LETTER

Antenna Array Self-Calibration Algorithm with Location Errors for MUSIC

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SUMMARY The characteristics of antenna array, like sensor location, gain and phase response are rarely perfectly known in realistic situations. Location errors usually have a serious impact on the DOA (direction of arrival) estimation. In this paper, a novel array location calibration method of MUSIC (multiple signal classification) algorithm based on the virtual interpolated array is proposed. First, the paper introduces the antenna array positioning scheme. Then, the self-calibration algorithm of FIR-Winner filter based on virtual interpolation array is derived, and its application restriction are also analyzed. Finally, by simulating the different location errors of antenna array, the effectiveness of the proposed method is validated.
key words: *direction of arrival, location error, virtual interpolated array, self-calibration*

1. Introduction

DOA estimation is one of the major research direction in signal processing. It is widely used in the field of radar, sonar, radio astronomy, and seismology to oceanography [1]. The DOA estimation methods, such as MUSIC (multiple signal classification), are based on accurate antenna array locations. While, there are often errors between the actual and the measured antenna array locations in practical systems, which will lead to the performance deterioration of these DOA estimation methods. Moreover, unlike gain and phase errors, perturbed array manifolds are nonlinear with respect to the location errors of the antenna array. This nonlinearity makes the analysis of the effect of antenna array location errors on DOA estimation methods is more difficult [2], [3].

Due to its high resolution properties and accurate performance, the MUSIC, which is based on the specific theory, attracts practical interests. It is implemented in spatial domain by decomposing the signal into two orthogonal subspaces (signal subspace and noise subspace) and could be used for parameter estimation, such as the DOA of superposed radio signals on the antenna array. Meanwhile, compared with the DFT (discrete fourier transform), MUSIC method is able to be applied to the DOA estimation and the spatial sampling of the wavefront both for uniform and non-uniform arrays. However, the effects of array perturbations caused by the non-uniform array is severe even for micro perturbations [4]. Thus, the antenna array location error is necessary to be calibrated for MUSIC.

There are several solutions have been proposed for dealing with the antenna array location error [5], [6]. A improved algorithm (I-NSF) in [7], using the maximum a posteriori noise subspace fitting (MAP-NSF) algorithm results as the input to the iterative approach, is proposed to eliminate sensor location error and remove the sensitivity to initialization. While, these methods commonly use the auxiliary source whose location is known in advance or self-calibrated, which estimates the source DOA and perturbed array response vector parameters iteratively. As is revealed in [8], virtual baseline with the flexible length based on a two-dimensional (2-D) antenna array is proposed for UWB (ultra wide band) interferometer DF (direction finding) system. It could compensate the fluctuations of the antenna phase center and provide unambiguous AOA (angle of arrival) during the detection. Furthermore, inspired by the idea of using antenna array interpolation for virtual baselines, virtual interpolated antenna array are expected to be able to calibrate the antenna array location error without auxiliary sources or parameter iterations.

The remainder of the paper is organized as follows. Section 2 provides the antenna array locating scheme, and derives the interpolated array calibration algorithm and application restrictions. Then, the effectiveness of the proposed method under different antenna array location errors is verified by simulations in Sect. 3. Finally, the conclusion is presented in Sect. 4.

2. Virtual Interpolated Array Algorithm

2.1 Antenna Array Location Scheme

Suppose N incoherent plane waves are incident on an uniform linear array of M $(N < M)$ elements. The received signal vector $r(t)$ is an $M \times 1$ complex vector given by

$$r(t) = A * s(t) + n(t) \tag{1}$$

where $N \times 1$ vector $s(t)$ is given by

$$s(t) = [s_1(t), s_2(t), \ldots\ldots, s_N(t)]^T \tag{2}$$

with $s_k(t)$ being a complex narrow-band amplitude of the k-th signal, and $n(t)$ is $M \times 1$ complex noise vector. $s(t)$ is N-dimensional complex Gaussian distribution with mean zero and covariance matrix R_s. $n(t)$ is a vector Gaussian process, independent of the signals, with mean zero and covariance

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σ_n^2.

