Performance Analysis of Block MSN Algorithm with Pseudo-Noise Control in Multi-User MIMO System

Nobuyoshi KIKUMA[†a], *Fellow*, **Kousuke YONEZU**[†], *Student Member*, *and* **Kunio SAKAKIBARA**[†], *Senior Member*

SUMMARY MU-MIMO (Multi-User Multiple Input and Multiple Output) has been considered as a fundamental technology for simultaneous communications between a base station and multiple users. This is because it can generate a large virtual MIMO channel between a base station and multiple user terminals with effective utilization of wireless resources. As a method of implementing MU-MIMO downlink, Block Diagonalization (BD) was proposed in which the transmission weights are determined to cancel interference between multiple user terminals. On the other hand, Block Maximum Signal-to-Noise ratio (BMSN) was proposed which determines the transmission weights to enhance the gain for each user terminal in addition to the interference cancellation. As a feature, BMSN has a pseudo-noise for controlling the null depth to the interference. In this paper, to enhance further the BMSN performance, we propose the BMSN algorithm that has the pseudo-noise determined according to receiver SNR. As a result of computer simulation, it is confirmed that the proposed BMSN algorithm shows the significantly improved performance in evaluation of bit error rate (BER) and achievable bit rate (ABR).
key words: Multi-User MIMO, block beamforming, BD algorithm, BMSN algorithm, pseudo-noise control

1. Introduction

Currently, we can see the explosive expansion of cellular networks and wireless LANs (WLANs) along with the growing popularity of smart phones and tablets. At the same time, it has presented the demand for achieving broadband wireless transmission within a limited frequency band. As is well known, the most effective and most attractive technology for a high transmission rate is multiple-input and multiple-output (MIMO) transmission [1]–[7]. MIMO promises to increase the channel capacity compared to single-input and single-output (SISO) systems. Therefore, MIMO has been incorporated into many of the latest wireless communication standards such as Long-Term Evolution (LTE) [8], WiFi [9], and Wi-MAX [10]. Moreover, MIMO communcations has shifted from point-to-point links to multiuser broadcast links, and multi-user MIMO (MU-MIMO) systems have recently attracted even more attention as a technology that enhances the total system capacity. It is because it can generate a large virtual MIMO channel between a base station and multiple user terminals (UTs) with effective utilization of wireless resources [11]–[14].

MU-MIMO transmission realizes communication with multiple terminal stations with a limited number of antennas called space division multiple access (SDMA) [15]–[17]. Actually, MU-MIMO transmission has been incorporated into the IEEE802.11ac standard [18] and LTE-Advanced standard [19], and also you can see the commercial products based on these standards. The standardization of next-generation WLAN also aims to achieve further high performance and high efficiency by using MU-MIMO transmission technology. From this technological background, MIMO/MU-MIMO transmissions are key technologies for the next-generation mobile radio network and WLAN systems.

Block Diagonalization (BD) is known as one representative of precoding algorithms with moderate complexity in MU-MIMO broadcast channel [20]–[22]. The BD algorithm creates the transmit weights so as to ensure zero inter-user and inter-stream interference in the received signals of each user. In addition, Block Maximum Signal-to-Noise ratio (BMSN) algorithm has been proposed as a modified method of BD algorithm [23]–[26]. The BMSN algorithm aims at achieving positive block beamforming for each user as well as reducing inter-user interference (IUI).

We pay attention here to BMSN algorithm, which features a pseudo-noise (diagonal loading) control that can vary the null depth to other users in MU-MIMO broadcast transmission. Therefore, it is expected that the performance of BMSN can be enhanced by using the pseudo-noise optimized for propagation environments. The Minimum Mean Square Error (MMSE) channel inversion algorithm [27]–[30] also has a similar concept to the BMSN in the sense that the diagonal loading is utilized. However, the BMSN realizes the user-by-user beamforming while the MMSE employs the total channel inversion of all users, and so the influence of other users in the BMSN is supposed to be different from that in the MMSE. In this paper, we examine the effect of pseudo-noise on BMSN performance, and demonstrate that the BMSN algorithm with the pseudo-noise adapted according to receiver SNR is effective compared to the BD and other BMSN algorithms with fixed values of pseudo-noise.

This paper is organized as follows. In Sect. 2, we explain the system model and linear transmission control algorithms of BD and BMSN, followed by typical ways of determining the pseudo-noise in BMSN. In Sect. 3, the bit error rate (BER) and the achievable bit rate (ABR) are evaluated for the algorithms presented here. The discussion about

the effect of pseudo-noise in BMSN is provided in Sect. 4. Finally, the concluding remarks are presented in Sect. 5.

2. Linear Transmission Control Algorithms in MU-MIMO Broadcast Channel

2.1 System Model

In this paper, we focus on the linear transmission control because of its low complexity of computation [22]. Figure 1 shows the system model for MU-MIMO broadcast channel. The numbers of transmit antennas, receive antennas, and users are N_T, N_R, and N_U, respectively, and the case where $N_R = 2$ is depicted in Fig. 1.

The total channel matrix is $H \in \mathbb{C}^{N_R \cdot N_U \times N_T}$ composed of individual user channel matrices denoted by $H^{(k)} \in \mathbb{C}^{N_R \times N_T}$ $(k = 1 \sim N_U)$, and $H^{(k)}$ has elements as follows:

$$H^{(k)} = \begin{bmatrix} h_{11}^{(k)} & \cdots & h_{1N_T}^{(k)} \\ \vdots & & \vdots \\ h_{N_R 1}^{(k)} & \cdots & h_{N_R N_T}^{(k)} \end{bmatrix} \tag{1}$$

where $h_{il}^{(k)}$ is the channel response for the l-th transmit antenna and i-th receive antenna for user k, and it is assumed in this paper that $h_{il}^{(k)}$ is the zero-mean unit-variance complex-Gaussian fading gain, which is called independent identically distributed (i.i.d.) Rayleigh flat fading.

The transmit signal vector at the t-th symbol is $s(t) \in \mathbb{C}^{N_R \cdot N_U \times 1}$, and it consists of the transmit signal vectors for all users, denoted by $s^{(k)}(t) \in \mathbb{C}^{N_R \times 1}$ $(k = 1 \sim N_U)$. The transmit weight matrix is $W \in \mathbb{C}^{N_T \times N_R \cdot N_U}$, and similarly it is constructed by $W^{(k)} \in \mathbb{C}^{N_T \times N_R}$ $(k = 1 \sim N_U)$ each of which denotes the weight matrix for user k.

