

Evaluation of Impact on Digital Radio Systems by Measuring Amplitude Probability Distribution of Interfering Noise

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SUMMARY This paper presents a method for evaluating the maximum bit error probability (BEP) of a digital communication system subjected to interference by measuring the amplitude probability distribution (APD) of the interfering noise. Necessary conditions for the BEP evaluation are clarified both for the APD measuring receiver and the communication receiver considered. A method of defining emission limits is presented in terms of APD so that the worst BEP of a communication system does not exceed a required permissible value. The methods provide a theoretical basis for a wide variety of applications such as emission requirements in compliance testing, dynamic spectrum allocations, characterization of an electromagnetic environment for introducing new radio systems, and evaluation of intra-system interference.

key words: *amplitude probability distribution, electromagnetic interference, electromagnetic noise, bit error probability, non-Gaussian noise*

1. Introduction

The potential for interference in radio communication systems has increased because of the widespread use of high-speed and high-performance electronic appliances that emit broadband electromagnetic noise. It is important to establish appropriate emission limits to avoid the risk of interference. Current emission limits using a quasi-peak (QP) detector were established by the International Special Committee on Radio Interference (CISPR) on the basis of degradation in the reception quality of AM broadcasting signals subjected to a regularly repeated impulsive interference. It has been pointed out that the limits associated with the QP-detector are not adequate to protect digital radio services, and a new method and limits are needed for protecting digital radio services.

Since the waveform of electromagnetic noise is random in nature and greatly depends on the emitting source and its condition, which are unknown in many cases, a statistical approach is essential for evaluating the impact of the interfering noise on radio systems. Amplitude probability distribution (APD) is one of the statistical properties of random signals, and was used to characterize manmade and natural noise in the late 1960s and 1970s [1]–[3]. It was also demonstrated that the bit error probability (BEP) of some digital modulation signals was well estimated using the APD obtained through measurement [4], [5].

On the basis of the demonstrated good correlation between APD and BEP, CISPR began to develop a measurement method by means of APD in 2001 [6] for protection of digital radio services. As a result from this work, CISPR published the standards on the APD measuring receiver specifications [7] and definition of emission limits [8] in 2005 and 2006, respectively. Moreover, a project was started to introduce an APD-based emission limit to a product standard [9], and a new edition of the international standard will be published in 2015.

In parallel with the standardization in CISPR, research and development was conducted for implementing a real-time APD measuring function [10], [11]. In 2006, an EMI receiver having a CISPR-compliant (single channel) APD measurement function became available on the market. It was still inconvenient to use a single-channel APD measuring receiver since the user had to conduct the measurement on a channel-by-channel basis to find the worst frequency at which the interference impact became maximum. In fact, conversion of an APD to be valid with another measurement frequency is impossible except for special cases [12]. To overcome this drawback, prototype multichannel APD measuring receivers were developed using an FIR filter bank [13] and fast Fourier transform (FFT) [14]. Furthermore, a commercially available multichannel APD receiver was developed in 2011 [15].

To specify a limit value as an emission requirement, it is necessary to find a criterion for the interfering signal level that gives the maximum permissible BEP in a system being subjected to interference. The estimation of BEP in [4], [5] required numerical integration of the joint probability density function (PDF) of the amplitude and phase of the interfering signal, which was assumed or measured in practice. It is time consuming to find an amplitude criterion that gives the permissible BEP.

It was demonstrated that the maximum symbol-error probability (SEP) of a coherent binary phase shift keying (BPSK) receiver subjected to interference can be expressed exactly by the APD of an interfering signal, and a simple expression for the maximum BEP in terms of APD was developed [16]. The concept was shown to be applicable to estimation of the approximated maximum BEP of a coherent receiver with a very simple expression for various uncoded modulation signals under certain general conditions [17]–[19]. The expression was extended to provide the exact upper bound of the BEP for multi-level modulations [20].

According to the simple BEP-APD expression, an APD

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curve plotted on a log-log scaled graph can be converted to the maximum BEP curve just by linearly shifting the vertical and horizontal scales. Since the conversion of an APD to the maximum BEP can be conducted without numerical calculation, the emission limit can be defined very simply in terms of APD to keep the maximum BEP below a given value [17].

Since many current radio communication systems use forward error correcting (FEC) codes to improve communication performance by reducing the BEP, investigations have also been conducted to develop a method for protecting coded systems by means of APD. An emission limit can be defined by considering the effect of the coding, known as the coding gain [21]. For interfering noise that can be modeled by Middleton's class A noise [22], the coding gain of a BPSK system was evaluated through numerical simulations, and it was found that the coding gain for Gaussian noise is generally smaller than for impulsive interference [23]. A simple expression of the upper bound of the BEP for a coded BPSK system under class A interference was developed [24], and a definition of an emission limit was discussed in [25].

By using the good correlation of APD with BEP, various applications of APD measurement have been proposed and demonstrated. The APD was used for characterizing an electromagnetic environment to estimate the communication performance of newly introduced radio systems [26]–[33]. APD-based analyses were conducted to evaluate co-channel and adjacent channel intersystem interference [14], [34]–[36] and to predict the impact of noise from household appliances on digital TV reception [37], [38]. The usefulness of APD measurement was also demonstrated for resolving intra-system interference issues, e.g., evaluating the self jamming effect on communication performance and mapping the noise intensity on a printed circuit board [39]–[43].

On the basis of the results published by the authors [16]–[21], [23]–[25], this paper provides an overview, from the aspect of measurement, of the method of evaluating the maximum BEP from APD clarifying necessary conditions both on the measuring receiver and communication receiver, and the method of defining APD-based emission limits in a systematic manner. Whereas most of the contents of the paper has been already published by the authors, some materials are newly provided for deriving necessary conditions for expressing the maximum BEP by APD. Discussions on the maximum BEP of a spread spectrum system and spreading gain are also added.

The paper is organized as follows. In Sect. 2, definition of the APD and the basic construction of APD measuring equipment are described. In Sect. 3, the expression of the maximum BEP for an uncoded radio communication system is derived. On the basis of this expression, a method of defining an emission limit for protecting a radio receiver is presented in Sect. 4. Derivation of emission limits for protecting a coded system is discussed in Sect. 5 with an example of a BPSK system that uses a convolutional code. The

paper is concluded with future perspectives in Sect. 6.

2. Definition of the APD and the Measurement Equipment

2.1 Definition of the APD

The APD of interfering noise is defined as the cumulative distribution of the probability that the envelope amplitude A of the received interfering noise exceeds specified threshold level a as [7]

$$\text{APD}_A(a) \equiv \text{Prob}[A > a]. \quad (1)$$

The APD of a measured sample of a noise envelope $a(t)$ is defined by the sum of the intervals $W_i(a_k)$ during which $a(t)$ exceeds a certain threshold a_{th} , as shown in Fig. 1, normalized by the total measurement time T_0 .

$$\text{APD}_A(a_{th}) = \sum_{i=1}^{n(a_{th})} W_i(a_{th}) / T_0, \quad (2)$$

where $n(a_{th})$ denotes the total number of intervals $W_i(a_{th})$.

2.2 Measuring Receiver

A simplified block diagram of a single-channel APD measuring receiver is shown in Fig. 2. The RF noise $n(t)$ into the receiver (pulse internal receiver noise in practice) is band-limited using a bandpass filter (BPF) having a specified bandwidth called resolution bandwidth (RBW). Note that this band limitation is usually conducted in an intermediate frequency (IF) band in a heterodyne receiver. The bandpass signal $n_b(t)$ is then envelope-detected, sampled at a certain

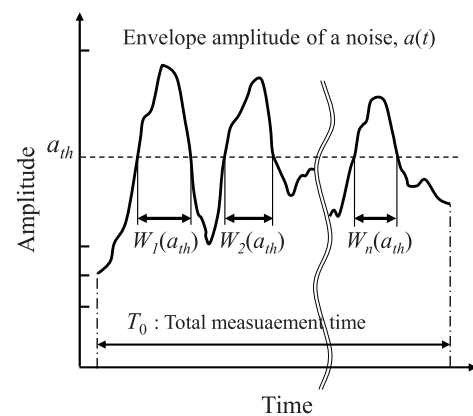


Fig. 1 Measurement of APD.