The steering matrix A is an $M \times N$ matrix such that

$$\mathrm{A} = [\mathrm{a}(\theta_1), \mathrm{a}(\theta_2), \ldots\ldots, \mathrm{a}(\theta_N)] \tag{3}$$

with steering vector $\mathrm{a}(\theta_i)$ being the $M \times 1$ signal direction vector at angle θ_i from the array's boresight, given by

$$\mathrm{a}(\theta_i) = [1, e^{2\pi(d/\lambda)\sin(\theta_i)}, \ldots\ldots, e^{2\pi(M-1)(d/\lambda)\sin(\theta_i)}]^T \tag{4}$$

where T denotes the transpose, d denotes the uniform element spacing, and λ denotes the wavelength of signals.

When there is only the antenna array location error, it can be considered as an orientation dependent phase perturbation $\Gamma(\theta_i)$, that is

$$\hat{W}(\theta_i) = [1 \quad e^{j2\pi(\Delta d_2/\lambda)\sin(\theta_i)} \quad \cdots \quad e^{j2\pi(\Delta d_M/\lambda)\sin(\theta_i)}]^T \tag{5}$$

$\Delta d_m, 1 \leq i \leq M$ is the location error between actual and measured positions of the i-th antenna element. Assume that Δd_m is Gaussian distribution, $\sigma_{d_m}^2$ is the variance of Δd_m, the probability of $\Delta d_m \in [-3\sigma_{d_m}, 3\sigma_{d_m}]$ is 99.74%.

where the steering matrix $\tilde{A}$ with perturbation matrix W is

$$\tilde{A} = [\tilde{\mathrm{a}}(\theta_1), \tilde{\mathrm{a}}(\theta_2), \ldots\ldots, \tilde{\mathrm{a}}(\theta_N)] \tag{6}$$

and

$$\tilde{\mathrm{a}}(\theta_i) = \mathrm{a}(\theta_i) \cdot \hat{W}(\theta_i) \tag{7}$$

and $d'_m = (m-1)d + \Delta d_m, 1 \leq m \leq M$ is the actual spacing based on nonuniform linear array between the first antenna element and the m-th.

The covariance matrix of the received signal vector from the signals model and the above assumptions is

$$R_x = E[r(t)r(t)^H] = \tilde{A}R_s(\tilde{A})^H + \sigma_n^2 I \tag{8}$$

with the superscript H denoting the Hermitian transpose. It also follows that the rank of $WAR_s(WA)^H$ is N, with the smallest $M-N$ of its eigenvalues being zero. Therefore, if the eigenvalues of R_x are ordered in descending order of magnitudes such that $\lambda_1 \geq \lambda_2 \geq \cdots$, then

$$\lambda_{N+1} = \lambda_{N+2} = \cdots = \sigma_n^2 \tag{9}$$

The MUSIC spectrum [8], [9] can be expressed in terms of the signal eigenvectors, as given by

$$P(\theta) = \frac{1}{(\tilde{a}(\theta))^H (\sum_{n=1}^{N} v_n v_n{}^H) \tilde{a}(\theta)} \tag{10}$$

where $v_1, v_2, \cdots, v_M$ are the noise eigenvectors.

2.2 Interpolated Array Calibration Algorithm

To deal with the problem mentioned above, a virtual interpolation self-calibration algorithm based on Wiener filter (W-VISC) is proposed based on the nonuniform linear array, to construct a virtual uniform linear array and calibrate the antenna array location error. Assume that, the number of the virtual elements is M. Thus, the set of the actual elements coordinate in the x-direction can be expressed as $\left[x_0 + d'_1, x_0 + d'_2, \ldots, x_0 + d'_M\right]$, and the coordinate of virtual elements is $x_0 + d \times (0, 1, \ldots\ldots, M-1)$. The signal of the virtual interpolated array with minimum mean square error (MMSE) interpolation [7] is

$$x(t) = P^{\mathrm{T}}\mathrm{R}^{-T}r(t) \tag{11}$$

where R is $M \times M$ matrix, and

$$R = [R_{ij}] = [J_0(\frac{2\pi(d'_i - d'_j)}{\lambda})], 1 \leq i, j \leq M \tag{12}$$

where P denote the cross-correlation $M \times M$ matrix between the received signal vector $r(t) = [r_1(t), r_2(t), \cdots, r_M(t)]^{\mathrm{T}}$ and the desired vector $x(t)$.

$$P = [P_{ij}] = [J_0(\frac{2\pi(d'_i - (j-1)d)}{\lambda})], 1 \leq i, j \leq M \tag{13}$$

where J_0 denotes the first-class Bessel function with zero order. Equation (11) is actually a M-th order FIR−Winner filter, with the MMSE

$$\sigma^2{}_{\min} = J_0(0) - P^T R^{-1} P \tag{14}$$

The interpolation output $x(t)$ can be considered as a virtual uniform antenna array with M antenna elements.