Therefore, we have following relations:

$$H = \begin{bmatrix} H^{(1)} \\ \vdots \\ H^{(N_U)} \end{bmatrix}, \tag{2}$$

$$W = \begin{bmatrix} W^{(1)}, \cdots, W^{(N_U)} \end{bmatrix}, \tag{3}$$

and

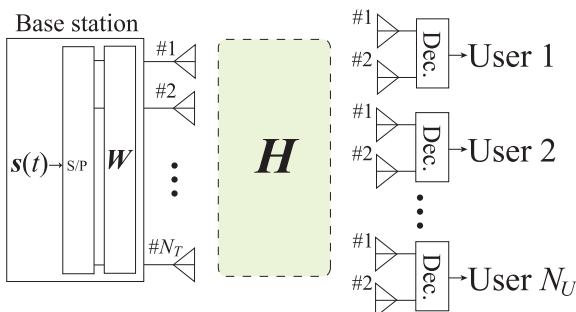**Fig. 1** System model of MU-MIMO broadcast channel ($N_R = 2$).

$$s(t) = \left[\left(s^{(1)}(t) \right)^T, \cdots, \left(s^{(N_U)}(t) \right)^T \right]^T. \tag{4}$$

At the receiver side, the receive signal of user k at the t-th symbol is denoted by $y^{(k)}(t) \in \mathbb{C}^{N_R \times 1}$ $(k = 1 \sim N_U)$, and so the receive signal vector for all users, $y(t) \in \mathbb{C}^{N_R \cdot N_U \times 1}$, is given by

$$y(t) = \left[\left(y^{(1)}(t) \right)^T, \cdots, \left(y^{(N_U)}(t) \right)^T \right]^T. \tag{5}$$

As a result, the receive signals $y^{(k)}(t)$ and $y(t)$ are expressed as follows:

$$y^{(k)}(t) = H^{(k)} W s(t) + n^{(k)}(t) \tag{6}$$

$$y(t) = H W s(t) + n(t) \tag{7}$$

$$n(t) = \left[\left(n^{(1)}(t) \right)^T, \cdots, \left(n^{(N_U)}(t) \right)^T \right]^T \tag{8}$$

where $n^{(k)}(t) \in \mathbb{C}^{N_R \times 1}$ denotes the internal noise vector at the receiver of user k and the elements of $n^{(k)}(t)$ are zero-mean Gaussian noise with variance of σ^2.

When the receive weight $W_r^{(k)} \in \mathbb{C}^{N_R \times N_R}$ is used at user k, the receive signal of user k, denoted by $y_r^{(k)}(t)$, can be expressed as follows:

$$y_r^{(k)}(t) = W_r^{(k)} \left(H^{(k)} W s(t) + n^{(k)}(t) \right). \tag{9}$$

2.2 Block Diagonalization (BD) Algorithm

In the multi-user broadcast links, the signal of user k should not be transmitted to all users except user k $(k = 1 \sim N_U)$, and thus we firstly prepare the matrix, $\bar{H}^{(k)}$, defined as,

$$\bar{H}^{(k)} = \begin{bmatrix} H^{(1)} \\ \vdots \\ H^{(k-1)} \\ H^{(k+1)} \\ \vdots \\ H^{(N_U)} \end{bmatrix} \in \mathbb{C}^{(N_U - 1) \cdot N_R \times N_T} \tag{10}$$

where $\bar{H}^{(k)}$ is a channel matrix excluding the channel matrix of user k, i.e., $H^{(k)}$, from H. Figure 2 represents the spatial channel conditions when the transmit weight for user k is observed in the system model of Fig. 1. Next, singular value decomposition (SVD) is applied to the matrix $\bar{H}^{(k)}$, resulting in

$$\bar{H}^{(k)} = \bar{U}^{(k)} \bar{D}^{(k)} (\bar{V}^{(k)})^H$$
$$= \bar{U}^{(k)} \left[\bar{D}_s^{(k)} \quad 0_{N_R \cdot (N_U - 1) \times (N_T - N_R \cdot (N_U - 1))} \right]$$
$$\cdot \left[\bar{V}_s^{(k)} \quad \bar{V}_n^{(k)} \right]^H \tag{11}$$

where $\bar{U}^{(k)} \in \mathbb{C}^{(N_U - 1)N_R \times (N_U - 1)N_R}$ and $\bar{V}^{(k)} \in \mathbb{C}^{N_T \times N_T}$ are unitary matrices consisting of all left singular vectors and of all right singular vectors, respectively. $\bar{D}^{(k)} \in$
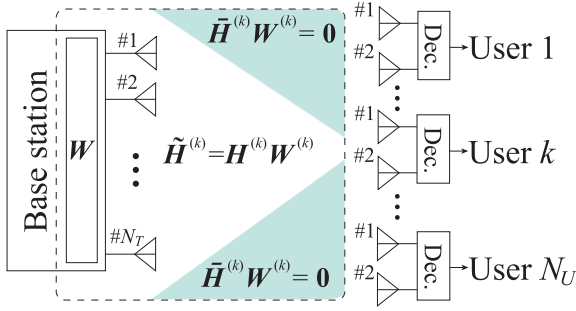
Fig. 2 Spatial channel conditions using transmit weight for user k in BD algorithm ($N_R = 2$).

$\mathbb{R}^{(N_U-1)N_R \times N_T}$ is the partial diagonal matrix consisting of all singular values. Also, $\bar{V}_s^{(k)} \in \mathbb{C}^{N_T \times (N_U-1)N_R}$ and $\bar{V}_n^{(k)} \in \mathbb{C}^{N_T \times (N_T-(N_U-1)N_R)}$ denote the right singular matrices, which consist of the singular vectors corresponding to nonzero singular values and zero singular values, respectively, and $\bar{D}_s^{(k)} \in \mathbb{R}^{(N_U-1)N_R \times (N_U-1)N_R}$ is the diagonal matrix consisting of nonzero singular values only. To suppress in advance the interferences of all users except user k, we choose the matrix $\bar{V}_n^{(k)}$ as the transmit weight for user k, which yields the following relationship between $\bar{V}_n^{(k)}$ and $\bar{H}^{(k)}$:

$$H^{(1)}\bar{V}_n^{(k)} = \cdots = H^{(k-1)}\bar{V}_n^{(k)} = H^{(k+1)}\bar{V}_n^{(k)} = \cdots$$
$$= H^{(N_U)}\bar{V}_n^{(k)} = \mathbf{0}_{N_R \times (N_T-(N_U-1)N_R)}. \quad (12)$$

Hence, user-by-user block diagonalization of channel matrix, i.e., inter-user interference cancellation, can be realized when $W^{(k)} = \bar{V}_n^{(k)}$.