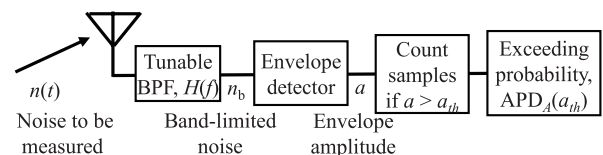


Fig. 2 Simplified block diagram of APD measuring receiver.

interval, and inputted into the counter.

The counter counts the number of samples if the sampled value of the amplitude is greater than a_{th} . The $APD_A(a_{th})$ is given by the total number of samples that are greater than a_{th} divided by the total number of samples.

When an unmodulated sine wave is received at the tuned frequency, the receiver needs to have the maximum gain and display the amplitude in root mean square (rms) voltage. For example, when an unmodulated wave with an envelope amplitude of $\sqrt{2}$ volts is received, the reading is 1 volt, and the signal power is calculated as $1^2/Z_0$ in watts. Note that Z_0 denotes the input impedance of the receiver. Thus, the maximum gain of the filter is considered to be unity at its center frequency f_c , and the gain of the envelope detector is $1/\sqrt{2}$.

$$\max |H(f)| = |H(f_c)| = 1, \quad H(f) \equiv F[h(t)], \quad (3)$$

where $F[h(t)]$ represents the Fourier transform of the filter's impulse response $h(t)$.

There are definitions to specify the RBW. The most common is the -3 -dB bandwidth B_{-3} . The noise bandwidth B_n is used to measure signal power with an rms detector. When the maximum filter gain is unity, as expressed by Eq. (3),

$$B_n \equiv \frac{\int_0^\infty |H(f)|^2 df}{\max |H(f)|^2} = \int_0^\infty |H(f)|^2 df = \frac{1}{2} \int_{-\infty}^\infty h(t)^2 dt. \quad (4)$$

If the value of B_n differs from its reference $B_{n,ref}$, the measured rms amplitude value will deviate by $10 \log(B_n/B_{n,ref})$ in dB.

To detect the peak of an impulsive signal, the impulse bandwidth is specified [7] as

$$B_{imp} \equiv \max |h(t)| / (2 \max |H(f)|). \quad (5)$$

If the B_{imp} differs from its reference value $B_{imp,ref}$, the reading of the peak amplitude for an impulsive signal will deviate by $20 \log(B_{imp}/B_{imp,ref})$ in dB. For an ideal Gaussian filter, $B_n = B_{imp}/\sqrt{2}$ and $B_{imp} = \sqrt{\pi/(2 \log_e 2)} B_{-3} \approx 1.5 B_{-3}$. The ratio B_{imp}/B_n indicates the linearity of the phase of the filter's transfer function $H(f)$ [7].

3. Maximum Bit Error Probability

3.1 Assumptions and Conditions

The upper bound of symbol error probability (SEP) and BEP for a coherent receiver without error correction coding is derived in terms of APD. To do this, the following assumptions are made:

- 1) The communication system is memory-less between decision instances. Hence, the communication receiver makes a symbol-by-symbol decision.
- 2) The interfering noise does not affect the carrier and symbol timing recovery.

The following conditions are also required for APD measurement.

- 3) The APD measurement is conducted at the same frequency as the carrier frequency of the communication signal.
- 4) The APD measuring receiver and communication receiver have (approximately) the same internal noise level (i.e. the noise figure). This means the power ratio of incoming noise to the internal noise of the APD measuring receiver is the same as that of the communication receiver.
- 5) The amplitude-frequency response of the BPF in an APD measuring receiver (shown in Fig. 2) is (approximately) the same as the amplitude spectrum of the waveform difference between any two symbols of the communication system. This condition is detailed in the following subsection.

3.2 Symbol Error Probability of Coherent Receiver

The probability of a symbol error in a coherent demodulator is now discussed. Suppose that there exists a set of M ($= 2^m$) signals $\{s_k(t)\}$ ($k = 1$ to M) having a symbol duration of T_s . A matched filter demodulation optimized for additive white Gaussian noise (AWGN) is conducted by selecting the symbol $s_k(t)$ that maximizes the following metric [44], as shown in Fig. 3,

$$C_{ik} \equiv 2(r_i(t) \otimes s_k(T_s - t))_{t=T_s} - (s_k(t) \otimes s_k(T_s - t))_{t=T_s}, \quad (6)$$

$$r_i(t) \equiv s_i(t) + n(t),$$

where $\otimes$ represents a convolution integral, and $s_k(T_s - t)$ represents the impulse response of a matched filter to detect the symbol $s_k(t)$. Furthermore, $r_i(t)$ denotes the received signal that is the sum of a received symbol $s_i(t)$ and an interfering noise $n(t)$. In the above demodulation scheme, maximizing C_{ik} in Eq. (6) is equivalent to minimizing the integration of $|r_i(t) - s_k(t)|^2$. This yields $s_k(t)$ as the most "similar" symbol to the received signal $r_i(t)$.

If $C_{ik} - C_{ii} > 0$, a symbol error ε_{ik} occurs (subscript ik indicates that the received symbol s_i is incorrectly demodulated as s_k). With substitution of Eq. (6), the probability of this symbol error ε_{ik} is expressed by

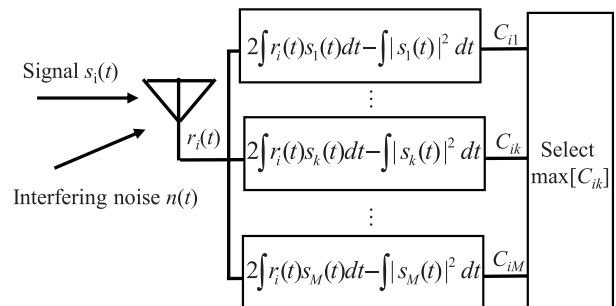


Fig. 3 Simplified block diagram of coherent receiver with matched filter demodulator.

$$P_{s,ik} = \Pr[C_{ik} > C_{ii}] = \Pr\left[n_{ik}(T_s) > \left(\frac{d_{ik}^2}{2}\right)Z_0\right], \quad (7)$$

$$n_{ik}(t) \equiv n(t) \otimes (s_k(T_s - t) - s_i(T_s - t))$$

$$\equiv a_{ik}(t) \cos(2\pi f_c t + \phi_{ik}(t)).$$

Note that d_{ik} represents the Euclidian distance between the symbols $s_k(t)$ and $s_i(t)$.

$$d_{ik}^2 \equiv \frac{1}{Z_0} \int_0^{T_s} (s_k(t) - s_i(t))^2 dt. \quad (8)$$

The division by Z_0 is for the distance to have a unit of energy. The bandpass signal $n_{ik}(t)$ given by Eq. (7) is regarded as the noise after band limitation by a filter having the selectivity $S_k(f) - S_i(f)$. Note that $S_k(f)$ and $S_i(f)$ respectively denote the Fourier transform of the symbol waveforms $s_k(T_s - t)$ and $s_i(T_s - t)$.

3.3 Maximum Symbol Error Probability

From Eq. (7), the symbol error ε_{ik} does not occur with any value of the phase $\phi_{ik}(T_s)$ if the envelope amplitude of $a_{ik}(T_s)$ does not exceed $d_{ik}^2 Z_0 / 2$. In other words, the maximum occurrence probability of ε_{ik} is expressed by the exceeding probability of the envelope amplitude $a_{ik}(t)$, as given by Eq. (9).

$$P_{s,ik,max} = \Pr\left[a_{ik}(t) > \left(\frac{d_{ik}^2}{2}\right)Z_0\right]. \quad (9)$$

This is essentially the same principle as previously presented [16], [17], except for the unit and scale factor of the variable, which is discussed as follows.

