LETTER

Doppler Resilient Waveforms Design in MIMO Radar via a Generalized Null Space Method

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SUMMARY To mitigate the interference caused by range sidelobes in multiple-input multiple-output (MIMO) radar, we propose a new method to construct Doppler resilient complementary waveforms from complete complementary code (CCC). By jointly designing the transmit pulse train and the receive pulse weights, the range sidelobes can vanish within a specified Doppler interval. In addition, the output signal-to-noise ratio (SNR) is maximized subject to the Doppler resilience constraint. Numerical results show that the designed waveforms have better Doppler resilience than the previous works.

key words: *Doppler resilient complementary waveforms, complete complementary code (CCC), range sidelobes, MIMO radar*

1. Introduction

Multiple-input multiple-output (MIMO) radar systems utilize multiple transmit antennas to transmit independent waveforms and multiple receive antennas to receive target echoes. Compared with the traditional phased array radar, MIMO radar has a better performance in target detection and parameter estimation due to the configuration of antenna arrays and waveform diversity [1]–[3].

In radar, phase coding [4] is commonly used to generate waveforms with impulse-like auto-correlation functions for localizing targets in range. A phase coded waveform is phase coded by a unimodular sequence and the auto-correlation function of the coded waveform is controlled by the auto-correlation function of the unimodular sequence. An ideal auto-correlation function is an impulse function. However, it is impossible to reach sidelobes annihilation with one sequence [5]. Thus, complementary sequence sets (CSSs) [6] are focused, which are widely used in waveform design for detection in radar systems [4].

Golay complementary pairs (GCPs), firstly discovered by Golay, have zero aperiodic auto-correlation sums at nonzero lags [7]. In radar systems, Golay complementary waveforms phase coded by GCPs are coherently transmitted to a point target, and the received signals are coherently processed through the matched filter. If the target is stationary, the sum of the matched filter outputs will be free of range sidelobes. However, the ideal auto-correlation property of Golay complementary waveforms is extremely sensitive to Doppler shift, nonzero Doppler shift will destroy the ideal complementary property. In other words, off the zero Doppler axis, the sum of the matched filter outputs has quite large range sidelobes. This means that a weak target near a strong target with different Doppler shift may be masked by the range sidelobes of the strong target [5]. Therefore, it is of great significance for radar to design waveforms with low range sidelobes at modest Doppler shifts.

More recently, some construction methods of Doppler resilient waveforms of single-channel radar, full polarimetric radar and MIMO radar have been presented. Suvorova et al. [8] first proposed to design the transmission of complementary waveforms according to the first order Reed-Müller codes, for which the range sidelobes at a specific Doppler shift were suppressed. Pezeshki et al. [5] used Prouhet-Thue-Morse (PTM) sequences to construct Doppler resilient Golay complementary waveforms, and the range sidelobes near zero Doppler were annihilated. On the basis of [5], Chi et al. [9] designed the transmission scheme of Doppler resilient Golay complementary waveforms by using oversampled PTM sequence, so that the range sidelobes vanished near a rational Doppler shift $\theta = 2\pi l/m$, where $l \neq 0$ and $m \neq 1$ are co-prime integers. Tang et al. [10] extended the fully polarimetric radar scene in [5] to MIMO radar and designed the Doppler resilient complete complementary code (CCC) [11] for MIMO radar based on the generalized Prouhet-Thue-Morse (GPTM) sequence, where the range sidelobes can vanish at modest Doppler shifts. Thereafter, Nguyen et al. [12] used Equal Sums of Powers (ESP) sequences to provide the same Doppler resilience as PTM sequences, but multiple antennas were needed to transmit the pulse trains. Dang et al. [13]–[15] proposed a method to jointly design the transmit and receive pulse trains in order to annihilate the range sidelobes at small Doppler. However, the decrease in the signal-to-noise ratio (SNR) occured. As a trade-off, the idea of maximizing SNR under the Doppler resilience was proposed. Wu et al. [16] constructed Doppler resilient Golay complementary waveforms which realized range sidelobes suppression in multiple flexibly-adjustable Doppler zones, and the controllable loss between Doppler resolution and SNR. The development in [17] was to propose a new class of sequence pairs, called "quasi-orthogonal Z-complementary pairs (QOZCPs)", which could be applied to fully polarimetric radar systems. In [18], a null space ap-

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proach for designing Golay complementary waveforms with Doppler resilience was proposed, and the range sidelobes were cleared in a specified Doppler interval of interest or even an overall Doppler interval.