As shown in Fig. 2, the channel matrix $\tilde{H}^{(k)} = H^{(k)}\bar{V}_n^{(k)} \in \mathbb{C}^{N_R \times (N_T-(N_U-1)N_R)}$ is regarded as that of single-user MIMO for user k. In the BD algorithm, the eigenmode transmission beamforming (EM-BF) [31], [32] is employed for the matrix $\tilde{H}^{(k)}$. Namely, applying SVD to $\tilde{H}^{(k)}$ gives

$$\tilde{H}^{(k)} = \tilde{U}^{(k)}\tilde{D}^{(k)}(\tilde{V}^{(k)})^H$$
$$= \tilde{U}^{(k)} \begin{bmatrix} \tilde{D}_s^{(k)} & \mathbf{0}_{N_R \times (N_T-N_R \cdot N_U)} \end{bmatrix} \begin{bmatrix} \tilde{V}_s^{(k)} & \tilde{V}_n^{(k)} \end{bmatrix}^H \quad (13)$$

where $\tilde{U}^{(k)} \in \mathbb{C}^{N_R \times N_R}$ and $\tilde{V}^{(k)} \in \mathbb{C}^{(N_T-(N_U-1)N_R) \times (N_T-(N_U-1)N_R)}$ are the left singular matrix and the right singular matrix of $\tilde{H}^{(k)}$, respectively, and $\tilde{D}^{(k)} \in \mathbb{R}^{N_R \times (N_T-(N_U-1)N_R)}$ is the partial diagonal singular value matrix. $\tilde{V}_s^{(k)} \in \mathbb{C}^{(N_T-(N_U-1)N_R) \times N_R}$ and $\tilde{V}_n^{(k)} \in \mathbb{C}^{(N_T-(N_U-1)N_R) \times (N_T-N_R \cdot N_U)}$ denote the right singular matrices corresponding to nonzero singular values and zero singular values, respectively, and $\tilde{D}_s^{(k)} \in \mathbb{R}^{N_R \times N_R}$ is the diagonal matrix consisting of nonzero singular values $\sqrt{\tilde{\lambda}^{(k)}(1)}, \cdots, \sqrt{\tilde{\lambda}^{(k)}(N_R)}$. We call here $\tilde{\lambda}^{(k)}(1), \cdots, \tilde{\lambda}^{(k)}(N_R)$ channel eigenvalues.

Finally, the total transmit weight of BD algorithm is given by

$$W_{BD}^{(k)} = \bar{V}_n^{(k)}\tilde{V}_s^{(k)}. \quad (14)$$

When using $W = [W_{BD}^{(1)}, \cdots, W_{BD}^{(N_U)}]$ and $W_r^{(k)} = (\tilde{U}^{(k)})^H$ in Eq. (9), the receive signal of user k, $y_r^{(k)}(t)$, is expressed as

$$y_r^{(k)}(t) = (\tilde{U}^{(k)})^H \cdot \left(H^{(k)}W_{BD}^{(k)}s^{(k)}(t) + n^{(k)}(t) \right)$$
$$= \tilde{D}_s^{(k)}s^{(k)}(t) + (\tilde{U}^{(k)})^H n^{(k)}(t) \quad (15)$$

considering Eq. (12). In this way, inter-substream interference is eliminated in the multi-substream transmission of each user.

2.3 Block Maximum SNR (BMSN) Algorithm

2.3.1 Basic Principle

BD algorithm is understood to obtain the transmit weight $W^{(k)}$ for user k from the following constrained minimization:

$$\min_{W^{(k)}} \|\bar{H}^{(k)}W^{(k)}\|_F^2 \quad (16)$$
$$\text{subject to } \|W^{(k)}\|_F^2 = \text{constant}$$

where $\bar{H}^{(k)}$ is the channel matrix of Eq. (10) and $\|\cdot\|_F$ stands for Frobenius norm. It is found from the above equation that BD algorithm is devoted to reduce inter-user interference to other users. This is similar to the criterion of the power inversion adaptive array [33]–[35].

On the other hand, the BMSN algorithm is based on the minimization of interference to other users while maintaining the high gain of one's own channel [23]–[26]. Therefore, it is expected that this algorithm brings about performance improvement of the eigenmode transmission of each user. The principle of BMSN algorithm to obtain the transmit weight $W^{(k)}$ for user k is described as follows:

$$\min_{W^{(k)}} \|\bar{H}^{(k)}W^{(k)}\|_F^2 \quad (17)$$
$$\text{subject to } H^{(k)}W^{(k)} = T^{(k)}$$

where $T^{(k)} \in \mathbb{C}^{N_R \times N_R}$ is a constant matrix corresponding to the desired channel matrix of each user. Equation $H^{(k)}W^{(k)} = T^{(k)}$ is referred to as the beamforming condition for transmit weights. Furthermore, this problem is mathematically equivalent to the maximum transmission DUR (Desired signal to Undesired signal power Ratio) provided by

$$\max_{W^{(k)}} \left(\text{DUR}^{(k)} \right)$$

$$\text{with } \text{DUR}^{(k)} = \frac{\|H^{(k)}W^{(k)}\|_F^2}{\|\bar{H}^{(k)}W^{(k)}\|_F^2} \quad (18)$$

$$= \frac{\text{trace}\{(W^{(k)})^H (H^{(k)})^H H^{(k)} W^{(k)}\}}{\text{trace}\{(W^{(k)})^H (\bar{H}^{(k)})^H \bar{H}^{(k)} W^{(k)}\}}$$

where trace$\{\cdot\}$ is the sum of diagonal elements. This is similar to the maximum SNR adaptive array [33], [34], [36]. Therefore, this algorithm is referred to as block maximum SNR, shortly, BMSN.

The solution of this problem is obtained by differentiating the $\text{DUR}^{(k)}$ of Eq. (18) with $(W^{(k)})^*$ and equating the resultant to zero. Thereby, we have the following equation:

$$(\bar{H}^{(k)})^H \bar{H}^{(k)} W^{(k)}$$

$$= \frac{\text{trace}\{(W^{(k)})^H (\bar{H}^{(k)})^H \bar{H}^{(k)} W^{(k)}\}}{\text{trace}\{(W^{(k)})^H (H^{(k)})^H H^{(k)} W^{(k)}\}} (H^{(k)})^H H^{(k)} W^{(k)}$$

$$= \frac{1}{\text{DUR}^{(k)}} (H^{(k)})^H H^{(k)} W^{(k)}. \tag{19}$$

The above equation means a generalized eigenvalue problem of $(\bar{H}^{(k)})^H \bar{H}^{(k)}$ and $(H^{(k)})^H H^{(k)}$ with eigenvalues equal to $1/\text{DUR}^{(k)}$. However, we use here the beamforming condition $H^{(k)} W^{(k)} = T^{(k)}$ in Eq. (19) for reducing the computational complexity. Consequently, we have

$$(\bar{H}^{(k)})^H \bar{H}^{(k)} W^{(k)} = \frac{1}{\text{DUR}^{(k)}} (H^{(k)})^H T^{(k)}, \tag{20}$$

and we obtain the following solution for transmit weight with a scalar $\mu^{(k)} = 1/\text{DUR}^{(k)}$:

$$W_{\text{opt}}^{(k)} = \mu^{(k)} \{ (\bar{H}^{(k)})^H \bar{H}^{(k)} + \alpha^{(k)} I \}^{-1} (H^{(k)})^H T^{(k)} \tag{21}$$

where $\alpha^{(k)}$ is a diagonal loading of positive scalar for regularizing the inverse and obtaining the inverse matrix with stability. In addition, $\alpha^{(k)}$ has a function of controlling the null depth to other users [24]–[26]. We call $\alpha^{(k)}$ the pseudo-noise because it is quite similar to the noise in MMSE (Minimum Mean Square Error) algorithm [27], [28], [37]. The scalar $\mu^{(k)}$ is determined from constant transmit power condition.