On the basis of the above-mentioned procedure, the maximum probability of the specific symbol error $P_{s,ik,max}$ is obtained by receiving $n(t)$ using a receiver whose diagram is depicted in Fig. 4(a). The received noise $n(t)$ is band limited using a BPF. The BPF's selectivity ($S_k(f) - S_i(f)$) is regarded as optimal for extracting the noise component that contributes to the symbol error ε_{ik} . This is because the above selectivity is matched with the worst noise, $n(t) = (s_k(t) - s_i(t))/2$, which causes the symbol error ε_{ik} with

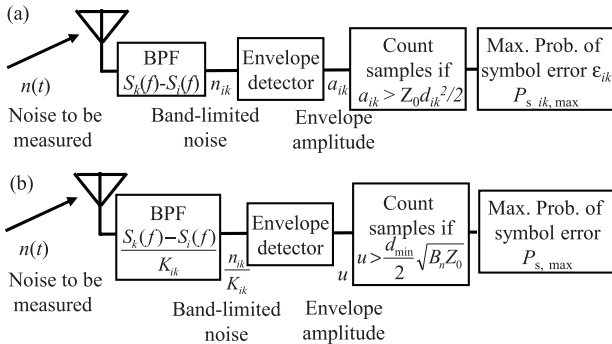


Fig. 4 (a) Diagram for measuring maximum probability of specific symbol error ε_{ik} based on Eq. (9). (b) Diagram for measuring maximum probability of symbol error based on Eq. (13) with conditions of Eqs. (10) and (11) satisfied.

the minimum energy of $(d_{ik}/2)^2$. Note that this selectivity of the BPF, $(S_k(f) - S_i(f))$, depends on both symbol waveforms $s_k(t)$ and $s_i(t)$. After the filtering, the noise is envelope-detected and sampled at the interval of the symbol duration T_s . The number of samples whose amplitude exceeds $Z_0 d_{ik}^2 / 2$ is then counted. The result is divided by the total number of samples to give the maximum probability of the symbol error ε_{ik} .

When comparing the diagrams shown in Figs. 2 and 4(a), one can see that the structure of the two receivers is the same. This means that the maximum SEP can be determined from an APD measured by the receiver shown in Fig. 2 if it provides the probability identical to the exceeding probability of the envelope amplitude obtained by the receiver in Fig. 4. This requires the following equalities on the frequency selectivity and amplitude gain of the receivers for all pairs of i and k ($i \neq k$) [19].

- 1) The noise and impulse bandwidths of the APD measuring receiver, denoted by B_{n_APD} and B_{imp_APD} respectively, need to be (approximately) equal to those of the BPF in Fig. 4, namely,

$$\begin{aligned} B_{n,ik} &= B_n \cong B_{n_APD} \quad (i \neq k), \\ B_{imp,ik} &= B_{imp} \cong B_{imp_APD} \quad (i \neq k). \end{aligned} \quad (10)$$

Note that $B_{n,ik}$ and $B_{imp,ik}$ represent the noise and impulse bandwidths of the BPF that has the selectivity of $S_k(f) - S_i(f)$.

- 2) Since the BPF shown in Fig. 2 has the maximum gain of unity, the amplitude-frequency selectivity of the BPF, $|S_k(f) - S_i(f)|$ in Fig. 4 needs to be able to be normalized to have the maximum gain of unity.

By applying the definition of noise bandwidth given by Eq. (4) to the normalized selectivity of $(S_k(f) - S_i(f))/K_{ik}$, and substituting its maximum gain of unity, the normalize factor K_{ik} is determined as

$$K_{ik} = d_{ik} \sqrt{\frac{Z_0}{2B_n}}. \quad (11)$$

- 3) As the receiver shown in Fig. 2, the gain of the envelope detector in Fig. 4 needs to be $1/\sqrt{2}$.

By dividing by the normalize factor K_{ik} and multiplying the detector gain of $1/\sqrt{2}$ to both sides of the inequality within the probability expression in Eq. (9),

$$P_{s,ik,max} = \Pr\left[u > \frac{d_{ik}}{2} \sqrt{B_n Z_0}\right] = \text{APD}_U\left(\frac{d_{ik}}{2} \sqrt{B_n Z_0}\right). \quad (12)$$

Note that $u \equiv a_{ik}/K_{ik} \sqrt{2}$ and the APD of the normalized amplitude u becomes independent of i and k ($i \neq k$).

The probability given by Eq. (12) is a function of the symbol distance d_{ik} , and takes the maximum for the symbol pair $(s_k(t)$ and $s_i(t))$ that gives the minimum distance d_{min} . The maximum SEP is represented by

$$P_{s,max} = \text{APD}_U\left(\frac{d_{min}}{2} \sqrt{B_n Z_0}\right). \quad (13a)$$

Figure 4(b) shows a diagram for measuring the maximum SEP based on Eq. (13a). In terms of the rms signal amplitude A_s (in volts) and average bit energy E_b (in joules), Eq. (13a) can be rewritten as

$$P_{s,\max} = \text{APD}_U \left(\frac{\beta A_s}{\sqrt{m}} \sqrt{B_n T_s} \right), \quad \beta \equiv \frac{d_{\min}}{2\sqrt{E_b}}, \quad (13b)$$

$$E_b = \frac{E_s}{m} = \frac{1}{m} \left(\frac{1}{MZ_0} \sum_{k=1}^M \int_0^{T_s} s_i(t)^2 dt \right) = \frac{A_s^2 T_s}{mZ_0}.$$

3.4 Maximum Bit Error Probability

For binary modulation with $m = 1$ (1 bit/symbol), such as BPSK, the maximum BEP is equal to the maximum SEP, and is given by Eq. (13). For a multilevel modulation signal (with $m > 1$), there are $M = 2^m$ symbols, and each symbol represents m bits. Here, Gray encoded symbols are assumed. Let the number of different bits between two symbols i and j be denoted by $m_d(i, j)$. In a Gray coded system, the following condition on symbol distance d_{ij} is satisfied.

$$d_{ij} < d_{ik} \quad \text{if } m_d(i, j) < m_d(i, k). \quad (14)$$

Furthermore, let the minimum value of the distance d_{ij} that satisfies $m_d(i, j) = k$ be represented by d_k .

$$d_k \equiv \min(d_{ij}) \text{ when } (i, j) \text{ satisfies } m_d(i, j) = k. \quad (15)$$

It is obvious that $d_{\min} = d_1$ for a Gray coded system.

If it is assumed that one symbol error always causes one bit error, the maximum BEP is approximated by the maximum SEP divided by the number of bits transmitted by one symbol, m ,

$$P_{b,\max} \cong \frac{1}{m} \text{APD}_U \left(\frac{\beta_1 A_s}{\sqrt{m}} \sqrt{B_n T_s} \right), \quad \beta_1 \equiv \frac{d_1}{2\sqrt{E_b}}. \quad (16)$$

This is identical to the expression of BEP (Eq. (21)) presented in [17] with modification [18], except for the term $(B_n T_s)^{1/2}$ multiplied by the signal amplitude A_s . While this term is unity for linearly modulated signals (such as M -PSK and quadrature amplitude modulation (QAM)) with a rectangular envelope of symbol waveform, the effect of the term is discussed in the following subsection.