Motivated by the work of Wang et al. [18], we extend the fully polarimetric radar scene to MIMO radar and propose a generalized null space method to jointly design the transmission of the transmit pulse train and the receive pulse weights. The received pulse weights in the proposed method are complex-valued. Compared with the integer-valued weights in [10] and [15], the proposed received pulse weights are more flexible. In addition, based on the proposed method, the range sidelobes of resulting waveforms are annihilated in a wider Doppler band than the previous works.

The rest of this paper is organized as follows. In Sect. 2, some basic definitions are provided. In Sect. 3, we propose a method of constructing Doppler resilient waveforms in MIMO radar. In Sect. 4, some numerical results are given. Finally, the paper is concluded in Sect. 5.

2. Preliminaries

Let $\mathbf{0}$, $\mathbf{O}$, $j = \sqrt{-1}$, and Null($\mathbf{G}$) denote null vector, null matrix, imaginary unit, and null space of matrix $\mathbf{G}$, respectively. Let $(\cdot)^T$, $(\cdot)^*$, $\|\cdot\|_p$, $\lfloor\cdot\rfloor$, and Diag$(\cdot)$ denote the transpose, complex conjugate, l_p-norm, greatest integer no more than a real variable, and diagonal matrix created from a vector, respectively.

Definition 1. *Let* $\mathbf{x} = (x[0], x[1], \cdots, x[L-1])$ *and* $\mathbf{y} = (y[0], y[1], \cdots, y[L-1])$ *be two complex-valued sequences of length-L. The aperiodic cross-correlation function (ACCF) of* $\mathbf{x}$ *and* $\mathbf{y}$ *at shift k (−L<k<L) is defined as*

$$C_{\mathbf{x},\mathbf{y}}[k] = \begin{cases} \sum\limits_{i=0}^{L-1-k} x[i]y^*[i+k], & 0 \leq k<L, \\ \sum\limits_{i=0}^{L-1+k} x[i-k]y^*[i], & -L<k<0. \end{cases}$$

When $\mathbf{x} = \mathbf{y}$*, the above definition becomes aperiodic autocorrelation function (AACF), denoted by* $C_{\mathbf{x}}[k]$.

Definition 2. *A set with size D of length-L unimodular sequences* $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_{D-1}\}$ *is called a complementary sequence set (CSS) if the sum of aperiodic auto-correlation functions satisfies*

$$\sum_{d=0}^{D-1} C_{\mathbf{x}_d}[k] = DL\delta_k,$$

where δ_k *is the Kronecker delta function.*

When $D = 2$*, the CSS degenerates to a Golay complementary pair (GCP).*

Definition 3 ([11])**.** *Two CSSs* $R_0 = \{\mathbf{x}_{0,0}, \mathbf{x}_{0,1}, \cdots, \mathbf{x}_{0,D-1}\}$ *and* $R_1 = \{\mathbf{x}_{1,0}, \mathbf{x}_{1,1}, \cdots, \mathbf{x}_{1,D-1}\}$ *are called mutually orthogonal if*

$$\sum_{d=0}^{D-1} C_{\mathbf{x}_{0,d},\mathbf{x}_{1,d}}[k] = 0, \forall k.$$

Definition 4 ([11])**.** *Let* $R_p = \left\{\mathbf{x}_{p,0}, \mathbf{x}_{p,1}, \cdots, \mathbf{x}_{p,D-1}\right\}$ *be a CSS with size D of length-L for* $0 \leq p \leq D-1$*, and a matrix* $\mathbf{R}$ *is given by*

$$\mathbf{R} = \begin{bmatrix} \mathbf{x}_{0,0} & \mathbf{x}_{0,1} & \cdots & \mathbf{x}_{0,D-1} \\ \mathbf{x}_{1,0} & \mathbf{x}_{1,1} & \cdots & \mathbf{x}_{1,D-1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{x}_{D-1,0} & \mathbf{x}_{D-1,1} & \cdots & \mathbf{x}_{D-1,D-1} \end{bmatrix}.$$