Concerning the constant matrix $T^{(k)}$,

$$T^{(k)} = I_{N_R} \tag{22}$$

is a simple and most likely candidate from considering the eigenmode transmission of each user [23]. In this paper, $T^{(k)} = I_{N_R}$ of Eq. (22) is adopted consistently for BMSN algorithm.

In this way, user-by-user block MSN can be realized when $W^{(k)} = W_{\text{opt}}^{(k)}$. Afterwards, we follow the same process as the BD algorithm.

Similar to Fig. 2 for BD algorithm, the channel matrix $\tilde{H}^{(k)} = H^{(k)} W_{\text{opt}}^{(k)}$ is regarded as that of single-user MIMO for user k. To employ the eigenmode transmission beamforming (EM-BF) for the matrix $\tilde{H}^{(k)}$, we apply SVD to $\tilde{H}^{(k)}$, which leads to Eq. (13). Ultimately, the transmit weight of BMSN algorithm is given by

$$W_{\text{BMSN}}^{(k)} = W_{\text{opt}}^{(k)} \tilde{V}_s^{(k)} \tag{23}$$

where $\tilde{V}_s^{(k)}$ is the right singular matrix corresponding to

nonzero singular values of $\tilde{H}^{(k)}$.

When using $W = [W_{\text{BMSN}}^{(1)}, \cdots, W_{\text{BMSN}}^{(N_U)}]$ and $W_r^{(k)} = (\tilde{U}^{(k)})^H$ ($\tilde{U}^{(k)}$: the left singular matrix of $\tilde{H}^{(k)}$) in Eq. (9), the receive signal of user k, $y_r^{(k)}(t)$, is expressed as

$$y_r^{(k)}(t) = (\tilde{U}^{(k)})^H \cdot \left(H^{(k)} \sum_{l=1}^{N_U} W_{\text{BMSN}}^{(l)} s^{(l)}(t) + n^{(k)}(t) \right)$$

$$= y_S^{(k)}(t) + y_I^{(k)}(t) + y_N^{(k)}(t) \tag{24}$$

$$y_S^{(k)}(t) = (\tilde{U}^{(k)})^H H^{(k)} W_{\text{BMSN}}^{(k)} s^{(k)}(t) = \tilde{D}_s^{(k)} s^{(k)}(t) \tag{25}$$

$$y_I^{(k)}(t) = \sum_{\substack{1 \le l \le N_U \\ l \ne k}} (\tilde{U}^{(k)})^H H^{(k)} W_{\text{BMSN}}^{(l)} s^{(l)}(t) \tag{26}$$

$$y_N^{(k)}(t) = (\tilde{U}^{(k)})^H n^{(k)}(t) \tag{27}$$

where $y_S^{(k)}(t)$ is the desired signal, $y_I^{(k)}(t)$ is the interference from other users, and $y_N^{(k)}(t)$ is the internal noise. In the BMSN algorithm, it is noted that the relation $y_I^{(k)}(t) = 0$ does not always hold according to pseudo-noise $\alpha^{(k)}$, which is different from BD algorithm. On the other hand, it is found from $y_S^{(k)}(t)$ of Eq. (25) that inter-substream interference is eliminated in the multi-substream transmission of each user.

While the MMSE and generalized MMSE (GMMSE) [27]–[30] are channel inversion algorithms, BMSN is regarded as a modified version of BD which carries out block beamforming user by user. Therefore, unlike the MMSE, BMSN can be applied to the case where each user terminal has multiple antennas [27], [29]. In addition, the optimum diagonal loading (pseudo-noise) for the BMSN transmit weight can be realized user by user when the average SNRs of individual users are different. This means that BMSN can achieve user-by-user control of the residual interference suppression by setting the appropriate pseudo-noise individually. It is essentially different from MMSE and GMMSE criteria that obtain total channel inversion for all users with a common diagonal loading.

2.3.2 Performance Control by Pseudo-Noise

Since we assume that all channel responses have the same variance, we can provide the same value for $\alpha^{(k)}$ of all users, and hence $\alpha^{(k)}$ is simply expressed as α.

One method of determining the pseudo-noise α is to use a constant value for it. As a result of brief parameter studies using Eq. (21), we have here two typical values; one is a larger one α_1 and the other is a smaller one α_2 as follows:

$$\alpha_1 = \frac{10^{-2}}{(N_U - 1)N_R N_T} \text{trace} \left\{ E \left[(\bar{H}^{(k)})^H \bar{H}^{(k)} \right] \right\} \tag{28}$$

$$= \frac{10^{-2}}{(N_U - 1)N_R N_T} E \left[\sum_{\substack{k'=1 \\ k' \ne k}}^{N_U} \sum_{i=1}^{N_R} \sum_{l=1}^{N_T} \left| h_{il}^{(k')} \right|^2 \right] \tag{29}$$

$$\alpha_2 = \frac{10^{-6}}{(N_U - 1)N_R N_T} \text{trace} \left\{ E \left[(\bar{H}^{(k)})^H \bar{H}^{(k)} \right] \right\} \tag{30}$$

$$= \frac{10^{-6}}{(N_U - 1)N_R N_T} E\left[\sum_{\substack{k'=1 \\ k' \neq k}}^{N_U} \sum_{i=1}^{N_R} \sum_{l=1}^{N_T} \left| h_{il}^{(k')} \right|^2 \right] \quad (31)$$

where $E[\cdot]$ stands for expectation. The factor of 10^{-2} in α_1 is supposed to be equivalent to the average SNR=20dB. On the other hand, the small factor of 10^{-6} in α_2 is derived from the matrix invertible condition of single precision numeral. In this paper, BMSN with α_1 is called BMSN1 and BMSN with α_2 is called BMSN2. Considering the performance of MSN adaptive array, BMSN2 makes deeper nulls to other users than BMSN1.

On the other hand, we can determine the pseudo-noise α in the same manner as MMSE which determines α depending on SNR at user terminal [27]. Let $\gamma_i^{(k)}$ denote the SNR of the receive antenna i ($i = 1 \sim N_R$) at the user terminal k ($k = 1 \sim N_U$) when signals with unit total transmit power are transmitted without transmit weights, then $\gamma_i^{(k)}$ is expressed as

$$\gamma_i^{(k)} = \frac{E\left[\sum_{l=1}^{N_T} \left| h_{il}^{(k)} \right|^2 \right]}{N_T \sigma^2}, \quad (32)$$

where σ^2 is the noise power at each user terminal. In this case, the pseudo-noise, denoted by α_3, is determined as follows:

$$\alpha_3 = \frac{N_T}{\frac{1}{N_R} \sum_{i=1}^{N_R} \gamma_i^{(k)}}. \quad (33)$$

BMSN with α_3 is called BMSN3 in this paper. BMSN3 controls the null depth to other users depending on SNR. On the assumption that $h_{il}^{(k)}$ is the zero-mean unit-variance complex-Gaussian fading gain, we have $\alpha_1 = 10^{-2}$, $\alpha_2 = 10^{-6}$, and $\alpha_3 = N_T \sigma^2$ ($\gamma_i^{(k)} = 1/\sigma^2$).