Considering multiple bit errors in one symbol error, the maximum BEP is derived as follows:

Suppose a situation in which the amplitude of the noise u is larger than $d_1/2$ but equal to or less than $d_2/2$. From the definition of d_1 and d_2 (Eq. (15)), a symbol error that causes two bit errors never occurs in such a case. In other words, the maximum possible number of bit errors caused by one symbol error is one. The maximum probability of a symbol error causing a single bit error is expressed by

$$P_{b1,\max} = \frac{1}{m} \Pr \left[\frac{d_2}{2} \sqrt{B_n Z_0} \geq u > \frac{d_1}{2} \sqrt{B_n Z_0} \right]$$

$$= \frac{1}{m} \left[\text{APD}_U \left(\frac{\beta_1 A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) - \text{APD}_U \left(\frac{\beta_2 A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) \right],$$

$$\beta_1 \equiv \frac{d_1}{2} \frac{1}{\sqrt{E_b}}, \quad \beta_2 \equiv \frac{d_2}{2} \frac{1}{\sqrt{E_b}}. \quad (17)$$

Similarly, the maximum probability of a symbol error causing k bit errors is expressed by

$$P_{bk,\max} = \frac{k}{m} \left[\text{APD}_U \left(\frac{\beta_k A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) - \text{APD}_U \left(\frac{\beta_{k+1} A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) \right] \quad (k = 1 \sim m-1),$$

$$P_{bm,\max} = \frac{m}{m} \text{APD}_U \left(\frac{\beta_m A_s}{\sqrt{m}} \sqrt{B_n T_s} \right),$$

$$\beta_k \equiv \frac{d_k}{2} \frac{1}{\sqrt{E_b}} \quad (k = 1 \sim m). \quad (18)$$

Since Eq. (18) with different values of k represents the probabilities of mutually exclusive events, the total maximum BEP is given by summing Eq. (18) with respect to k , to give the following equation:

$$P_{b,\max} = \sum_{k=1}^m P_{bk,\max} = \frac{1}{m} \sum_{k=1}^m \text{APD}_U \left(\frac{\beta_k A_s}{\sqrt{m}} \sqrt{B_n T_s} \right). \quad (19)$$

It is worth noting that Eq. (19) still holds even if $d_k = d_{k+1}$ because the APDs for β_k and β_{k+1} become the same.

If the APDs in Eq. (19) with $k > 1$ are negligibly smaller than the one with $k = 1$, Approx. (16) of the maximum BEP is accurate. However, for an impulsive noise having a heavy-tailed distribution, the occurrence probability of multiple bit errors in one symbol cannot be ignored [20].

Figure 5 shows numerically simulated average BEPs compared with the maximum BEPs evaluated from the APD with Approx. (16) and Eq. (19) for a single-carrier 64-QAM transmission system under interference. The applied interfering signal consists of repetitive tone signal with a bursts duration of 5/3 of the symbol duration of the QAM signal

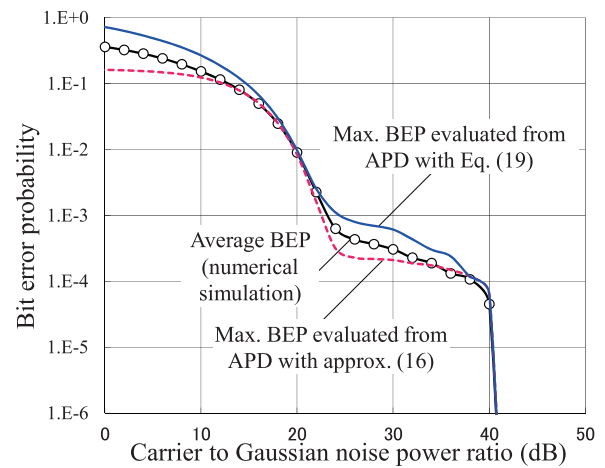


Fig. 5 Comparison of simulated average BEP (plotted with circles), maximum BEP evaluated from APD with approximation (16) (dotted line), and maximum BEP from same APD from Eq. (19) for 64-QAM transmission with interfering signal of repetitive tone bursts.

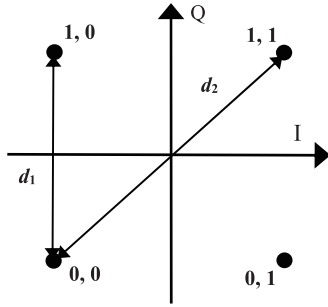


Fig. 6 Signal constellation of QPSK signal.

[20]. Assuming that the bandwidth and internal Gaussian noise level of the APD measuring receiver were the same as those of the QAM receiver, the APD of the sum of the interfering repetitive tone signal and receiver noise was numerically calculated. Figure 5 also shows that the maximum BEPs obtained with Approx. (16) (plotted with a dotted line) are lower than the average BEP (plotted with a solid line and circles) for the ranges where the BEP curves are relatively less steep. This is caused by ignoring the effect of the terms with $k > 1$ in Eq. (19).

3.5 Application to Typical Modulations

In the following, Eq. (19) is applied to typical digital communication signals.

1) PSK and QAM signals

The symbol distances d_k for PSK and QAM signals can be derived from its signal constellation.

The signal constellation of a QPSK signal is shown in Fig. 6 as an example, where,

$$\beta_1 \equiv \frac{d_1}{2} \frac{1}{\sqrt{E_b}} = 1, \quad \frac{\beta_2}{\beta_1} = \frac{d_2}{d_1} = \sqrt{2} \quad (\text{for QPSK}). \quad (20)$$

Note that β_1 in Eq. (20) is identical to β for a QPSK signal that is specified in Table 1 of [17].

Next, for a 16-QAM signal shown in Fig. 7, it can be found that [20]

$$\beta_1 = \sqrt{\frac{2}{5}}, \quad \frac{\beta_2}{\beta_1} = \sqrt{2}, \quad \frac{\beta_3}{\beta_1} = \sqrt{5}, \quad \frac{\beta_4}{\beta_1} = \sqrt{8} \quad (\text{for 16-QAM}). \quad (21)$$

Furthermore, for a 64-QAM signal constellation defined for IEEE 801.11a/g wireless LANs [45],

$$\beta_1 = \sqrt{\frac{1}{7}}, \quad \frac{\beta_2}{\beta_1} = \sqrt{2}, \quad \frac{\beta_3}{\beta_1} = \sqrt{5}, \quad \frac{\beta_4}{\beta_1} = \sqrt{8}, \quad \frac{\beta_5}{\beta_1} = \sqrt{29}, \quad \frac{\beta_6}{\beta_1} = \sqrt{34} \quad (\text{for 64-QAM}). \quad (22)$$

For an 8-PSK signal shown in Fig. 8,

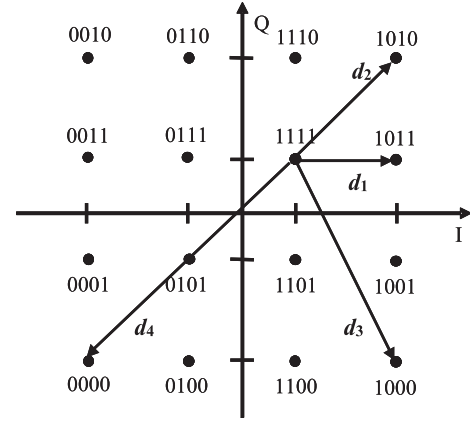


Fig. 7 Constellation of 16-QAM signal.

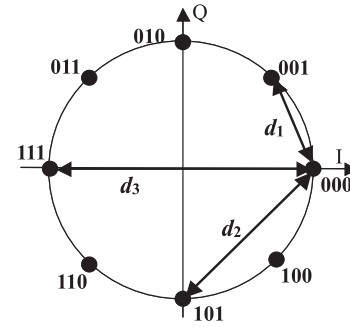


Fig. 8 Constellation of 8-PSK signal.

$$\beta_1 = \sqrt{3} \sin(\pi/8), \quad \frac{\beta_2}{\beta_1} = \frac{d_2}{d_1} = \frac{\sin(\pi/4)}{\sin(\pi/8)}, \quad \frac{\beta_3}{\beta_1} = \frac{d_3}{d_1} = \frac{1}{\sin(\pi/8)} \quad (\text{for 8-PSK}). \quad (23)$$

For an M -PSK, M -pulse amplitude modulation (PAM), or M -QAM symbol waveform that has a rectangular envelope (i.e. without band limitation), the waveform is expressed as

$$s_i(t) = a_i h_a(t) \cos(2\pi f t + \phi_i), \quad a_i, \phi_i : \text{const.} \quad h_a(t) \equiv \begin{cases} 1 & (0 \leq t \leq T_s) \\ 0 & \text{otherwise} \end{cases}. \quad (24)$$

Since any signal difference $s_k(t) - s_i(t)$ ($i \neq k$) can be written in the same form as Eq. (24) having a rectangular envelope, it can be derived that B_n and B_{imp} are equal to the symbol rate $1/T_s$.

$$B_n = B_{\text{imp}} = 1/T_s \quad (\text{for PSK, PAM, and QAM without band limitation}). \quad (25)$$

