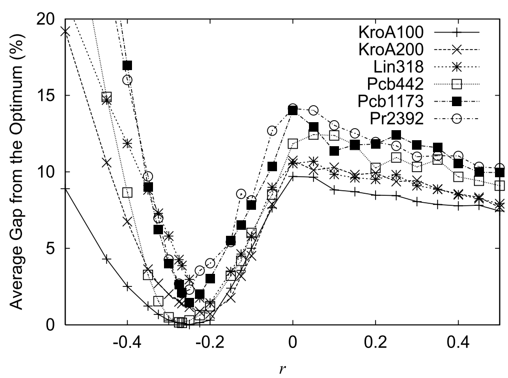


**Fig. 11** Performance of 2-opt method with LSF.

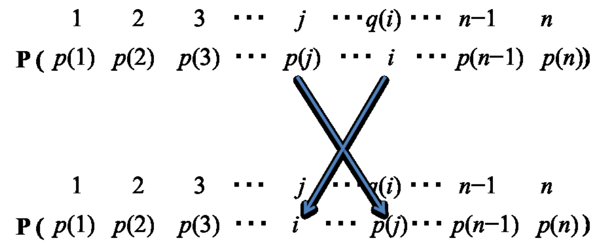


**Fig. 12** 2-exchange method for QAP.

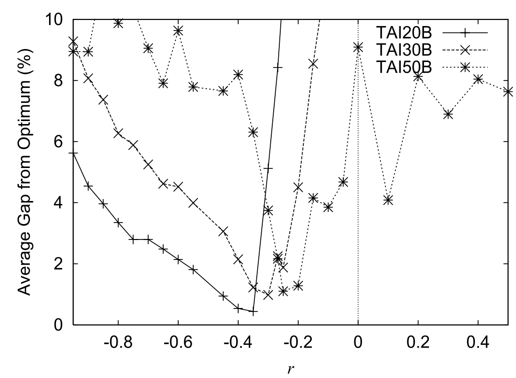


**Fig. 13** 2-exchange method with the LSF.

space of the TSP, 2-opt will improve the solution on each axis asynchronously. For such asynchronous algorithm, we can apply the ideal low cross-correlation to improve the solution, similarly to the case of the Hopfiled-Tank neural networks.

In order to apply the LSF to the 2-opt method, we use an update equation which has been defined in Ref. [7]. $\Delta_{ij}(t)$ is the improvement of the solution when the 2-opt flip connecting the city $i$ to $j$ was applied. The original 2-opt method can be realized by defining $x_{ij}(t+1) = \Delta_{ij}(t+1)$ and updating the solution by the corresponding flip only the case $x_{ij}(t+1) > 0$. The LSF can be applied to this 2-opt method as follows,

$$\hat{x}_{ij}(t+1) = \sum_{u=0}^{M} r^u x_{ij}(t-u) = \sum_{u=0}^{M} r^u \Delta_{ij}(t-u). \tag{23}$$

When $\hat{x}_{ij}(t+1) > 0$, the 2-opt flip connecting the city $i$ with $j$ will be applied. The autocorrelation of each flip will be close to $C(\tau) \approx C \times r^\tau$ and the lowest cross-correlation among the flip will be realized. We use the following form of the 2-opt with LSF suitable for the numerical simulations,

$$\hat{x}_{ij}(t+1) = r\hat{x}_{ij}(t) + \Delta_{ij}(t). \tag{24}$$

Figure 11 show the results on the 100-city TSP (KroA100), 200-city TSP (KroA200), 318-city TSP (Lin318), 442-city TSP (Pcb442), 1173-city TSP (Pcb1173) and 2394-city TSP (Pr2394) [18]. The performances are evaluated by the difference of the obtained average solutions from the optimum solution. The average solution is obtained by 100 runs with different initial conditions, and cutoff time of each run is 50000 iterations.

From Fig. 11, the best performance could be obtained by negative $r$. The best performance could be realized around $-2+\sqrt{3}$, which has been clarified to make the minimum cross-correlation.

## 5.2 Improving 2-Exchange for QAP by LSF

Performance improvement by the LSF is evaluated also on the QAP. As the basic flip of the solution, we usually apply the exchange of two elements as shown in Fig. 12, We can use the same update rule as the case of the 2-opt defined in Eq. (24).

Figure 13 shows the performances of 2-exchange with LSF on 5 benchmark QAPs from the QAPLIB [17], Tai20b, Tai30b, Tai50b, Tai100b and Tai150b. The results also show that the performance becomes best when $r$ is negative. Also from these results, it could be confirmed that the performance of the heuristic algorithms can be improved by the low-cross correlation realized by the LSF.