3. Analysis by Computer Simulation

3.1 Simulation Conditions

By computer simulation, we evaluate the bit error rate (BER) and the achievable bit rate (ABR) for BD and three BMSN algorithms. Through the evaluation, we demonstrate BMSN3 shows superiority over other algorithms. Common parameters for computer simulation are described in Table 1.

3.2 Evaluation with Bit Error Rate and Channel Eigenvalues

First, BER performance is evaluated for BD and BMSN algorithms. The bit rate is assumed to be 4 bits/symbols/user and adaptive modulation shown in Table 2 is employed according to the channel eigenvalues of all algorithms. The notation [2, 2], [3, 1] and [4, 0] in Table 2 denote the combinations of bits/symbol/user for each data stream. Hence, for

Table 1 Common simulation parameters.

Number of transmit antennas (N_T)	16
Number of receive antennas (N_R)	2
Number of users (N_U)	8
Propagation channel	i.i.d. Rayleigh flat fading (zero-mean and unit-variance)
α_1 of BMSN1	10^{-2}
α_2 of BMSN2	10^{-6}
α_3 of BMSN3	$N_T \sigma^2 = 16\sigma^2$
SNR	$-10 \log_{10} \sigma^2$ [dB]

Table 2 Adaptive modulation.

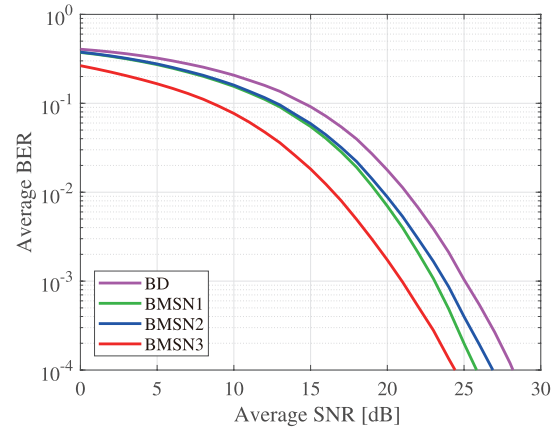| bits/symbol/user	modulation scheme 1	modulation scheme 2
[2,2]	QPSK	QPSK
[3,1]	8QAM	BPSK
[4,0]	16QAM	–

Fig. 3 Average BER versus average SNR of BD and BMSN algorithms.

total 4 bits/symbol/user, BPSK, QPSK, 8PSK and 16QAM are used as the modulation scheme for each bit rate and the modulation scheme with the minimum BER is selected for each transmission trial [32]. Figure 3 shows the comparison in the BER versus SNR among BD, BMSN1, BMSN2 and BMSN3. As can be seen in Fig. 3, three BMSN algorithms obviously outperform BD. Furthermore, BMSN3 achieves considerable improvement over BD, BMSN1, and BMSN2.

Figure 4 shows the channel eigenvalue distributions of the BD and BMSN algorithms when SNR = 15 dB. In this figure, λ_1 and λ_2 stand for channel eigenvalues $\tilde{\lambda}^{(k)}(1)$ and $\tilde{\lambda}^{(k)}(2)$, respectively. As shown in Fig. 4, the 2nd eigenvalues (λ_2) of BD, BMSN1 and BMSN2 are very small compared to their individual 1st eigenvalues (λ_1). Particularly, the 2nd eigenvalues of BMSN1 and BMSN2 are extremely small. Therefore, it leads to the fact that an actual modulation might not be assigned for the second data stream in the adaptive modulation scheme for BD, BMSN1 and BMSN2. However, the 1st eigenvalues of BMSN1 and BMSN2 are larger than BD, and eventually the BERs of BMSN1 and BMSN2 are both lower than BD as shown in Fig. 3.

On the other hand, both the 1st and 2nd eigenvalues of BMSN3 are much increased compared to other algorithms,
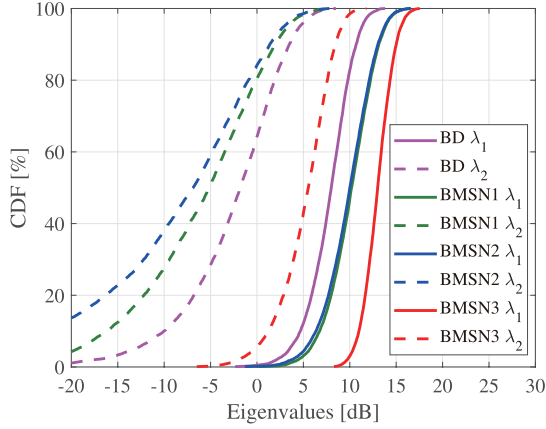
Fig. 4 Comparison of channel eigenvalue distributions (SNR = 15 dB).

Fig. 5 CDF of achievable bit rate (SNR = 15 dB).

resulting in more effective operation of adaptive modulation and thus lower BER as shown in Fig. 3.

3.3 Evaluation with Achievable Bit Rate

Here, we evaluate ABR for BD and BMSN algorithms. In the case of $N_R = 2$, the ABRs of user k by BD and BMSN algorithms ($C_{BD}^{(k)}$ and $C_{BMSN}^{(k)}$) are obtained as

$$C_{BD}^{(k)} = \sum_{i=1}^{2} \log_2\left(1 + \frac{\tilde{\lambda}_{BD}^{(k)}(i)}{N_T \sigma^2}\right), \tag{34}$$

$$C_{BMSN}^{(k)} = \sum_{i=1}^{2} \log_2\left(1 + \frac{\tilde{\lambda}_{BMSN}^{(k)}(i)}{P_I^{(k)}(i) + N_T \sigma^2}\right) \tag{35}$$

where $\tilde{\lambda}_{BD}^{(k)}(i)$ and $\tilde{\lambda}_{BMSN}^{(k)}(i)$ are i-th channel eigenvalues of user k which are obtained by BD and BMSN algorithms, respectively. $P_I^{(k)}(i)$ is the interference power to i-th receive antenna of user k from other users in BMSN and it is obtained from Eq. (26) as follows.

$$P_I^{(k)}(i) = \Bigg[\sum_{\substack{1 \le l \le N_U \\ l \neq k}} \left((\tilde{U}^{(k)})^H H^{(k)} W_{BMSN}^{(l)} \right) \cdot$$

$$\left((\tilde{U}^{(k)})^H H^{(k)} W_{BMSN}^{(l)} \right)^H \Bigg]_{ii} \tag{36}$$

where $[A]_{nm}$ denotes the (n, m) component of matrix A.