Then $\left\{\mathbf{x}_{p,d}\right\}_{0\leq p,d\leq D-1}$ *is a (D,L)-CCC if and only if any two rows of* $\mathbf{R}$ *are mutually orthogonal.*

3. Signal Model

Let $x_{p,d}(t)$ denotes the analog signal phase coded by the length-L sequence $\mathbf{x}_{p,d}$. In order to analyse the waveform transmission scheme of a MIMO radar with D transmit antennas and S receive antennas, a complete complementary code waveform matrix $\mathbf{W}$ is given by

$$\mathbf{W} = \begin{bmatrix} x_{0,0}(t) & x_{0,1}(t) & \cdots & x_{0,D-1}(t) \\ x_{1,0}(t) & x_{1,1}(t) & \cdots & x_{1,D-1}(t) \\ \vdots & \vdots & \vdots & \vdots \\ x_{D-1,0}(t) & x_{D-1,1}(t) & \cdots & x_{D-1,D-1}(t) \end{bmatrix}.$$

We use the signal mode given in [15]. Let $\mathbf{U} = \{u_n\}_{n=0}^{N-1}$ be the order of the transmitted pulses. To allow for indexing of D different waveforms, we take $\mathbf{U}$ to be a D-ary sequence; that is, defined over the alphabet $\mathcal{D} = \{0, 1, \ldots, D-1\}$. Hence, for $p = 0, 1, \cdots, D-1$, the length-N coherent transmit pulse train $z_{p,\mathbf{U}}(t)$ of the pth antenna is

$$z_{p,\mathbf{U}}(t) = \sum_{n=0}^{N-1}\left(\sum_{d=0}^{D-1}\left(\frac{1}{D}\sum_{r=0}^{D-1}\omega^{r(u_n-d)}\right)x_{p,d}(t-nT)\right),$$

where T is the pulse repetition interval (PRI), and

$$\frac{1}{D}\sum_{r=0}^{D-1}\omega^{r(u_n-d)} = \begin{cases} 1, & u_n = d \\ 0, & u_n \neq d, \end{cases}$$

in which $\omega = e^{j2\pi/D}$ and $d \in \{0, 1, \cdots, D-1\}$. At the nth PRI of $z_{p,\mathbf{U}}(t)$, $x_{p,d}(t)$ is transmitted if $u_n = d$.

Consider a point target with a Doppler shift f_d in Hz. Also, consider the received signal in the sth antenna to illustrate the signal processing operation at the receiver, where $s = 0, 1, \cdots, S-1$. The received signal of antenna s is given by

$$y_s(t) = \left(\sum_{p=0}^{D-1} h_{s,p} z_{p,\mathbf{U}}(t)\right) e^{j2\pi f_d t},$$

where $h_{s,p}$ denotes the target scattering coefficient from the

pth transmit antenna to the sth receive antenna.