From condition 1) in Sect. 3.3, the noise and impulse bandwidths of the APD measuring receiver need to be approximately the same as B_n and B_{imp} , respectively.

$$B_{n_APD} \cong B_n = 1/T_s, \quad B_{\text{imp_APD}} \cong B_{\text{imp}} = 1/T_s. \quad (26)$$

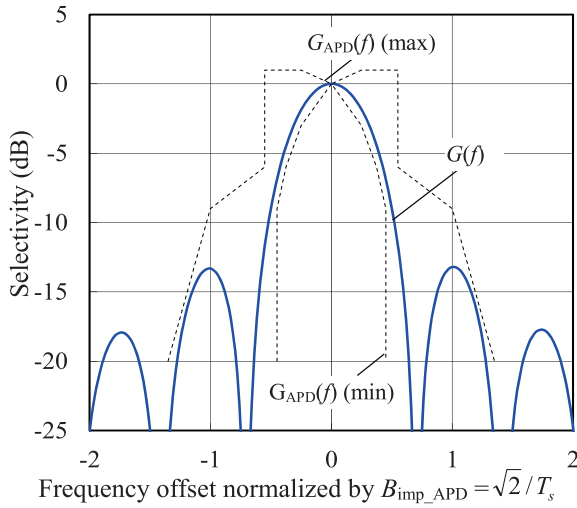


Fig. 9 Frequency selectivity $G(f) = |(S_k(f) - S_i(f))|/K_{ik}$ for measuring maximum BEP of PSK, PAM, or QAM signal with rectangular envelope of symbol waveform with duration of T_s . Scale of horizontal axis is defined by $(f - f_c)T_s/\sqrt{2}$. Dotted lines show example of tolerable deviation (maximum and minimum gain) of frequency selectivity for APD measuring receiver $G_{APD}(f)$ [7] in case of $B_{imp_APD} = \sqrt{2}/T_s$.

A Gaussian filter with $B_{n_APD} (\cong B_{-3_APD}) = 1/T_s$ and $B_{imp_APD} = \sqrt{2}/T_s$ provides a good approximation of Eq. (26) for an APD measuring receiver.

Figure 9 plots the optimal amplitude-frequency selectivity $G(f) = |S_k(f) - S_i(f)|/K_{ik}$ for a PSK, PAM, or QAM signal with a rectangular envelope of symbol waveform. The scale of the horizontal axis is defined by $(f - f_c)T_s/\sqrt{2}$. Note that f_c denotes the filter center frequency. For comparison, the dotted lines indicate the tolerable deviations of the amplitude selectivity for the APD measuring receiver $G_{APD}(f)$ specified by [7] for noise measurement above 1 GHz. The scale of the horizontal axis is $(f - f_c)/B_{imp_APD}$, which is equal to $(f - f_c)T_s/\sqrt{2}$ under the above-mentioned conditions of $B_{n_APD} (\cong B_{-3_APD}) = 1/T_s$ and $B_{imp_APD} = \sqrt{2}/T_s$. It is shown that $G(f)$ fits the selectivity of a standardized APD measuring receiver, which means that the BEP of a PSK, PAM, or QAM signal can be well determined by APD measurement using the -3-dB bandwidth equal to the symbol rate $1/T_s$.

2) M-FSK signals

Assume an M -FSK signal with a rectangular envelope symbol waveform with an amplitude of $\sqrt{2}A_s$ and minimum frequency separation $f_d = 1/2T_s$. The signal can be expressed as

$$\begin{aligned} s_i(t) &\equiv \sqrt{2}A_s \cos[2\pi(f_0 + f_i)t] \quad (0 \leq t \leq T_s), \\ f_i &\equiv if_d \quad (i = 1, 2, \dots, M). \end{aligned} \quad (27)$$

Each signal $s_i(t)$ has a symbol energy of $E_s = A_s^2 T_s / Z_0$, and signal distance d_{ik} is equal to $(2A_s^2 T_s / Z_0)^{1/2}$ for all combinations of i and k ($i \neq k$) [44].

For $|i - k| \leq 2$ (i.e., the frequency deviation $\Delta f = |i - k|f_d \leq 1/T_s$), the selectivity $G(f) = |(S_k(f) - S_i(f))|/K_{ik}$

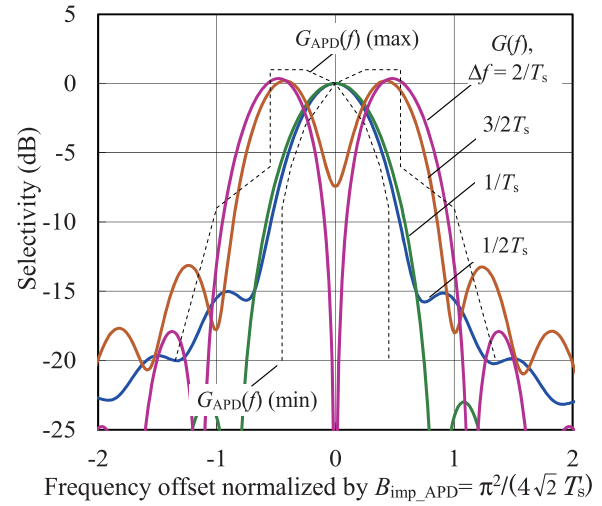


Fig. 10 Frequency selectivity $G(f) = |(S_k(f) - S_i(f))|/K_{ik}$ for measuring maximum BEP of FSK signals with several frequency deviations Δf and rectangular envelope of symbol waveform. Scale of horizontal axis is defined by $(f - f_c)(4\sqrt{2}T_s/\pi^2)$. Dotted lines show tolerable deviation of frequency selectivity for APD measuring receiver, $G_{APD}(f)$ [7] in case of $B_{imp_APD} = \pi^2/(4\sqrt{2}T_s)$.

has a single peak with a unit gain at $f_c = f_0 + (i + k)f_d/2$, as shown in Fig. 10. Note that the scale of the horizontal axis is $(f - f_c)(4\sqrt{2}T_s/\pi^2)$. It can be also derived from Eq. (27) that $B_n = \pi^2/(8T_s) \cong 1.2/T_s$ and $B_{imp} = \pi/(2T_s) \cong 1.6/T_s$ [19]. A Gaussian filter that has $B_{n_APD} (\cong B_{-3_APD}) = \pi^2/(8T_s)$ and $B_{imp_APD} = \sqrt{2}B_{n_APD} = \pi^2/(4\sqrt{2}T_s)$ is suitable for APD measurement for determining the maximum BEP of a binary FSK with $\Delta f \leq 1/T_s$. In the figure, the tolerable deviations of the amplitude selectivity for the APD measuring receiver [7] are also plotted with the assumption of $B_{imp_APD} = \pi^2/(4\sqrt{2}T_s)$.

The maximum BEP (=SEP) is given by

$$\begin{aligned} P_{s,\max} &= P_{b,\max} = \text{APD}_U(\beta_1 A_s \sqrt{B_n T_s}), \\ \beta_1 &= \sqrt{1/2}, \quad \sqrt{B_n T_s} = \pi/\sqrt{8} \end{aligned} \quad (\text{for 2-FSK}). \quad (28)$$

By comparing the maximum BEP of the FSK (Eq. (28)) to that for a BPSK (Eq. (13b) with $\beta = 1$ and $B_n T_s = 1$), the latter is slightly better against a narrowband interference having the same APD due to the terms multiplied by the signal amplitude A_s , that is, $\beta = 1$ ($\sqrt{2} = 3$ dB larger than the value for FSK) and $\sqrt{B_n T_s} = 1$ ($\pi/\sqrt{8} \cong 1$ dB smaller than the value for FSK).

In the case of $|k - i| \geq 3$ (i.e., $\Delta f \geq 3/2T_s$), however, the amplitude selectivity $G(f)$ exhibits two separate major peaks, as shown in Fig. 10. Such selectivity cannot be approximated using a filter installed in a standard APD measuring receiver. This means the APD obtained with a standard (single channel) measuring receiver cannot be applied to the determination of the maximum BEP of multi-frequency FSK signals using large frequency deviations.