In the MU-MIMO system with SNR = 15 dB, the CDFs (Cummulative Distribution Functions) of $C_{BD}^{(k)}$ and $C_{BMSN}^{(k)}$ per user are shown in Fig. 5. Also, ABRs of CDF equal to 10% versus average SNR are shown in Fig. 6. It is seen in Figs. 5 and 6 that three BMSN algorithms outperform BD algorithm in terms of the ABR. Furthermore, BMSN3 attains very high ABR compared to BMSN1 and BMSN2 regardless of SNR, which means the positive effect of pseudo-noise determined according to SNR. The low SNR corresponds to long transmission distance from the base station, and therefore BMSN3 is effective from a point of view in enlarging the service area where the improvement of ABR at the cell
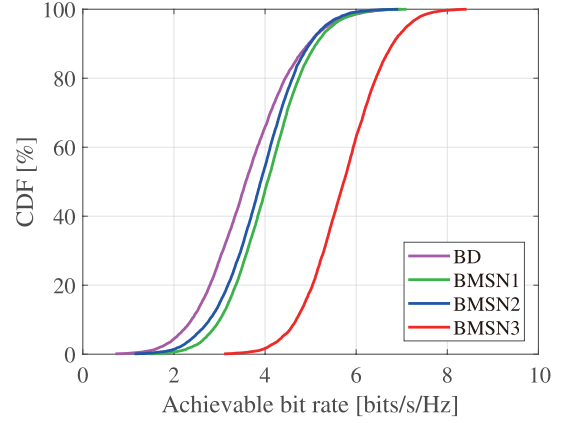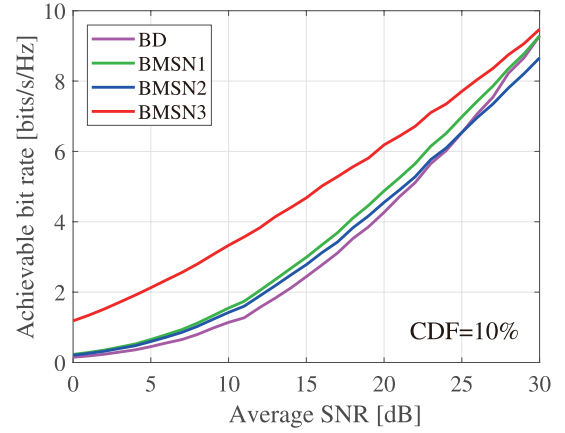

Fig. 6 ABR of CDF equal to 10% versus average SNR.
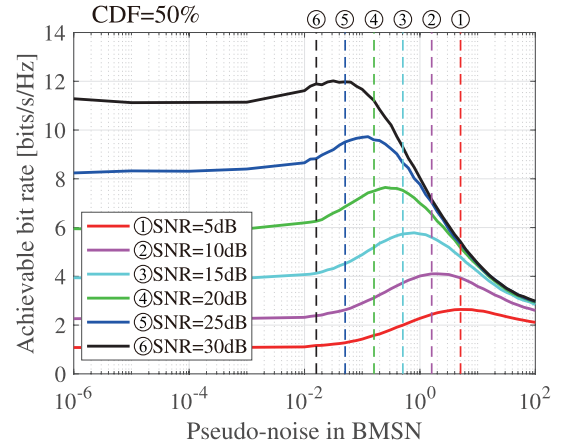
Fig. 7 ABR of CDF equal to 50% versus pseuso-noise in BMSN (Dotted lines show $\alpha = \alpha_3 = N_T \sigma^2$ in each SNR).

edges is more important than that in the neighborhood of base station.

To investigate the optimum value of pseudo-noise in BMSN, we show in Fig. 7 the ABR of CDF = 50% versus the pseudo-noise in each SNR. In this figure, dotted lines represent the values of $\alpha = \alpha_3 = N_T \sigma^2$ in each SNR. It is

found from Fig. 7 that there is an apparent optimum value of pseudo-noise of BMSN in each SNR. In addition, it is confirmed that the optimum value is close to $\alpha = \alpha_3 = N_T \sigma^2$ although you can observe the larger difference between the optimum value and α_3 for the higher SNR. Thus, we consider the value of α_3 is available to enhance substantially the performance of BMSN. More analytical discussion about the optimum value of pseudo-noise for BMSN will be our future work.

4. Discussion on Pseudo-Noise in BMSN

As above-mentioned, the BMSN algorithm is based on the minimization of interference to other users while maintaining the high gain of each one's own channel. When the pseudo-noise is considered, the principle of BMSN algorithm is also described as follows:

$$\min_{\boldsymbol{W}^{(k)}} \left(\|\bar{\boldsymbol{H}}^{(k)} \boldsymbol{W}^{(k)}\|_F^2 + \alpha^{(k)} \|\boldsymbol{W}^{(k)}\|_F^2 \right) \tag{37}$$

$$\text{subject to } \boldsymbol{H}^{(k)} \boldsymbol{W}^{(k)} = \boldsymbol{T}^{(k)}.$$

Furthermore, this problem is mathematically equivalent to the maximum transmission DUR provided by

$$\max_{\boldsymbol{W}^{(k)}} \left(\text{DUR}^{(k)} \right)$$

$$\text{with } \text{DUR}^{(k)} = \frac{\|\boldsymbol{H}^{(k)} \boldsymbol{W}^{(k)}\|_F^2}{\|\bar{\boldsymbol{H}}^{(k)} \boldsymbol{W}^{(k)}\|_F^2 + \alpha^{(k)} \|\boldsymbol{W}^{(k)}\|_F^2}. \tag{38}$$

As found from Eq. (38), the BMSN3 with $\alpha^{(k)} = \alpha_3 = N_T \sigma^2$ is based on the maximum DUR including the internal noise effect, thus bringing about the optimum performance according to the receiver SNR. We can say that DUR including the receiver noise is equivalent to SINR (Signal to Interference plus Noise power Ratio).

