PAPER

A New Subsample Time Delay Estimation Algorithm for LFM-Based Detection

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SUMMARY This paper investigated a Subsample Time delay Estimation (STE) algorithm based on the amplitude of cross-correlation function to improve the estimation accuracy. In this paper, a rough time delay estimation is applied based on traditional cross correlator, and a fine estimation is achieved by approximating the sampled cross-correlation sequence to the amplitude of the theoretical cross-correlation function for linear frequency modulation (LFM) signal. Simulation results show that the proposed algorithm outperforms existing methods and can effectively improve time delay estimation accuracy with the complexity comparable to the traditional cross-correlation method. The theoretical Cramér–Rao Bound (CRB) is derived, and simulations demonstrate that the performance of STE can approach the boundary. Eventually, four important parameters discussed in the simulation to explore the impact on Mean Squared Error (MSE).

key words: *subsample time delay, cross-correlation, LFM*

1. Introduction

It is significant for the field of nondestructive testing to estimate the time delay, phase and amplitude of the ultrasonic echo signal. Particularly, as one of the core parameter estimation technologies, time delay estimation arises in many fields, including signal detection, target classification, wireless communication and transportation system for location, biomedicine ultrasound and precision measurement. Therefor a more accurate time delay estimation algorithm with low computational cost is inclusively required to complete these operations.

The time delay estimation algorithm based on basic correlation has a wide range of applications due to its simplicity in principle, low computational cost and the unrestricting of signal compared to other algorithms. However, the correlation method has a poor performance in the case of low Signal to Noise Ratio (SNR) and requires independence between signals and noise. Then the generalized cross-correlation method with different weight functions as prefilters is proposed in [1]. Qualitatively, the role of the prefilters is to accentuate the signal passed to the correlator and to suppress the noise power simultaneously. Many different weight functions were proposed to preprocess the signal, which were confirmed to improve estimation accuracy effectively in low SNR [2]–[4].

Time delay in the time domain produces linear phase change in the frequency domain. Therefore, a class of estimators perform time delay estimation procedures based on phase spectrum, which called “phase correlators”. In Phase-Only Correlator (POC) [5], the Fourier spectrum of both the received echo signal and the local template are first normalized by their respective spectral amplitudes and then multiplied. The principle of those estimators is to multiply the Fourier spectrum of the echo signal and the transmitted signal, and then perform inverse Fourier transform to obtain the phase to estimate the time delay [6]. Because the Fourier spectrum of both transmitted and echo signals are flattened, the POC output correlation peak much narrower than that of the conventional correlator, which is impulse-like. Furthermore, many improved phase correlators featured by different weight functions in frequency domain are proposed to suppress noise under different noise environment [7]–[12]. In order to suppress the amplified noise, the kurtosis-based strategy is proposed for resolving overlapping pulses in multiple echo cases [13].

The estimation algorithms mentioned above are with no restrictions about the form of detection signals. When actual application system uses a specific signal form, the estimation accuracy should be further improved, such as LFM signal. The LFM has a wide application in ultrasound detection fields because of its high doppler tolerance and superiority detection abilities in reverberation comparing to the traditional pulse signal [14]. LFM exhibits energy gathering at the optimal fractional order in the fractional Fourier domain and has been proven to have better time delay estimation accuracy than the traditional pulse compression [15], [16].

The matched filtering which is now widely used in ultrasonic detection applications is essentially based on cross-correlation. The unit impulse response of the matched filter is the time reversal of the transmitted signal. Therefore, its output is actually a correlation function between the transmitted signal and the received signal. However, in the practical discrete digital application systems, peak searching of the discrete correlation function certainly introduces discretized time delay estimation error, which leads to limited time de-

Manuscript received March 2, 2022.
Manuscript revised July 21, 2022.
Manuscript publicized September 8, 2022.

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DOI: 10.1587/transfun.2022EAP1025

lay estimation accuracy given an fixed sampling rate. A better time delay estimation performance can be obtained by increasing the sampling rate ideally, which requires higher technology in hardware equipment. In order to obtain a continuous time delay estimation which has an estimate error smaller than the digital sampling interval, some algorithms proposed about interpolate discrete signals or interpolate discrete correlation series. In general, interpolation functions such as pattern matching functions [17], sinc convolution method [18] and parabolic function [19], [20] are used to approximate the peak of correlation function, but those interpolation functions are not the exactly correlation functions of LFM in theory.