3) Spread spectrum signals

The applicability of the expression of the maximum BEP, Eq. (13), to direct sequence spread spectrum (DS-SS) signals generated by multiplying a pseudo noise (PN) sequence by the original symbol waveform is now discussed. The signal bandwidth is spread to N times by multiplying a PN sequence by a chip duration of T_s/N .

If an APD measuring receiver satisfies the conditions given by Eqs. (10) and (11), the measured APD can be applied to determine the maximum SEP and the maximum BEP respectively by Eqs. (13) and (19) without limitations on the type of noise. It is worth noting that the term $(B_n T_s)^{1/2}$ in the above expression of the maximum BEP is increased by the ratio of $N^{1/2}$, which means the improvement of the maximum BEP in terms of signal amplitude by a factor of $10 \log(N)$ in dB for a narrowband interference (known as the spreading gain).

Care must be taken for the condition given by Eq. (10). For a DS-SS signal, the bandwidth of each symbol waveform is increased by the ratio of N , as mentioned above; hence, the noise bandwidth B_n of the selectivity $(S_k(f) - S_i(f))/K_{ik}$ (in Fig. 4(b)) also becomes N times wider. However, the impulse bandwidth B_{imp} does not increase by the ratio of N but on the order of $N^{1/2}$ because the spectrum is spread incoherently.

On the other hand, an APD measuring receiver usually has an impulse bandwidth $B_{\text{imp_APD}}$ nearly equal to the noise bandwidth B_{n_APD} [7]. For example, if an APD measuring receiver in which $B_{n_APD} = B_{\text{imp_APD}} = N/T_s$ is used, the measured APD of a narrowband noise or wideband incoherent noise (as white Gaussian noise) can be directly applied to Eq. (19) for determining the maximum BEP. However, when a wideband impulsive noise is inputted to the APD measuring receiver, the noise at the output of the BPF with $B_{\text{imp_APD}} = N/T_s$ has a peak amplitude $N^{1/2}$ times as high as that with $N^{1/2}/T_s$. This results in an overestimation of the degradation effect in the maximum BEP.

4) Multicarrier signals

Consider a multicarrier signal that consists of N subcarriers. The maximum BEP of the total system is given by averaging the maximum BEP of each subcarrier as

$$P_{b,\max} = \frac{1}{N} \sum_{j=1}^N \frac{1}{m_j} \sum_{k=1}^m \text{APD}_U \left(\frac{\beta_{k,j} A_{s,j}}{\sqrt{m_j}} \sqrt{B_{n,j} T_s}, f_j \right), \quad (29)$$

where m , β_k , A_s , and B_n with subscript j represent the parameters (whose definition are the same as those in Eq. (19)) with respect to the modulation to the j th subcarrier, and f_j denotes the APD measured at the j th subcarrier frequency. Note that the energy allocated to a cyclic prefix and pilot symbol, which do not convey information, should be excluded from the signal amplitude A_j in rms when substituted into Eq. (29).

In some cases, the contribution of the terms with $k > 1$ in Eq. (29) can be ignored because the subchannel bandwidth of a multicarrier system is usually much narrower than

the total bandwidth, which reduces the impulsiveness of the input noise [37], [38], [46].

To evaluate the maximum BEP by using Eq. (29), APD measurement is necessary for all subchannels. It should be noted that conversion of an APD to be valid with another measurement frequency is impossible except for special cases [12]. For the purpose of evaluating the interference impact on multicarrier systems, FFT-based multichannel APD measuring equipment is a useful tool [14], [15], [37], [39].

4. Definition of Emission Limits for Protecting Uncoded Systems

In this section, the definition of emission limit for protecting an uncoded system is discussed. The aim is to find a condition on the APD of interfering noise so that a communication system can keep the maximum BEP below a permissible upper bound P_{req} with a given signal level A_s .

In the case in which the maximum BEP can be approximated by Eq. (16), i.e., the noise is not highly impulsive, the requirement can be written very simply as

$$\begin{aligned} P_{b,\max} &\cong \frac{1}{m} \text{APD}_U \left(\frac{\beta_1 A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) < P_{\text{req}} \\ \Rightarrow \text{APD}_U \left(\frac{\beta_1 A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) &< m P_{\text{req}}. \end{aligned} \quad (30)$$

This condition is satisfied if the APD curve of the interfering noise lies below the limit point $(u_{\text{limit}}, P_{\text{limit}})$ defined by

$$(u_{\text{limit}}, P_{\text{limit}}) \equiv \left(\frac{\beta_1 A_s}{\sqrt{m}} \sqrt{B_n T_s}, m P_{\text{req}} \right). \quad (31)$$

To avoid the risk of underestimation of the maximum BEP caused by impulsive noise, Eq. (19) is applied. The condition of keeping the maximum BEP lower than P_{req} is given by

$$P_{b,\max} = \frac{1}{m} \sum_{k=1}^m \text{APD}_U \left(\frac{\beta_k A_s}{\sqrt{m}} \sqrt{B_n T_s} \right) < P_{\text{req}}. \quad (32)$$

Since Eq. (32) constrains the APD curve at multiple points, namely, $\beta_1 A_s/m^{1/2}$, $\beta_2 A_s/m^{1/2}$, $\dots$, $\beta_m A_s/m^{1/2}$, it is inconvenient that the emission requirement cannot be simply expressed by a single point on the APD graph, as given by Eq. (31). A possible approach to defining an appropriate limit is to restrict the gradient (or steepness) of the APD curve within the probability range of interest. If the gradient of the APD curve is limited to be steeper than 10L dB per decade ($L \leq 0$), the following relationship can be added to Eq. (32) as

$$\text{APD}_U \left(\frac{\beta_k A_s}{\sqrt{m}} \right) \leq \left(\frac{\beta_k}{\beta_1} \right)^L \text{APD}_U \left(\frac{\beta_1 A_s}{\sqrt{m}} \right) \quad (k = 1, 2, \dots, m). \quad (33)$$

The upper bound for the maximum BEP is given by

$$P_{b,\max} = \sum_{k=1}^m \frac{1}{m} \text{APD}_U \left(\frac{\beta_k A_s}{\sqrt{m}} \right) \leq \frac{\gamma}{m} \text{APD}_U \left(\frac{\beta_1 A_s}{\sqrt{m}} \right),$$

$$\gamma \equiv 1 + \left(\frac{\beta_2}{\beta_1} \right)^L + \dots + \left(\frac{\beta_m}{\beta_1} \right)^L. \quad (34)$$

In Eq. (34), the term $(\gamma/m)\text{APD}_U(A_s\beta_1/m^{1/2})$ represents an approximation of the maximum BEP. Since the maximum SEP is given by $\text{APD}_U(A_s\beta_1/m^{1/2})$, the factor γ can be regarded as the maximum number of bit errors caused by one symbol error.

Based on the expression of the maximum BEP (Eq. (34)), the emission requirement can be defined as follows:

The APD curve of the interfering signal must lie below a limit-line segment with a slope of 10 dB per decade in a log-log scaled graph. Note that the line segment connects the following two points:

$$\left(\frac{\beta_1 A_s}{\sqrt{m}}, \frac{m}{\gamma} P_{\text{req}} \right) \text{ and } \left(\frac{\beta_m A_s}{\sqrt{m}}, \frac{m}{\gamma} \left(\frac{\beta_m}{\beta_1} \right)^L P_{\text{req}} \right),$$

$$\gamma \equiv \sum_{k=1}^m \left(\frac{\beta_k}{\beta_1} \right)^L. \quad (35)$$

From Eq. (34), it is clear that $\gamma = m$ for a very impulsive noise, which gives an APD curve having a horizontal part (i.e., $L = 0$), and the limit is given as follows as the most conservative case.

$$(u_{\text{limit}}, P_{\text{limit}}) \equiv \left(\frac{\beta_1 A_s}{\sqrt{m}}, \sqrt{B_n T_s} P_{\text{req}} \right). \quad (36)$$

This limit requires lower (more stringent) APD by a factor of $1/m$ in comparison to the limit given by Eq. (31). In contrast, for a non-impulsive noise that gives an APD curve with a steep slope, γ approaches unity, and the emission limit can be approximated by Eq. (31).