## 6. Conclusion

This paper applied the theory of minimizing the asynchronous cross-correlation to improvement of heuristic searching algorithms for combinatorial optimization problems. We have introduced the LSF to generate the ideal searching dynamics and applied it to improvement of the Hopfield-Tank neural networks, the 2-opt and the 2-exchange algorithms. By minimizing the cross-correlation in these asynchronously updating heuristic algorithms, their performance could be improved.

The theory of minimizing the asynchronous cross-correlation was originally investigated on the chaotic CDMA

[1]. By using the sequences having the autocorrelation with dumped oscillation, $C(\tau) \approx C \times r^{\tau}, r = -2 + \sqrt{3}$, asynchronous cross-correlation becomes minimum. This paper clarified effectiveness of such sequences also on combinatorial optimization.

The theory of chaotic CDMA may also improve performance of other methods in various fields, in which minimization of the cross-correlation is needed. There are several ways to generate such lowest cross-correlation such as the Kalman map, the LSF. and so on. In our future works, we try this theory also on other applications to improve performance by lowest cross-correlation.

## References

[1] T. Kohda and H. Fujisaki, "Pursley's aperiodic cross-correlation functions revisited," IEEE Trans. Circuits Syst. I, vol.50, no.6, pp.800–805, 2003.

[2] G. Mazzini, R. Rovatti, and G. Setti, "Interference minimisation by auto-correlation shaping in asynchronous DS-CDMA systems: Chaos-based spreading is nearly optimal," Electron. Lett., vol.35, no.13, pp.1054–1055, 1999.

[3] K. Umeno and A. Yamaguchi, "Construction of optimal chaotic spreading sequence Using Lebesgue Spectrum Filter," IEICE Trans. Fundamentals, vol.E85-A, no.4, pp.849–852, April 2002.

[4] H. Nozawa, "A neural network model as a globally coupled map and applications based on chaos," Chaos, vol.2, no.3, pp.377–386, 1992.

[5] T. Yamada and K. Aihara, "Nonlinear neurodynamics and combinatorial optimization in chaotic neural networks," J. Intelligent and Fuzzy Systems, vol.5, pp.53–68, 1997.

[6] L. Chen and K. Aihara, "Chaotic simulated annealing by a neural network model with transient chaos," Neural Networks, vol.8, no.6, pp.915–930, 1995.

[7] M. Hasegawa, T. Ikeguchi, and K. Aihara, "Combination of chaotic neurodynamics with the 2-opt algorithm to solve traveling salesman problems," Phys. Rev. Lett., vol.79, no.12, pp.2344–2347, 1997.

[8] M. Hasegawa, T. Ikeguchi, K. Aihara, and K. Itoh, "A novel chaotic search for quadratic assignment problems," Eur. J. Oper. Res., vol.139, no.3, pp.543–556, 2002.

[9] M. Hasegawa, T. Ikeguchi, and K. Aihara, "Solving large scale traveling salesman problems by chaotic neurodynamics," Neural Networks, vol.15, no.2, pp.271–283, 2002.

[10] M. Hasegawa, T. Ikeguchi, T. Matozaki, and K. Aihara, "Solving combinatorial optimization problems by nonlinear neural dynamics," Proc. International Conference on Neural Networks (ICNN'95), pp.3140–3145, 1995.

[11] Y. Hayakawa, A. Marumoto, and Y. Sawada, "Effects of the chaotic noise on the performance of a neural network model for optimization problems," Phys. Rev. E, vol.51, no.4, pp.R2693–R2696, 1995.

[12] M. Hasegawa, T. Ikeguchi, T. Matozaki, and K. Aihara, "An analysis on additive effects of nonlinear dynamics for combinatorial optimization," IEICE Trans. Fundamentals, vol.E80-A, no.1, pp.206–212, Jan. 1997.

[13] Y. Horio and K. Aihara, "Analog computation through high-dimensional physical chaotic neuro-dynamics," Physica D: Nonlinear Phenomena, vol.237, no.9, pp.1215–1225, 2008.

[14] J.J. Hopfield and D.W. Tank, "Neural computation of decision in optimization Problems," Biological Cybernetics, vol.52, pp.141–152, 1985.

[15] K. Aihara, T. Takabe, and M. Toyoda, "Chaotic neural networks," Phys. Lett. A, vol.144, no.6-7, pp.333–340, 1990.

[16] K. Aihara, "Chaotic neural networks," Bifurcation Phenomena in Nonlinear Systems and Theory of Dynamical Systems, H. Kawakami, ed., pp.143–161, World Scientific, 1990.

[17] R. Burkard, S. Karish, and F. Rendl, "QAPLIB-A quadratic assignment problem library," J. Global Optimization, vol.10, 391–403, 1997; http://www.seas.upenn.edu/qaplib

[18] G. Reinelt, TSPLIB, http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/

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