In this paper, we propose a STE algorithm for LFM signal which achieves an accuracy smaller than the sampling interval. The theoretical expression of cross-correlation function for LFM is deduced. The simulation results show that the proposed algorithm improves the delay performance significantly when the algorithm complexity is equivalent to the basic related algorithm. The characteristics of the STE algorithm can be used in applications such as defect location and precision measurement.

This paper is organized as follows: in Sect. 2, the problem is stated. Section 3 shows the derivation of the Cramér–Rao Bound of time delay estimation, and gives its specific expression. The proposed estimator is presented in Sect. 4. Then the simulation results are stated to prove that the MSE of the proposed algorithm can reach the CRB in Sect. 5, whereas Sect. 6 gives the conclusions.

2. Problem Statement

In this paper, we focus on a classic ultrasonic detection model, and its block diagram is shown in Fig. 1. The transmitted discretely sampled LFM signal is presented as follows:

$$x(n) = Ae^{j2\pi f_0 (n/F_s) + j\pi\mu(n/F_s)^2}, n = 0, \cdots, N-1 \quad (1)$$

where A is the amplitude of the sending signal, f_0 is the initial frequency, f_1 is the end frequency, F_s is the sampling rate, N is the number of signal samples in time period of T, μ is the chirp rate and $\mu = \frac{f_1 - f_0}{T} = \frac{B}{T} = \frac{BF_s}{N}$. After digital-to-analog conversion, the corresponding continuous signal $x(t)$ is transmitted by ultrasonic transducer.

The received signal $y(t)$ scattered by a point is digitized and can be presented as:

$$\begin{aligned} y(n) &= x(n - t_c F_s) \\ &= A' e^{j2\pi f_0 (n/F_s - t_c) + j\pi\mu(n/F_s - t_c)^2}, \\ &\quad n = 0, \cdots, N-1 \end{aligned} \quad (2)$$

where A' is the amplitude of the echo signal, t_c is the continuous time-delay. Therefore $y(n)$ can be regarded as a delayed copy of $x(n)$. The traditional method directly uses matched filtering to process the received signal. Perform cross-correlation operations on the transmitted and received signals to obtain the cross-correlation function as follows:

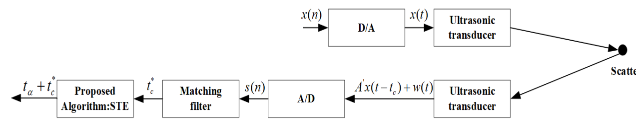

Fig. 1 Ultrasonic signal detection system.

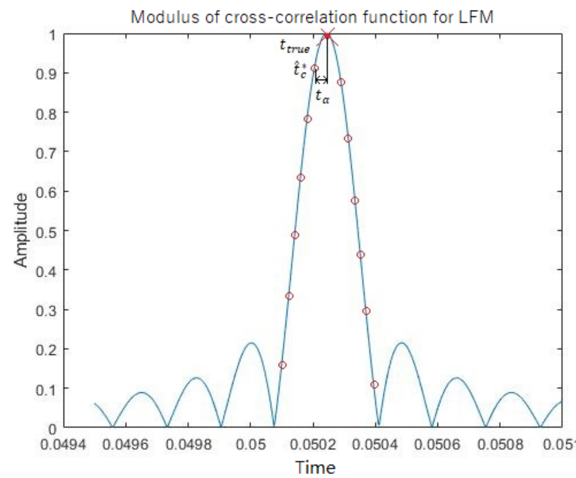

Fig. 2 Schematic diagram of time delay estimation based on the correlation function of the transceiver signal.

$$\begin{aligned} R(n, n_c) &= \sum_{m=0}^{N-1} x(m-n) y^*(m) \\ &= A'^2 e^{j2\pi \frac{f_0(n_c - n)}{F_s} - j\pi\mu \frac{(n_c^2 - n^2) + (N-1)(n_c - n)}{F_s^2}} \\ &\quad \frac{\sin(N\pi\mu(n - n_c)/F_s^2)}{\sin(\pi\mu(n - n_c)/F_s^2)} \end{aligned} \quad (3)$$

where $n_c = t_c F_s$. Therefor, time delay can be estimated by searching the abscissa of peak n_c^* corresponding to Eq. (2) [8] and presented as:

$$\hat{t}_c^* = \arg\{\max(R(n, n_c))\} \cdot T_s = n_c^* \cdot T_s \quad (4)$$

where $T_s = 1/F_s$ is the sampling interval. It can be seen from Eq. (4), $\arg\{\max[R(n, n_c)]\}$ is a discrete coordinate value, hence the time delay estimation $\hat{\boldsymbol{t}}_c^*$ can only be an integer multiple of the sampling interval. Fig. 2 clearly shows that the subsample time delay t_α which is smaller than the sampling interval is ignored. Meanwhile, the peak of the curve in Fig. 2 shows the true value (the location of the cross line) of time delay t_{true}, but it is hard to estimate with the precision of the traditional method. On this condition, a novel algorithm named STE is proposed in this paper to improve the performance of time delay estimation given by Eq. (4). The characteristics of the STE algorithm can be used in applications such as defect location and precision measurement.

$$J = \frac{2}{\sigma^2}\begin{bmatrix} N & 0 & 0 \\ 0 & A'^2\left(\omega_0^2 N - 2\omega_0\mu t_c N + 2\omega_0\mu T_s P + \mu^2 T_s^2 Q - 2\mu^2 T_s t_c P + \mu^2 t_c^2 N\right) & -A'^2\left(\omega_0 N + \mu T_s P - \mu t_c N\right) \\ 0 & -A'^2\left(\omega_0 N + \mu T_s P - \mu t_c N\right) & A'^2 N \end{bmatrix} \tag{10}$$

3. Derivation of CRB for Time Delay Estimation

As a criterion to evaluate the performance of parameter estimation algorithm, CRB reflects the minimum reachable error of parameter estimation. In this part, we firstly derive the CRB for chirp signals with unknown time delay, phase and amplitude, measured in additive Gaussian white noise.

The chirp signal is given in Eq. (2). Therefor the continuous echo signal with additive noise is

$$\begin{aligned} s(n) &= y(n) + w(n) \\ &= A' e^{j2\pi f_0 (n/F_s - t_c) + j\pi\mu(n/F_s - t_c)^2 + \varphi'} + w(n), \qquad (5) \\ &\quad n = 0, \cdots, N-1. \end{aligned}$$

The white noise $w(n)$ is circularly symmetric complex-valued Gaussian distributed with zero mean and variance σ^2. Then we have

$$u_n = A' \cos\left(2\pi f_0\left(\frac{n}{F_s} - t_c\right) + \pi\mu\left(\frac{n}{F_s} - t_c\right)^2 + \varphi'\right) \tag{6}$$

$$v_n = A' \sin\left(2\pi f_0\left(\frac{n}{F_s} - t_c\right) + \pi u\left(\frac{n}{F_s} - t_c\right)^2 + \varphi'\right) \tag{7}$$

where u_n and v_n are respectively the real part and the imaginary part of $y(n)$. Then logarithm of the joint probability density function of $s[n]$ is given by

$$\ln f(s \mid \theta) = -N \ln\left(\pi\sigma^2\right) - \frac{1}{\sigma^2}\sum_{n=0}^{N-1}\left((p_n - u_n)^2 + (q_n - v_n)^2\right) \tag{8}$$

where θ is the vector of parameters $[A', t_c, \varphi']$, p_n and q_n represent the real and imaginary part of $s[n]$, respectively.

Fisher information matrix can be expressed as follows:

$$\begin{aligned} J_{ij} &= -E\left(\frac{\partial^2}{\partial\theta_i \partial\theta_j} \ln f(s, \theta)\right) \\ &= \frac{1}{\sigma^2}\sum_{n=0}^{N-1}\left(\frac{\partial u_n}{\partial\theta_i}\frac{\partial u_n}{\partial\theta_j} + \frac{\partial v_n}{\partial\theta_i}\frac{\partial v_n}{\partial\theta_j}\right) \end{aligned} \tag{9}$$