An example of the emission limit for an impulsive in-

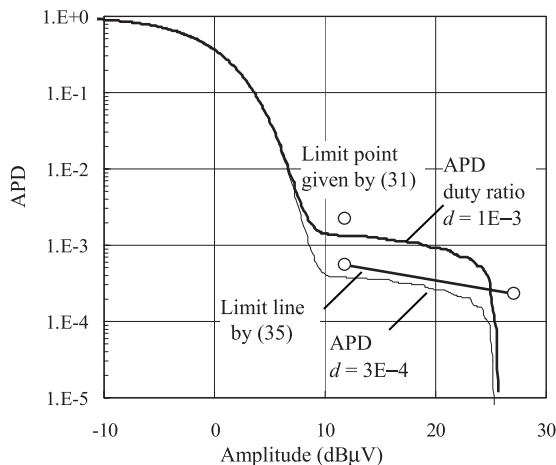


Fig. 11 Comparison of emission limit given by Eq. (31) with that given by Eq. (35) for 64QAM system. Gradient of limit line $10L = -5$ (dB/decade) [20].

terference for the protection of a 64-QAM signal is illustrated in Fig. 11. The figure shows an APD curve of an interfering signal with a duty ratio $d = 10^{-3}$ (which was assumed in the simulations of BEP shown in Fig. 5). Based on the simulation results, the average BEP was 3.7×10^{-4} at CNR of 28 dB, as shown in Fig. 5. To keep the maximum BEP below the permissible value of $P_{\text{req}} = 3.7 \times 10^{-4}$ with the CNR of 28 dB, the emission limit given by (31) and that by (35) are plotted in the graph [20]. It was assumed that the gradient of the APD curve at its plateau (nearly horizontal part) is -5 dB per decade (i.e., $L = -0.5$).

From Fig. 11, the APD curve of the interfering signal with the duty ratio 10^{-3} has some margin in relation to the limit point $(u_{\text{limit}}, P_{\text{limit}})$ given by Eq. (31). This means that an interfering signal with APD that just meets the limit without margin may result in a BEP larger than the permissible value. The limit line given by Eq. (35) is lower than the plateau of the above APD curve by approximately 40%. To meet the requirement for a maximum BEP, for example, the duty ratio of the interfering signal must be reduced by a factor of one-third, as shown by the thin line in the figure.

5. Emission Limits for Protecting Coded Systems

5.1 Coding Gain and Emission Requirement

Currently, many communication systems use error correction coding to improve BEP performance. The improvement by coding is called coding gain and defined as the difference in the signal to interference power ratio between the uncoded and coded systems necessary to achieve the same value of average BEP. It should be noted that a coding scheme is not necessarily implemented in a communication system intended to fully compensate for degradation in BEP caused by a specific interference.

To develop an emission limit to protect a coded system, there are two possible strategies depending on the purpose for which the coding gain should be used [23].

- 1) One is to relax the emission limit in accordance with the coding gain.
- 2) The other is not to relax the limit, and coding gain is kept as a margin to other degradation factors in the communication channel, such as fading and self jamming.

The first approach requires a priori knowledge of coding gain against the interference to be regulated.

However, exact expression on the maximum or average BEP for a coded system is not generally available and the coding gain is estimated only as an approximated value by using bounding techniques or by numerical simulations even if the interference is a Gaussian noise. It is much more complicated to express the BEP of a coded system for non-Gaussian interference. It should also be noted that an APD does not include any information of the time-domain waveform of the interfering signal, while error correction schemes work making use of a sequence of demodulated bits. Hence, it is usually needed to assume that a sequence

of noise samples follow a statistically independent and identical distribution, which is usually the case if interleaving is used.

A method for defining limits was discussed following the first strategy [23]. Since the coding gain is unknown in many actual situations, the use of the coding gain for AWGN is proposed on the basis of the assumption that the former is larger than the coding gain for non-Gaussian interference for small BEP values (e.g., $< 10^{-4}$). This assumption is supported by the results of numerical simulations for a coherent BPSK system with convolutional codes and Viterbi decoding in the presence of a white class A noise.

5.2 Coding Gain of BPSK System under Class A Interference

Theoretical discussions are now made regarding the coding gain for a simple BPSK system using a convolutional code and Viterbi decoding in a class A impulsive noise environment, as assumed in [23].

Middleton's class A model has the following PDF $p_Z(z)$ of the I or Q component z [22],

$$p_Z(z) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m! \sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{z^2}{2\sigma_m^2}\right),$$

$$\sigma_m^2 \equiv \sigma^2 \frac{m/A + \Gamma}{1 + \Gamma}, \quad \Gamma \equiv \frac{\sigma_G^2}{\sigma_I^2}, \quad \sigma^2 = \sigma_G^2 + \sigma_I^2$$

(noise model for coded system). (37)

and APD of the envelope amplitude as

$$\text{APD}_U(u) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \exp\left(\frac{-u^2}{2\sigma_m^2}\right)$$

(noise model for coded system). (38)

where A is the impulsive index given by the product of the average number of received pulses per unit time and the average pulse width, which corresponds to the duty ratio of impulsive interference, σ^2 denotes the total noise power, and Γ represents the ratio of the Gaussian noise power σ_G^2 to the impulsive noise power σ_I^2 . It is known that the model can represent the distribution of many types of actual noise from highly impulsive to purely Gaussian by changing the above-mentioned parameters.

It should be recalled that the bandwidth for modelling APD (given by Eq. (38)) needs to match with that for the communication signal, as given by Eq. (10). This is particularly important for wideband non-Gaussian noise in general because a measured APD depends on the band limitation for the measurement. For a BPSK signal with a rectangular symbol envelope waveform, the noise bandwidth B_n is equal to the bit rate R_b ($= 1/T_s$), as given by Eq. (25). For a coded BPSK having a code rate of R_c and the same information bit rate R_b as the uncoded system, the bandwidth is increased by the ratio of $1/R_c$. Hence, the class A noise parameters in Eq. (38) for a coded BPSK needs to be defined by a $1/R_c$

times wider bandwidth than that for the uncoded one.

To evaluate the BEP for convolutional coding and Viterbi decoding, approximation by truncated union bound is used for simplicity. The BEP is approximated as [44]

$$P_{b,\text{code}} \cong \frac{\beta_{d_{\text{free}}}}{k} P_2(d_{\text{free}}), \quad (39)$$

where k is defined by the code rate $R_c = k/n$ under the assumption that k and n have no common divisor; $\beta_{d_{\text{free}}}$ denotes the number of incorrectly decoded information bits for each possible incorrect path that merges with the correct path, and $P_2(d_{\text{free}})$ and d_{free} represent the pairwise error probability and minimum distance of the code, respectively. For soft decision decoding, $P_2(d_{\text{free}})$ is obtained as the probability that the sum of the coherent component of d_{free} samples of the interfering noise is equal to or larger than $d_{\text{free}}(E_c)^{1/2}$ in terms of energy ($=$ power per bandwidth) [44]. This probability is rewritten in terms of power (without normalizing by the bandwidth) as

$$P_2(d_{\text{free}}) = \Pr[X \geq d_{\text{free}}A_s]$$

$$= \int_{d_{\text{free}}A_s}^{\infty} p_X(x) dx, \quad X \equiv \sum_{l=1}^{d_{\text{free}}} z_l, \quad (40)$$

where z is the inphase component of the interfering noise, whose distribution is given by Eq. (37), and $p_X(x)$ is the PDF of the sum of the in-phase components (i.e. $X = z_1 + \dots + z_{d_{\text{free}}}$).