To confirm the maximized SINR of BMSN3, we show in Fig. 8 the average INR (Interference to Noise power Ratio), SNR and SINR per user antenna at receiver outputs. In Fig. 8(a), INR of BD is not depicted because the INR is equal to 0, i.e., $-\infty$ dB. We can actually see from a large difference among BMSN1, BMSN2 and BMSN3 in Fig. 8(a) that the pseudo-noise controls effectively the null depth to interferences from other users as is intended. Further, it is noticed that the receiver output INR of BMSN3 gets close to 0 dB, which means BMSN3 is quite efficient in making nulls to interferences in environments including internal noises. In contrast, the receiver output INRs of BD, BMSN1 and BMSN2 are very low because of their deeper interference nulls that are not always required. In addition, it is found from Fig. 8(b) that the receiver output SNR of BMSN3 is higher than those of BD, BMSN1 and BMSN2. In the lower SNR, particularly, the difference of receiver output SNRs is larger. This is because the BMSN3 successfully avoids making the unnecessary deep interference nulls and enlarges the beam gains of desired signals by increased degrees of freedom. As a result, BMSN3 has the total receiver output SINR significantly increased as shown in Fig. 8(c).
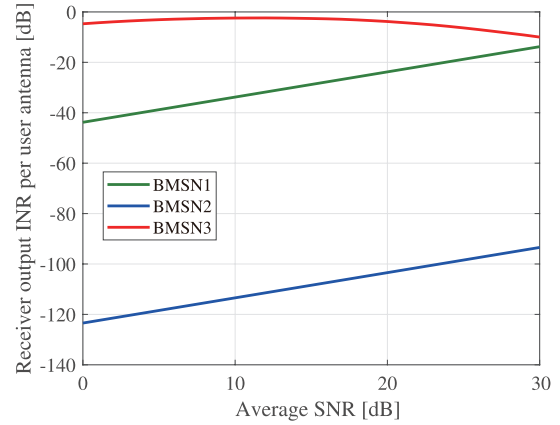
(a) Receiver output INR
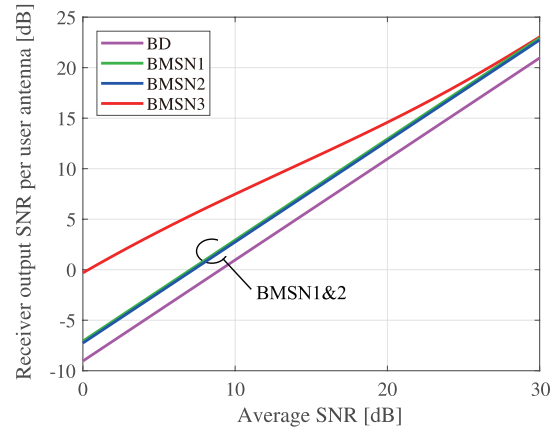
(b) Receiver output SNR
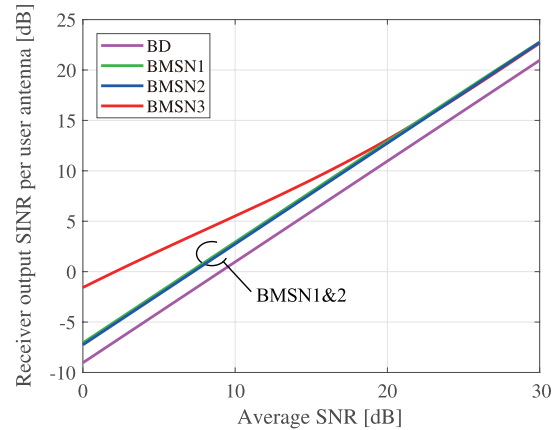
(c) Receiver output SINR

Fig. 8 Receiver output INR, SNR and SINR per user antenna vs. average SNR. In (a), INR of BD is not depicted because INR $=-\infty$ dB for BD.

5. Conclusion

We examined the performance control with a pseudo-noise of the BMSN algorithm that attempts to enhance the gain of the user signals in addition to the interference rejection in the downlink of multiuser MIMO (MU-MIMO) system. Since the pseudo-noise has a function of controlling the null

depth to other users, we must determine the pseudo-noise appropriately. In this paper, three methods of determining the pseudo-noise (α) are compared.

As a simple method, we put a constant value into the pseudo-noise such as $\alpha = 10^{-2}$ and $\alpha = 10^{-6}$ when the propagation channel is i.i.d. Rayleigh fading of the unit-variance. They are named BMSN1 and BMSN2, respectively. BMSN2 with $\alpha = 10^{-6}$ forms interference nulls that are deeper than BMSN1 with $\alpha = 10^{-2}$. Another method is the one of providing the pseudo-noise with $\alpha = N_T \sigma^2$ which is determined according to SNR of the receivers. It is called BMSN3 and it can change adaptively the null depth depending on SNR. The comparative evaluation of the three BMSN algorithms and BD algorithm is carried out through computer simulation under the condition of $(N_T, N_R, N_U) = (16, 2, 8)$. As a result, BMSN3 using the adapted pseudo-noise outperforms other algorithms in the BER characteristics. In addition, it is confirmed that both the first and the second channel eigenvalues of BMSN3 are large in comparison with other algorithms. It means that the adaptive modulation scheme operates more effectively in BMSN3. In the evaluation with CDFs of achievable bit rate (ABR), ABR of BMSN3 is found to be high regardless of SNR. Thus, it is shown that the performance of the BMSN algorithm is improved by using the adapted pseudo-noise according to SNR. Consequently, it can be said that BMSN3 is the most appropriate in applying BMSN algorithm in the MU-MIMO.

In the future, we will continue more analytical discussion about the optimum value of pseudo-noise for BMSN algorithm, and we will compare the BMSN, particularly BMSN3, with MMSE that also uses the pseudo-noise adapted by average SNR. Simultaneously, to improve further the performance of BMSN, we will investigate a technique that employs the generalized eigenvalue decomposition in obtaining the BMSN weights. In addition, we will discuss the situation where the receiver SNRs are different from each other, expecting that the pseudo-noises $\alpha^{(k)}$ are controlled individually for each user.

Acknowledgments

References

[1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Pers. Commun., vol.6, no.3, pp.311–335, 1998.

[2] I.E. Telatar, "Capacity of multiantenna Gaussian channels," Eur. Trans. Telecommun., vol.1, no.6, pp.585–595, Nov./Dec. 1999.

[3] A. Zelst and T.C.W. Schenk, "Implementation of a MIMO OFDM based wireless LAN systems," IEEE Trans. Signal Process., vol.52, no.2, pp.483–494, Feb. 2004.

[4] A. Paulaj, R. Nabar, and D. Gore, Introduction to Space-Time Wireless Communications, Cambridge University Press, 2003.

[5] T. Ohgane, T. Nishimura, and Y. Ogawa, "Applications of space division multiplexing and those performance in a MIMO channel,"

IEICE Trans. Commun., vol.E88-B, no.5, pp.1843–1851, May 2005.

[6] "MIMO implementation aspects," Proc. IEEE RAWCON,Workshop, WS2, Sept. 2004.

[7] H. Taoka, K. Dai, K. Higuchi, and M. Sawahashi, "Field experiments on ultimate frequency efficiency exceeding 30 Bit/Second/Hz using MLD signal detection in MIMO-OFDM broadband packet radio access," Proc. IEEE VTC2007-Spring, pp.2129–2134, April 2007.

[8] 3GPP LTE, http://www.3gpp.org/article/lte

[9] IEEE 802.11, http://www.ieee802.org/11/

[10] WiMAX, http://www.wimaxforum.org/

[11] Q.H. Spencer, C.B. Peel, A.L. Swindlehurst, and M. Haardt, "An introduction to the multi-user MIMO downlin," IEEE Commun. Mag., vol.42, no.10, pp.60–67, Oct. 2004.

[12] D. Gesbert, M. Kountouris, R.W. Heath, Jr., C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm," IEEE Signal Process. Mag., vol.24, no.5, pp.36–46, Sept. 2007.

[13] Y. Takatori and K. Nishimori, "Application of downlink multiuser MIMO transmission technology to next generation very high throughput wireless access systems," IEICE Trans. Commun. (Japanese Edition), vol.J93-B, no.9, pp.1127–1139, Sept. 2010.

[14] K. Nishimori, Basic Theory and Performance on Multi-user MIMO, CORONA PUBLISHING, 2014.