At the receiver with S antennas, each antenna has D temporal responses of the receiver filters, i.e., $z_{0,\mathbf{V}}(t), z_{1,\mathbf{V}}(t), \cdots, z_{D-1,\mathbf{V}}(t)$. For $q = 0, 1, \cdots, D-1$, $z_{q,\mathbf{V}}(t)$ is given by

$$z_{q,\mathbf{V}}(t) = \sum_{n=0}^{N-1} v_n^* \left(\sum_{d=0}^{D-1} \left(\frac{1}{D} \sum_{r=0}^{D-1} \omega^{r(u_n-d)} \right) x_{q,d}(t-nT) \right),$$

where $\mathbf{V} = \{v_n\}_{n=0}^{N-1}$ is a complex-valued weight sequence of the received pulses. If $p = q$, $z_{q,\mathbf{V}}(t)$ is the weighted version of $z_{p,\mathbf{U}}(t)$. For ease of presentation, $z_{p,\mathbf{U}}(t)$ and $z_{q,\mathbf{V}}(t)$ are abbreviated as $z_{p\mathbf{U}}(t)$ and $z_{q\mathbf{V}}(t)$, respectively.

The received signal of the sth antenna passes through the qth receiver filter, then the output of the receiver filter is given by

$$\begin{aligned} F_{s,q}(\tau, f_d) &= \int_{-\infty}^{+\infty} y_s(t) z_{q\mathbf{V}}^*(t-\tau) \mathrm{d}t \\ &= \sum_{p=0}^{D-1} h_{s,p} \chi_{p\mathbf{U},q\mathbf{V}}(\tau, f_d), \end{aligned}$$

where

$$\chi_{p\mathbf{U},q\mathbf{V}}(\tau, f_d) = \int_{-\infty}^{+\infty} z_{p\mathbf{U}}(t) z_{q\mathbf{V}}^*(t-\tau) e^{j2\pi f_d t} \mathrm{d}t.$$

By selecting the radar parameters carefully, both range and Doppler aliasing are avoidable, and intrapulse Doppler effect is negligible. The central lobe of $\chi_{p\mathbf{U},q\mathbf{V}}(\tau, f_d)$ depends on the discrete-time composite cross-ambiguity function (CCAF) [15], [16]:

$$\begin{aligned} \chi_{p\mathbf{U},q\mathbf{V}}(k, \theta) = &\frac{1}{D} \left(\sum_{n=0}^{N-1} v_n e^{jn\theta} \right) \left(\sum_{d=0}^{D-1} C_{\mathbf{x}_{p,d},\mathbf{x}_{q,d}}[k] \right) \\ &+ \frac{1}{D} \sum_{r=1}^{D-1} S_{\mathbf{U},\mathbf{V},r}(\theta) \Delta_{p,q,r}, \end{aligned} \tag{1}$$

where $\tau = kT_c$, and $\theta = 2\pi f_d T$ denotes normalized Doppler shift in radians,

$$S_{\mathbf{U},\mathbf{V},r}(\theta) = \sum_{n=0}^{N-1} \omega^{ru_n} v_n e^{jn\theta}, \tag{2}$$

and

$$\Delta_{p,q,r} = \sum_{d=0}^{D-1} \omega^{-rd} C_{\mathbf{x}_{p,d},\mathbf{x}_{q,d}}[k].$$

According to the definition of CCC, we have $\sum_{d=0}^{D-1} C_{x_{p,d},x_{q,d}}[k] = 0$. The second part of (1) determines the range sidelobes around the Doppler shift θ because of $\Delta_{p,q,r}$. Thus, if the key term (2) is approximately zero in a specified Doppler interval, then the range sidelobes of CCAF are suppressed in the specified Doppler interval.

When $p = q$, (1) is called the discrete-time composite auto-ambiguity function (CAAF) and can be analysed similarly.

Therefore, we wish to design the order of the transmitted pulses $\mathbf{U}$ and the weight sequence of the received pulses $\mathbf{V}$ such that $S_{\mathbf{U},\mathbf{V},r}(\theta)$ is almost to zero in a specified Doppler interval for $r = 1, 2, \cdots, D-1$.