Substituting Eq. (6) and Eq. (7) into Eq. (9) to calculate the elements of the Fisher information matrix, we can obtain the Fisher information matrix as Eq. (10). In Eq. (10), P, Q, ω_0 are defined by

$$P = \sum_{n=0}^{N-1} n = \frac{N(N-1)}{2} \tag{11}$$

$$Q = \sum_{n=0}^{N-1} n^2 = \frac{N(N-1)(2N-1)}{6} \tag{12}$$

$$\omega_0 = 2\pi f_0 \tag{13}$$

Then, it is not hard to obtain the inverse matrix, and the second element on the main diagonal of the inverse matrix is the CRB expression of time delay. The CRB could be rewritten as:

$$CRB = \frac{\sigma^2}{2\pi^2 A'^2}\frac{3}{\mu^2 T_s^2 N(N-1)(N+1)} \tag{14}$$

According to $SNR = \frac{A'^2}{\sigma^2}$, the CRB of chirp signal for the estimation of t_c can be written as

$$CRB = \frac{N}{2SNR}\frac{3}{\pi^2 B^2 (N-1)(N+1)} \tag{15}$$

In the Sect. 5, we demonstrated that the cross-correlation method has a performance gap comparing with the theoretical boundary given by Eq. (15), while the proposed algorithm in this paper can approach the CRB.

4. Algorithm Description

The square modulus of the cross-correlation in Eq. (3) can be represented as follows:

$$\begin{aligned} T(n, t_c) &= |R(n, n_c)|^2 \\ &= A'^4 \frac{\sin^2\left(N\pi\mu\left(nT_s - t_c\right)/F_s\right)}{\sin^2\left(\pi\mu\left(nT_s - t_c\right)/F_s\right)} = T(n, t_c), \\ &\quad n \in [0, \cdots, N-1] \end{aligned} \tag{16}$$

Given the sample sequence of echo signal, we can easily calculate the square modulus of the sampled correlation function of transmitted and echo signals:

$$\begin{aligned} Q(n, t_c) &= \left|\sum_{m=0}^{N-l} x(m-n) y^*(m)\right| \\ &= \left|\sum_{m=0}^{N-l} x(m-n) x^*\left(m - \frac{t_c}{T_s}\right)\right|, \\ &\quad n \in [0, \cdots, N-1] \end{aligned} \tag{17}$$

Approximating the theoretical correlation expression given by Eq. (16) to the sampled correlation sequence, which is calculated according to Eq. (17), we can achieve the following cost function:

$$T\left(n,t_c^*\right)=\begin{cases} A'^4\dfrac{\sin^2\left(N\pi\mu\left(nT_s-t_c^*\right)/F_s\right)}{\sin^2\left(\pi\mu\left(nT_s-t_c^*\right)/F_s\right)} & n\neq\dfrac{t_c^*}{T_s} \\ A'^4N^2 & n=\dfrac{t_c^*}{T_s}\end{cases} \tag{20}$$

$$T'(n,t_c^*)=\frac{2A'^4\pi\mu}{F_s}\cdot\frac{\Gamma}{2\sin(\pi\mu(nT_s-t_c^*)/F_s)-\sin 3(\pi\mu(nT_s-t_c^*)/F_s)+\sin(\pi\mu(nT_s-t_c^*)/F_s)} \tag{21}$$

$$\begin{aligned}\Gamma=&\;2\cos(\pi\mu(nT_s-t_c^*)/F_s)-\cos(2N+1)(\pi\mu(nT_s-t_c^*)/F_s)-\cos(2N-1)\pi\mu(nT_s-t_c^*)/F_s\\ &-N\cos(2N-1)(\pi\mu(nT_s-t_c^*)/F_s)+N\cos(2N+1)(\pi\mu(nT_s-t_c^*)/F_s)\end{aligned}$$

$$t_\alpha=\frac{\sum_{n\in C}Q^4\left(n,t_c\right)\sum_{n\in C}T\left(n,t_c^*\right)T'\left(n,t_c^*\right)-\sum_{n\in C}Q^2\left(n,t_c\right)T'\left(n,t_c^*\right)\sum_{n\in C}Q_1^2\left(n,t_c\right)T\left(n,t_c^*\right)}{\left(\sum_{n\in C}Q^2\left(n,t_c\right)T'\left(n,t_c^*\right)\right)^2-\sum_{n\in C}Q_1^4\left(n,t_c\right)\sum_{n\in C}\left(T'\left(n,t_c^*\right)\right)^2} \tag{24}$$

$$J=\sum_{n=-\Delta}^{\Delta}\left(T\left(n,t_c\right)-aQ^2\left(n,t_c\right)\right)^2 \tag{18}$$

where $T\left(n,t_c\right)$ is a Sinc function and its energy is concentrated in the main lobe. Then we select Δ in the main lobe for the square approximation given by Eq. (18), and a is an amplitude adjustment parameter on the cost function. In Eq. (18), $t_c = t_c^* + t_\alpha$, where $t_c^* = n_c^* T_s$ and $t_\alpha \in [-0.5T_s, 0.5T_s]$ stands for the subsample time delay, which is a small value. Mathematically, we can perform Taylor expansion of $T\left(n,t_c\right)$ at t_c^*, the first-order Taylor expansion can be presented as:

$$\begin{aligned}T\left(n,t_c\right)&=T\left(n,t_c^*\right)+T'\left(n,t_c^*\right)\left(t_c-t_c^*\right)\\&=T\left(n,t_c^*\right)+t_\alpha T'\left(n,t_c^*\right)\end{aligned} \tag{19}$$

where $T\left(n,t_c^*\right)$ and $T'\left(n,t_c^*\right)$ are given by Eq. (20) and Eq. (21) respectively.

We substitute Eq. (19) into Eq. (18) to achieve:

$$J\triangleq\sum_{n=n_c^*-\Delta}^{n=n_c^*+\Delta}\left(T(n,t_c^*)+t_\alpha\cdot T'(n,t_c^*)-aQ^2(n,t_c)\right)^2 \tag{22}$$

To minimize the value of J given by Eq. (22), we only have to resolve the partial derivatives in Eq. (23):

$$\begin{cases}\dfrac{\partial J}{\partial t_\alpha}=0\\ \dfrac{\partial J}{\partial a}=0\end{cases} \tag{23}$$

Therefore, it is easy to resolve the subsample time delay, which is presented by Eq. (24) and in Eq. (24) C represents $\left(n_c^*-\Delta, n_c^*+\Delta\right)$.

5. Simulations

In this section, the MSE of time delay estimation is calculated through 10000 separate simulations by MATLAB to compare the accuracy of the proposed algorithm in the case of white Gaussian noise. In the experiments, we use the default parameters as: $A' = 0.45$, $f_0 = 100$ KHz, $f_1 = 200$ KHz, $F_s = 1$ MHz $\left(T_s = 1\times10^{-6}\text{ s}\right)$, $T = 1\times10^{-3}$ s, $N = T/T_s = 1000$ and $\mu = \left(f_1 - f_0\right)/T = 1\times10^8$. In the following simulations, we demonstrate how the following parameters: subsample delay t_α, sampling rate F_s, least-squares approximation point range Δ and sample number N impact on the performance of the proposed algorithm.

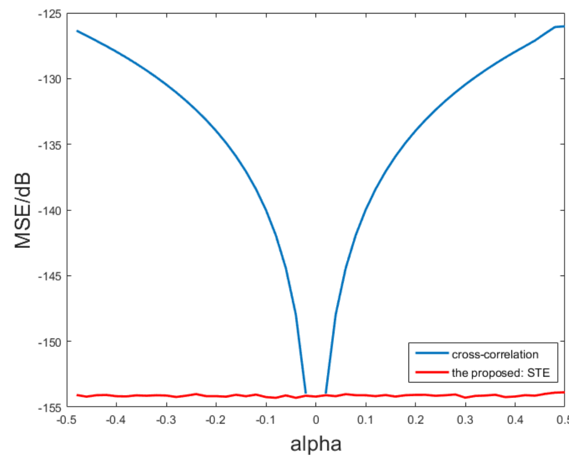

Fig. 3 MSE of total time delay of the proposed algorithm and the cross-correlation with α.

Firstly, we discuss the impact of subsample time delay t_α. Given $t_\alpha = \alpha T_s$. When α changes in the interval $[-0.5, 0.5]$, the MSE of the basic correlation and the novel algorithm is shown in Fig. 2. As can be observed from Fig. 3, the proposed algorithm is more accurate than the basic correlation method. When α changes within a certain range, MSE curve of the proposed algorithm almost unchanged overall. Basic correlation method can only reach the minimum MSE at a specific value of $\alpha = 0$ which causes a jump in the MSE curve, since the traditional method is an integer multiple of T_s. However, the proposed algorithm estimates t_α, the MSE curve of STE appears not related to α.