Assuming that each noise sample is identically class A distributed and independent, the sum of the noise samples then follows a new class A distribution with an impulsive index given by $d_{\text{free}}A$ and total power of $d_{\text{free}}\sigma^2$ as [24]

$$p_X(x) = e^{-Ad_{\text{free}}} \sum_{M=0}^{\infty} \frac{(Ad_{\text{free}})^M}{M! \sqrt{2\pi d_{\text{free}}\sigma_M^2}} \exp\left(-\frac{x^2}{2d_{\text{free}}\sigma_M^2}\right),$$

$$\text{APD}_R(r) = e^{-Ad_{\text{free}}} \sum_{M=0}^{\infty} \frac{(Ad_{\text{free}})^M}{M!} \exp\left(-\frac{r^2}{2d_{\text{free}}\sigma_M^2}\right), \quad (41)$$

$$\sigma_M^2 \equiv \sigma^2 \frac{M/(Ad_{\text{free}}) + \Gamma}{1 + \Gamma}.$$

Moreover, by considering the situation in which the sum of the noise is always out of phase with the desired signal, the maximum BEP for the coded BPSK system can be expressed in a similar way to that of the uncoded system by using the APD given by Eq. (41) with the substitution of $r = d_{\text{free}}A_s$.

$$P_{b,\text{code}} \max$$

$$\cong \frac{\beta_{d_{\text{free}}}}{k} e^{-Ad_{\text{free}}} \sum_{M=0}^{\infty} \frac{(Ad_{\text{free}})^M}{M!} \exp\left(-\frac{d_{\text{free}}^2 A_s^2}{2\sigma_M^2}\right),$$

$$\sigma_M^2 \equiv \sigma^2 \frac{M/(Ad_{\text{free}}) + \Gamma}{1 + \Gamma}$$

(for coded system). (42)

On the other hand, the uncoded BPSK system with the same bit rate occupies a narrower bandwidth by the factor of

R_c . Considering the condition given by Eq. (10), the APD of the noise needs to be modeled with an increased impulsive index by the ratio of $1/R_c$ and decreased total noise power by the ratio of R_c [46].

$$\text{APD}_U(u) = e^{-A/R_c} \sum_{M=0}^{\infty} \frac{(A/R_c)^M}{M!} \exp\left(-\frac{u^2}{2R_c\sigma_M^2}\right),$$

$$\sigma_M^2 \equiv \sigma^2 \frac{M/(A/R_c) + \Gamma}{1 + \Gamma} \quad (\text{for uncoded system}). \quad (43)$$

The maximum BEP of the uncoded BPSK with the same bit rate as the coded one is given by Eq. (19) with $m = 1$, $\beta_1 = 1$, and $B_n T_s = 1$. By substituting $u = A_s$ into Eq. (43)

$$P_{b,\text{uncode},\text{max}} = e^{-A/R_c} \sum_{m=0}^{\infty} \frac{(A/R_c)^m}{m!} \exp\left(\frac{-A_s^2}{2R_c\sigma_m^2}\right),$$

$$\sigma_m^2 \equiv \sigma^2 \frac{m/(A/R_c) + 1}{1 + \Gamma} \quad (\text{for uncoded system}). \quad (44)$$

By comparing Eq. (44) to Eq. (42), it is found that the effect of coding to the class A noise on the BPSK system is due to the following factors:

- 1) An increase in the BEP by the factor $\beta_{d_{\text{free}}}/k$ multiplied by the APD.
- 2) An equivalent increase in the signal to noise power ratio by the factor of $10 \log(R_c d_{\text{free}})$ in dB scale.
- 3) An increase in the impulsive index of the distribution by the ratio of $R_c d_{\text{free}}$ [25].

It should be noted that the first two factors are common to the coding effect on the BPSK system under AWGN [44], which can be regarded as the coding gain for Gaussian noise, denoted by G_{GN} . Thus, the total coding gain is expressed by

$$G = G_{\text{GN}} + G_{\text{IN}} \quad \text{in dB}, \quad (45)$$

where G_{IN} denotes the extra coding gain for an impulsive noise that is the effect of factor 3).

When the slope of a BEP curve is steep for a small BEP value, the effect of factor 1) becomes relatively small, and the asymptotic value of the coding gain G_{GN} [44] is given by

$$G_{\text{GN}} \cong 10 \log(R_c d_{\text{free}}) \quad \text{in dB}. \quad (46)$$

The effect of factor 3) is illustrated in Fig. 12. Curve a) in the graph shows the APD of class A noise ($\Gamma = 0.1$, $A = 0.001$, $\sigma^2 = 1.0$), and curve b) plots the APD with ($\Gamma = 0.1$, $A = 0.005$, $\sigma^2 = 1.0$). Both curves have a plateau, which indicates the impulsiveness of the noise. It is known that the width of the plateau of the APD curve (shown by W_a or W_b in the figure) is approximately given by [47]

$$W \cong 10 \log_{10} \left(1 + \frac{1}{\Gamma A} \right) \quad \text{in dB}. \quad (47)$$

As the impulsive index A increases (i.e. the noise becomes

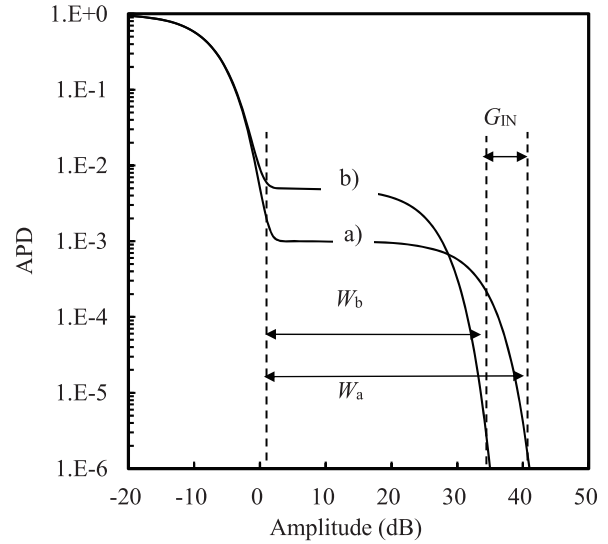


Fig. 12 Comparison of APD curves of class A noise with parameters of (a) $(\Gamma, A) = (0.1, 0.001)$, (b) $(\Gamma, A) = (0.1, 0.005)$.

less impulsive), the width W decreases and the plateau vanishes when the noise approaches Gaussian.

Since the maximum BEP curve plotted on a log-log scaled graph is the same as the APD curve (exactly for binary modulations as BPSK and approximately for multivalued modulations), the plateau width of the BEP curve also decreases by the same amount due to the increase in the impulsive index of the interfering noise. This decrease in the plateau width can be regarded as the improvement in the signal level for achieving the same value of the maximum BEP if the BEP of interest is below the plateau. In other words, the extra coding gain for an impulsive noise G_{IN} is approximately given by the decrease in the plateau width in the BEP (or APD) curve.

From Approx. (47), the extra coding gain due to the increase in the impulsive index from A to $R_c d_{\text{free}} A$ is approximated by

$$G_{\text{IN}} \cong 10 \log_{10} \left(\frac{1 + 1/(\Gamma A)}{1 + 1/(\Gamma A R_c d_{\text{free}})} \right). \quad (48)$$

For a nearly Gaussian noise (in the case of $\Gamma A \gg 4$ or $A > 10$ [48]), G_{IN} is negligible. In contrast, for a very impulsive noise, Approx. (48) is simplified as

$$G_{\text{IN}} \cong 10 \log_{10}(R_c d_{\text{free}}), \quad (49)$$

which is the same amount of the asymptotic value of G_{GN} (from the Approx. (46)). Moreover, the extra coding gain G_{IN} reduces as d_{free} increases and approaches zero. This can be understood as the reduction in the impulsiveness of the noise as a result from the summation of d_{free} independent noise samples (known as the central limit theorem).

The above-mentioned discussion on a coded BPSK system can be extended to other modulation schemes in principle by means of theoretical or numerical analysis.

6. Conclusions

A method of evaluating the maximum BEP by measuring APD and its application to emission requirements for protecting a communication system was presented together with necessary conditions. Discussions on the determination of BEP in the presence of fading and intersymbol interference were not presented, and remain for future work. Application of APD measurement is not limited to compliance testing, but also radio environment monitoring, dynamic spectrum allocation, and analysis of intrasystem interference. The key for widespread use of APD measurement is the availability of an FFT-based multichannel APD measuring function. It is expected that such functions will be implemented in common spectrum analyzers in the near future.

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