To make analysing the key term (2) much easier, we choose M different Doppler shifts as $\theta_m \in [0, D_I], m = 0, 1, \cdots, M-1$. Then a Doppler Vandermonde matrix $\mathbf{B}$ [18] is constructed as follows

$$\mathbf{B} = \begin{bmatrix} e^{j0\theta_0} & e^{j1\theta_0} & \cdots & e^{j(N-1)\theta_0} \\ e^{j0\theta_1} & e^{j1\theta_1} & \cdots & e^{j(N-1)\theta_1} \\ \vdots & \vdots & \vdots & \vdots \\ e^{j0\theta_{M-1}} & e^{j1\theta_{M-1}} & \cdots & e^{j(N-1)\theta_{M-1}} \end{bmatrix}, \tag{3}$$

where $\{\theta_0, \theta_1, \cdots, \theta_{M-1}\} \subset [0, D_I]$, and D_I is a positive real number.

In order to eliminate range sidelobes within the specified Doppler area, we force (2) to zero at all discrete Doppler shifts $\{\theta_0, \theta_1, \cdots, \theta_{M-1}\}$, i.e.,

$$\mathbf{B} \begin{bmatrix} \omega^{u_0} v_0 & \omega^{2u_0} v_0 & \cdots & \omega^{(D-1)u_0} v_0 \\ \omega^{u_1} v_1 & \omega^{2u_1} v_1 & \cdots & \omega^{(D-1)u_1} v_1 \\ \vdots & \vdots & \vdots & \vdots \\ \omega^{u_{N-1}} v_{N-1} & \omega^{2u_{N-1}} v_{N-1} & \cdots & \omega^{(D-1)u_{N-1}} v_{N-1} \end{bmatrix} = \mathbf{O}. \tag{4}$$

In fact, (4) can be converted to

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{BA} \\ \vdots \\ \mathbf{BA}^{D-2} \end{bmatrix} \mathbf{z} = \mathbf{0}, \tag{5}$$

where $\mathbf{z} = [z_0, z_1, \cdots, z_{N-1}]^T$, $z_n = w^{u_n} v_n$ for $n = 0, 1, \cdots, N-1$, and $\mathbf{A} = \mathrm{Diag}(\omega^{u_0}, \omega^{u_1}, \cdots, \omega^{u_{N-1}})$. Obviously that (5) has nontrivial solutions if $M \le \lfloor \frac{N-1}{D-1} \rfloor$.

Remark 1 ([18]). *In the specified Doppler interval* $[0, D_I]$, *we choose the discrete Doppler shifts as* $\theta_m = mD_I/(M-1)$ *for* $m = 0, 1, \cdots, M-1$. *If* $S_{\mathbf{U},\mathbf{V},r}(\theta_m) = 0$, *then the range sidelobes of* $\chi_{p\mathbf{U},q\mathbf{V}}(k, \theta)$ *can be annihilated for all* $\theta \in [0, D_I]$, *i.e.,* $\chi_{p\mathbf{U},q\mathbf{V}}(k, \theta)$ *tends to* 0.

Since it is difficult to solve $\mathbf{U}$ and $\mathbf{V}$ simultaneously in (5), we fix $\mathbf{U}$ and then solve $\mathbf{V}$. Choose $\mathbf{U}$ as an alternating sequence of length-N (N is an integer multiple of D), i.e., $u_0 = 0, u_1 = 1, \cdots, u_{D-1} = D-1, u_D = 0, \cdots, u_{N-1} = D-1$. Hence, $\mathbf{A}$ is determined. Noted that $\mathbf{U}$ can be arbitrarily chosen. Without loss of generality, a case of selecting $\mathbf{U}$ is given in this paper.

Let $\mathbf{G} = (\mathbf{B}^T, (\mathbf{BA})^T, \cdots, (\mathbf{BA}^{D-2})^T)^T$. After solving $\mathbf{Gz} = \mathbf{0}$, we find the null space of $\mathbf{G}$. Suppose that $\hat{\mathbf{z}} \in \mathrm{Null}(\mathbf{G})$, where $\hat{\mathbf{z}} = [\hat{z}_0, \hat{z}_1, \hat{z}_2, \cdots, \hat{z}_{N-1}]^T$. Then we can obtain $\mathbf{V} = \{v_n\}_{n=0}^{N-1}$ as $v_n = \hat{z}_n / w^{u_n}$.