Next, Fig. 4 compares the MSE of the basic correlation and the proposed algorithm, when sampling rate F_s changes from 1 MHz to 20 MHz, α randomly generated, SNR = 10 dB. As expected, the novel algorithm can achieve a bet-

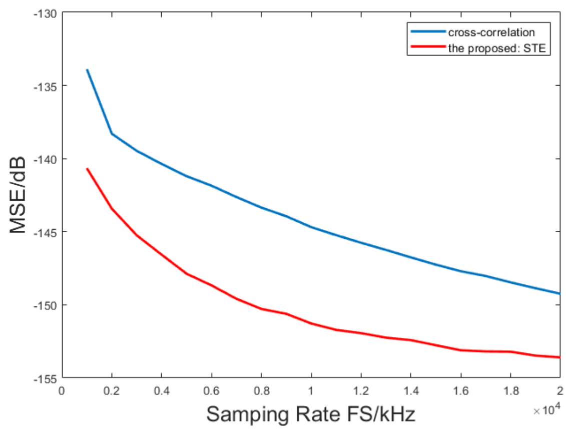

Fig. 4 MSE of total time delay of the proposed algorithm and the cross-correlation with F_s.

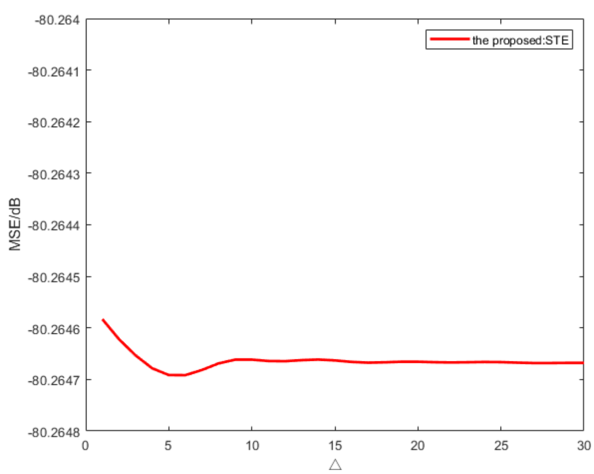

Fig. 5 MSE of total time delay of the proposed algorithm with Δ.

ter estimation accuracy than the basic correlation method overall. Especially, STE can obtain a good performance at low sampling rate. With the sampling rate increases, the MSE decline rate of the proposed method is also slower than the cross-correlation algorithm, which means the proposed method can effectively reduce the influence of F_s. The results in Fig. 4 shows that the proposed method can achieve the same error accuracy as the traditional method at a lower sampling rate. Therefore, the STE algorithm can reduce the computational complexity and the system cost of analog-to-digital conversion process.

Figure 5 shows the impact on the performance of the algorithm with square approximation point range Δ. From Fig. 5, we can know that when the estimated number of points is selected as 3, the MSE curve drops to the lowest point. Meanwhile, when $\Delta < 3$, the accuracy of the estimated results will be reduced. When $\Delta > 3$, noise has more impact on the accuracy of the approximation process than signals. Hence, we can come to the conclusion that the points out of the $\Delta = 3$ has little effect on the time delay estimation accuracy of the square approximation. Therefore, we suggest select $\Delta = 3$ for calculation as the best choice.

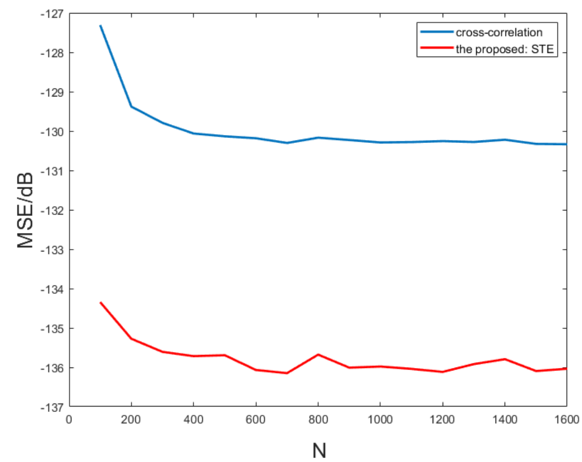

Fig. 6 MSE of total time delay of the proposed algorithm and the cross-correlation with N.

Figure 6 examines the impact of the total samples number. When the total samples number N changes from 100 to 1600, the MSE of the two algorithms decreased slowly. The simulation result shows that N has little effect on the performance of the algorithm.