2.3 Restriction

This algorithm of interpolated array calibration is restricted to the number and space of antenna elements. In order to evaluate the performance of the virtual antenna array above, RMSE (root mean square error) $\sigma_{\min}$ is used, considering $\sigma_{\min} \leq 10^{-2}$. Assume that, the amplitudes of received signals are normalized and the frequency is 10 GHz. As is showed in Fig. 1(a), where the RMSE with different number of antenna elements is depicted, it can be found that the RMSE of virtual antennas are all less than 10^{-2} when $M \geq 40$. From Fig. 1(b), the RMSE could be controlled to be less than 10^{-2} and varying from $x_0 + d'_1$ to $x_0 + d'_{40}$. Moreover, deploying the antenna array only in the x-direction could not satisfy the antenna array location error in any attitude. Therefore, the deployment of the antennas in y-direction is necessary. In addition, as aforementioned, the virtual interpolated array could be adjusted in real-time with the change of target frequency.

3. Performance Evaluation

Simulation experiments, compared with I-NSF and MAP-NSF, are conducted to evaluate the performances of the proposed interpolated array calibration algorithm under the

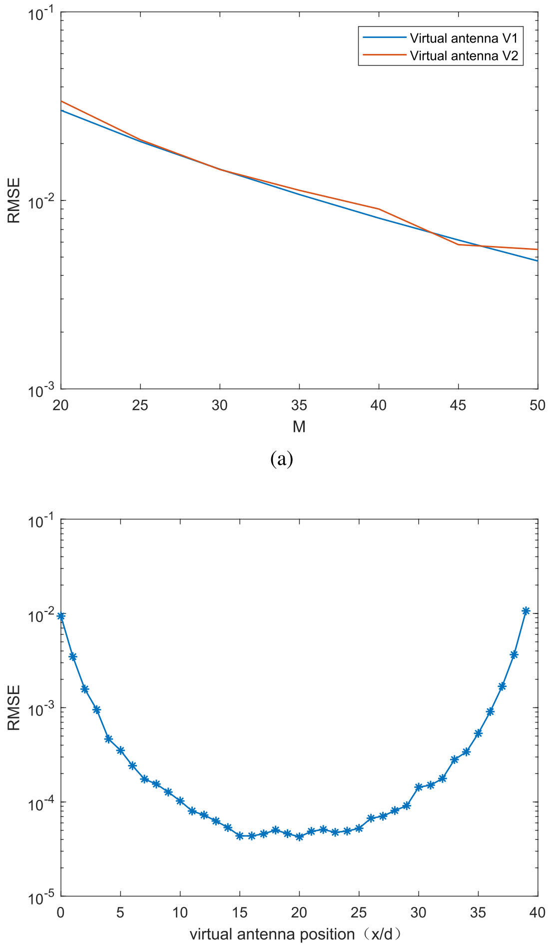

Fig. 1 Evaluation of RMSE. (a) RMSE with different M. (b) RMSE with d = 0.03 m and M = 40.

Table 1 Simulation parameters.

Parameter	Value
Frequency	10 GHz
M	40
N	1 ~ 2
d	0.0135m
λ	0.03 m
Incident angle	24° and 28°
Number of snapshots K	32 ~ 1024
SNR	0 ~ 32 dB

nonuniform antenna array. The parameters of the simulation are given in Table 1.