The proposed generalized null space method for range sidelobe suppression can be summarized as the following steps:

1. Input N, D_I, and $\theta_m = mD_I/(M-1)$.
2. Generate $\mathbf{B}$ as shown in (3).
3. Choose $\mathbf{U} = \{u_n\}_{n=0}^{N-1}$ as an alternating sequence of length-N, i.e., $\mathbf{A}$ is determined.
4. Select a solution $\hat{\mathbf{z}}$ from Null($\mathbf{G}$).
5. Obtain $\mathbf{V} = \{v_n\}_{n=0}^{N-1}$ as $v_n = \hat{z}_n / w^{u_n}$.

Remark 2. *When the transmit antennas number is $D = 2$, the method proposed in this paper is also applicable to fully polarimetric radar. In other words, the proposed method can take the waveform transmission scheme proposed in [18] for fully polarimetric radar as a special case.*

The SNR at the receiver output [13]–[15] is given by

$$\mathrm{SNR} = \frac{L\sigma_b^2}{N_0} \frac{\|\mathbf{V}\|_1^2}{\|\mathbf{V}\|_2^2},$$

where σ_b^2 is the power of the target and N_0 is the power of the white Gaussian noise. The SNR can be maximized by maximizing $\|\mathbf{V}\|_1^2/\|\mathbf{V}\|_2^2$ under the Doppler resilience constraint:

$$\mathbf{Gz} = \mathbf{0},$$

where $\mathbf{z} = [z_0, z_1, z_2, \cdots, z_{N-1}]^T$, and $z_n = w^{u_n} v_n$. In other words, the optimization problem is proposed as follows

$$\begin{aligned} \max_{\mathbf{U},\mathbf{V}} \quad & \frac{\|\mathbf{V}\|_1^2}{\|\mathbf{V}\|_2^2} \\ \text{s.t.} \quad & \mathbf{Gz} = \mathbf{0}. \end{aligned} \tag{6}$$

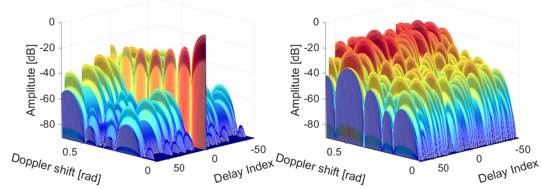
The optimization problem (6) can be converted to a unconstrained optimization problem [18] (Due to the page limitation, we omitted the detailed steps):

$$\min_{\boldsymbol{\lambda}} \frac{\|\mathbf{P}\boldsymbol{\lambda}\|_2^2}{\|\mathbf{P}\boldsymbol{\lambda}\|_1^2}, \tag{7}$$

where $\mathbf{P} = [\mathbf{z}_1 \ \mathbf{z}_2 \ \cdots \ \mathbf{z}_h]$, $\{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_h\}$ is an orthonormal basis of Null($\mathbf{G}$), and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_h]^T$ is an arbitrary complex vector. In this paper, we adopt the unconstrained Nelder-Mead simplex method [19] to solve the unconstrained optimization problem (7).

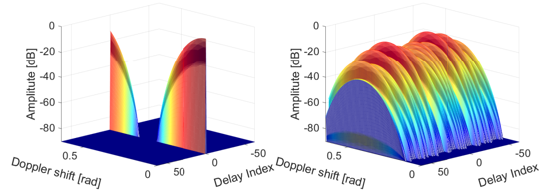
4. Numerical Result

To show the performance of range sidelobe suppression in the specified Doppler interval based on the proposed method in MIMO radar, the specified Doppler interval is set to $[0, 0.5]$ rad, the number of discrete Doppler shifts is $M = 21$, and $\theta_m = m/40$ rad, $m = 0, 1, \cdots, 20$. The radar parameters are set as follows: the carrier frequency is $f_c = 5$ GHz, the PRI is $T = 40\,\mu$s, and the signal bandwidth is $B = 20$ MHz. The range resolution is $R_r = 7.5$ m. To compare with the previous works, the number of pulses is set to $N = 64$ and the number of transmit antennas is set to $D = 4$. Besides, $\left\{\mathbf{x}_{p,d}\right\}_{0 \le p,d \le 3}$ is a (4,64)-CCC [20]. Therefore, $\mathbf{U}$ and $\mathbf{V}$ can be extracted based on the proposed method.