[15] G. Tsoulos, J. McGeehan, and M.A. Beach, "Space division multiple access (SDMA) field trials. Part 1: Tracking and BER performance," IEE Proc. Radar, Sonar Navig., vol.145, no.1, pp.73–78, Feb. 1998.

[16] Y. Doi, J. Kitakado, T. Ito, T. Miyata, S. Nakao, T. Ohgane, and Y. Ogawa, "Development and evaluation of the SDMA test bed for PHS in the field," IEICE Trans. Commun., vol.E86-B, no.12, pp.3433–3440, Dec. 2003.

[17] K. Nishimori and K. Cho, "Evaluation of SDMA employing vertical pattern and polarization control in actual cellular environment measurement," Proc. IEEE VTC 2004-Spring, vol.1, pp.244–248, May 2004.

[18] IEEE P802.11ac./D5.0, Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications.

[19] S. Parkvall, E. Dahlman, A. Furuskär, Y. Jading, M. Olsson, S. Wänstedt, and K. Zangi, "LTE-advanced - Evolving LTE towards IMT-Advanced," Proc. VTC2008-Fall, Sept. 2008.

[20] Q.H. Spencer, A.L. Swindlehurst, and M. Haardt, "Zero forcing methods for downlink spatial multiplexing in multiuser MIMO channels," IEEE Trans. Signal Process., vol.52, no.2, pp.461–471, Feb. 2004.

[21] K. Nishimori, T. Hiraguri, and H. Makino, "Transmission rate by user antenna selection for block diagonalization based multiuser MIMO system," IEICE Trans. Commun., vol.E97-B, no.10, pp.2118–2126, Oct. 2014.

[22] N. Kikuma, K. Nishimori, and T. Hiraguri, "Effect of User antenna selection on block beamforming algorithms for suppressing inter-user interference in multiuser MIMO system," IEICE Trans. Commun., vol.E101-B, no.7, pp.1523–1535, July 2018.

[23] T. Ooka, N. Kikuma, and K. Sakakibara, "BER performance of block MSN beamforming in multi-user MIMO transmission," IEICE Technical Report, A·P2016-11, April 2016.

[24] K. Yonezu, N. Kikuma, and K. Sakakibara, "BER performance improvement of block maximum SNR in multi-user MIMO transmission," IEICE Technical Report, A·P2017-45, July 2017 (in Japanese).

[25] K. Yonezu, N. Kikuma, and K. Sakakibara, "Improvement of block maximum SNR algorithm by adapted pseudo-noise in multi-user MIMO transmission," IEICE Technical Report, A·P2017-127, Nov. 2017 (in Japanese).

[26] K. Yonezu, N. Kikuma, and K. Sakakibara, "Improvement of block maximum SNR algorithm by adapted pseudo-noise in multi-user MIMO transmission," IEICE Society Conference, B-1-140, Sept. 2017 (in Japanese).

[27] C.B. Peel, B.M. Hochwald, and A.L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication: Part I: Channel inversion and regularization," IEEE

Trans. Commun., vol.53, no.1, pp.195–202, Jan. 2005.

[28] A.J. Tenenbaum and R.S. Adve, "Linear processing and sum through-put in the multiuser MIMO downlink," IEEE Trans. Wireless Commun., vol.8, no.5, pp.2652–2661, May 2009.

[29] H. Sung, S. Lee, and I. Lee, "Generalized channel inversion methods for multiuser MIMO systems," IEEE Trans. Commun., vol.57, no.11, pp.3489–3499, Nov. 2009.

[30] V. Stankovic and M. Haardt, "Generalized design of multi-user MIMO precoding matrices," IEEE Trans. Wireless Commun., vol.7, no.3, pp.953–961, March 2008.

[31] J.B. Andersen, "Array gain and capacity for known random channels with multiple element arrays at both ends," IEEE J. Sel. Areas Commun., vol.18, no.11, pp.2172–2178, Nov. 2000.

[32] K. Miyashita, T. Nishimura, T. Ohgane, Y. Ogawa, Y. Takatori, and K. Cho, "High data-rate transmission with eigenbeam-space division multiplexing (E-SDM) in a MIMO channel," Proc. Vehicular Technology Conference (VTC) 2002-Fall, vol.3, pp.1302–1306, Oct. 2002.

[33] R.A. Monzingo, R.L. Haupt, and T.W. Miller, Introduction to Adaptive Arrays, 2nd ed., Scitech Pub., 2010.

[34] N. Kikuma and M. Fujimoto, "Adaptive antennas," IEICE Trans. Commun., vol.E86-B, no.3, pp.968–979, March 2003.

[35] R.T. Compton, Jr., "The power inversion adaptive array: Concept and performance," IEEE Trans. Aerosp. Electron. Syst., vol.AES-15, no.6, pp.803–814, Nov. 1979.

[36] S.P. Applebaum, "Adaptive arrays," IEEE Trans. Antennas Propag., vol.AP-24, no.5, pp.585–598, Sept. 1976.

[37] B. Widrow, P.E. Mantey, L.J. Griffiths, and B.B. Goode, "Adaptive antenna systems," Proc. IEEE, vol.55, no.12, pp.2143–2159, Dec. 1967.

Kunio Sakakibara received the B.S. degree in electrical and computer engineering from Nagoya Institute of Technology, Nagoya, Japan, in 1991 and the M.S. and D.E. degrees in electrical and electronic engineering from Tokyo Institute of Technology, Tokyo, Japan, in 1993 and 1996, respectively. From 1996 to 2002, he worked at Toyota Central Research and Development Laboratories, Inc., Aichi, Japan, and was engaged in the development of antennas for automotive millimeter-wave radar systems. From 2000 to 2001, he was a Guest Researcher at the Department of Microwave Techniques, University of Ulm, Ulm, Germany. In 2002, he joined Nagoya Institute of Technology as a Lecturer, where in 2004, he became an Associate Professor and since 2012, he has been a Professor. His research interests have been millimeter-wave antennas and circuits. He is a senior member of IEEE.

Nobuyoshi Kikuma received the B.S. degree in electronic engineering from Nagoya Institute of Technology, Japan, in 1982, and the M.S. and Ph.D. degrees in electrical engineering from Kyoto University, Japan, in 1984 and 1987, respectively. From 1987 to 1988, he was a Research Associate at Kyoto University. In 1988 he joined Nagoya Institute of Technology, where he has been a Professor since 2001. His research interests include adaptive and signal processing arrays, radio source localization, and MIMO technology for wireless communications and radar sensing. He received the 4th Telecommunications Advancement Foundation Award in 1989, IEICE Best Paper Award in 2005, and IEICE Educational Service Award in 2016. Dr. Kikuma is a senior member of the IEEE.

Kousuke Yonezu received the B.E. degree in electrical and electronic engineering from Nagoya Institute of Technology, Nagoya, Japan in 2017. Currently, he is a graduate school student of electrical and mechanical engineering of Naoya Institute of Technology. He is doing research on muti-user MIMO systems.