In Fig. 2(a), suppose that DOAs of two targets are 24° and 28° with same SNR (signal to noise ratio) of 5 dB. Antenna array location errors with $\sigma_{d_i} = 20\% d$ obviously deteriorate the resolution of MUSIC spectrum, and the target with DOA of 28° could not be effectively distinguished. After applying the proposed self-calibration method, DOAs of the two targets could be accurately estimated and the resolution is greatly improved. As shown in Fig. 2(b), the relationship between the RMSE of different algorithms and the number of K is analyzed. in which K is 32, 64, 128, 256, 512

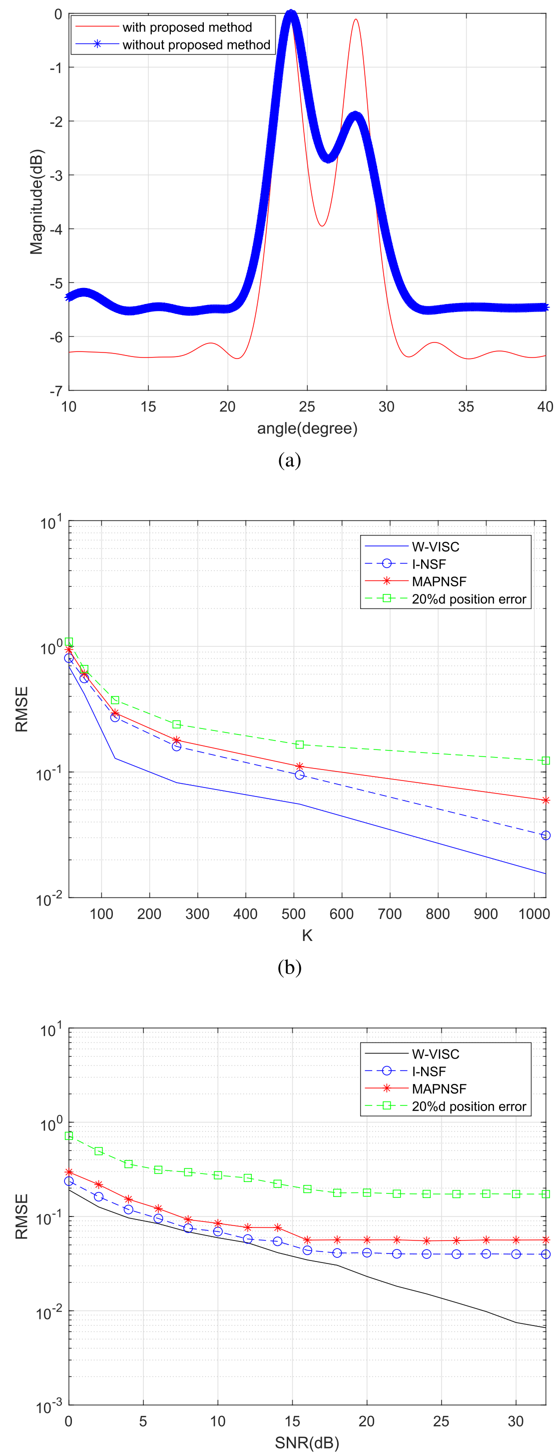

Fig. 2 Evaluation with different antenna array location errors. (a) MUSIC spectrum with two targets. (b) RMSE for different K of single target. (c) RMSE for different SNR of single target.

and 1024, the SNR is 5 dB. In general, with tne increase of K, the influence of location error decrease gradually. Compared with MAP-NSF and I-NSF, W-VISC method has more obvious performance improvement with the increase of K. In addition, the influence of different SNR for algorithms under same location error (20%d) is verified. Firstly, under the same SNR, W-VISC, MAP-NSF and I-NSF have calibrated the location error and obtained more accurate AOA estimation, but W-VISC has better performance under low SNR.

4. Conclusion

The perturbed array manifolds caused by the antenna array location error seriously deteriorates the DOA estimation, and even the MUSIC spectrum is difficult to reflect the actual angle of multi-target. Hence, it is necessary to calibrate the antenna array location error. In this paper, the antenna array models of the MUSIC algorithm with location error was first established. Then, a novel self-calibration algorithm for MUSIC, which is equivalent to an M-th order FIR−Winner filter, was proposed and derived based on virtual interpolated under the nonuniform antenna array. Next, the restrictions in terms of RMSE were verified through simulation, and the minimum number of antenna elements and the maximum space are determined. Finally, the simulation of the location errors of different antenna arrays, compared with MAP-NSF and I-NSF, proves the effectiveness of the proposed method.

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