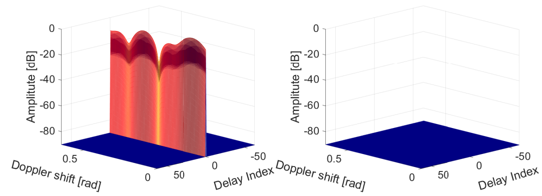
(a) CAAF $\left|\chi_{0\mathbf{U},0\mathbf{V}}(k,\theta)\right|$. (b) CCAF $\left|\chi_{0\mathbf{U},1\mathbf{V}}(k,\theta)\right|$.

Fig. 1 CAAF and CCAF based on the method in [10].



(a) CAAF $\left|\chi_{0\mathbf{U},0\mathbf{V}}(k,\theta)\right|$. (b) CCAF $\left|\chi_{0\mathbf{U},1\mathbf{V}}(k,\theta)\right|$.

Fig. 2 CAAF and CCAF based on the method in [15].



(a) CAAF $\left|\chi_{0\mathbf{U},0\mathbf{V}}(k,\theta)\right|$. (b) CCAF $\left|\chi_{0\mathbf{U},1\mathbf{V}}(k,\theta)\right|$.

Fig. 3 CAAF and CCAF based on the generalized null space method.

Figure 1 plots the CAAF and CCAF based on the method in [10], where the range sidelobes of CAAF and CCAF are eliminated (below −90 dB) inside the Doppler interval $[0, 0.05]$ rad. The corresponding velocity interval is $[0, 6]$ m/s according to normalized Doppler shift $\theta = 2\pi f_d T$ and velocity $v = cf_d/2f_c$. Here, $c = 3 \times 10^8$ m/s is the speed of light.

Figure 2 shows the CAAF and CCAF based on the method in [15]. The range sidelobes of CAAF are no more than −90 dB within the Doppler interval $[0, 0.5]$ rad (i.e., velocity interval $[0, 60]$ m/s) and the cleared region of CCAF is the Doppler interval $[0, 0.1]$ rad (i.e., velocity interval $[0, 12]$ m/s).

Figure 3 shows the CAAF and CCAF based on the proposed generalized null space method. This brings the range sidelobes below −90 dB inside the Doppler interval $[0, 0.5]$ rad (i.e., velocity interval $[0, 60]$ m/s). Therefore, within the Doppler interval $[0, 0.5]$ rad, the proposed waveforms can detect the moving target (see Fig. 3(a)) and have no inter-waveform interference (see Fig. 3(b)). As is shown in Fig. 3(a), the proposed method has a wider Doppler resilient interval than those shown in Fig. 1(a). Besides, the peak of CAAF in Fig. 3(a) exists, so that the moving target in the Doppler interval $[0, 0.5]$ rad can be detected. However, the CAAF in Fig. 2(a) lacks the peak in the Doppler interval of roughly $[0.3, 0.4]$ rad, which is problematic for detecting the moving target. It can be seen from Fig. 3(b) that the range sidelobes of CCAF are annihilated in a wider Doppler interval than those shown in Fig. 1(b) and Fig. 2(b). This means that the performance of suppressing inter-waveform interference is improved based on the proposed method.

5. Conclusion

In this paper, we proposed a new Doppler resilient transmit-receive design, which ensures the range sidelobes of CAAF and CCAF are eliminated within a specified Doppler interval. Besides, the SNR optimization problem is considered and solved by the unconstrained Nelder-Mead simplex method. The proposed method achieves range sidelobes suppression for a target with velocity in $[0, 60]$ m/s.

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