We compare the proposed algorithm with some common algorithms as shown in Fig. 7. In Fig. 7(a), the cross-correlation method, the generalized cross-correlation method with four weight functions [1], [2] and the quadratic-correlation method [21] are represented as comparison items. As can be seen from the Fig. 7(a), when $SNR > -20\,dB$, these contrast algorithms stay constant because the subsample time delay t_α is ignored. So that improving the SNB has no effect on the estimation accuracy. As can be noted from this figure, the MSE of the proposed STE has a linear downward trend and even can reached CRB. According to Fig. 7(b), we can conclude that the proposed STE algorithm has a better time delay estimation accuracy than the parabolic, cosine interpolation and Sinc convolution method in [17]–[20]. That's because Eq. (3) derived in this paper is exactly the interpolation function of LFM in theory.

To show how the proposed estimator works in actual applications, we carry out a precision measurement simulation for a specific metal detection. For metal detection simulation, we set the parameters as: $A' = 0.45$, $f_0 = 3\,\text{MHz}$, $f_1 = 4\,\text{MHz}$, $F_s = 10\,\text{MHz}$, $T = 25 \times 10^{-3}\,\text{s}$. Suppose that the ultrasonic signal transmit in metal with $v_m = 5000\,m/s$, and the delay time is $t_c = 670.4T_s$, therefore the metal detection range is $t_c v_m = 0.337\,m$. In the simulation results, we can see that given a fixed sample rate, the error of distance by the proposed estimator can achieve 10^{-7} order of magnitude, while the error of distance by the cross correlation estimator keeps at 10^{-4} order of magnitude even when the SNR increases.

6. Complexity Analysis

It can be proved by the simulation results that the proposed algorithm has a better time delay estimation accuracy than the

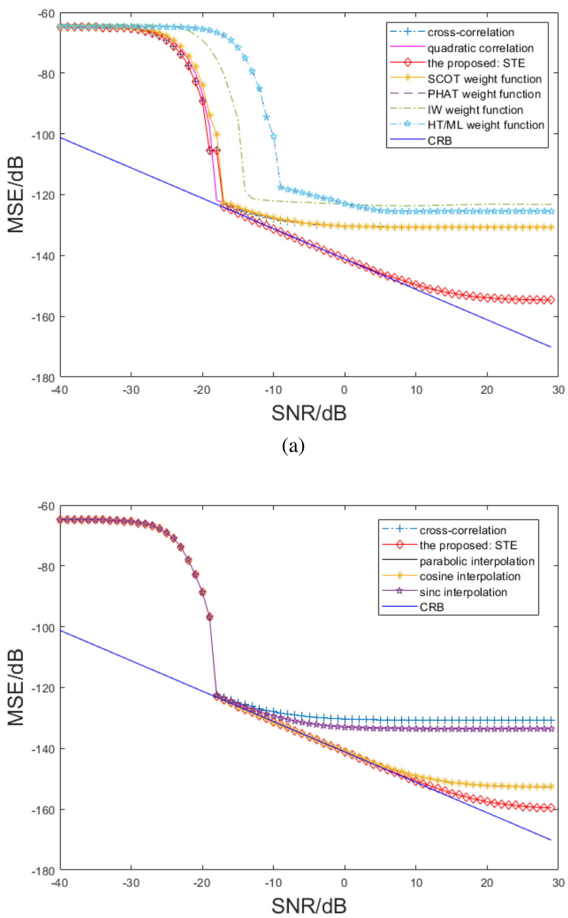

Fig. 7 Comparison of different algorithms: (a) MSE of time delay of basic correlation, quadratic correlation, generalized cross-correlation with weight function SCOT/PHAT/ML, the proposed STE and the Cramer-Rao Bound with SNR. (b) MSE of time delay of basic correlation, parabolic interpolation method, cosine interpolation, sinc function convolution, the proposed STE and the Cramer-Rao Bound with SNR.

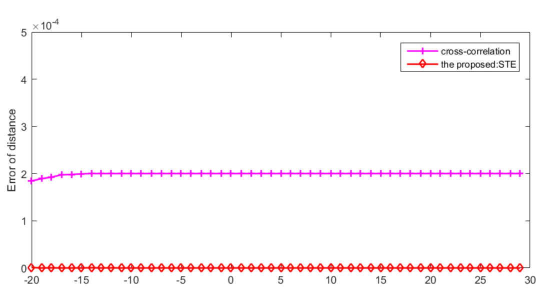

Fig. 8 Error of the distance in metal detection with fixed sample rate of $F_s = 10$ MHz.

traditional algorithms. In terms of algorithm complexity, we compare the proposed STE with some common algorithms and the results are given in Table 1. We select $\Delta = 3$ of the LFM cross-correlation function to make an estimation. Hence, the proposed STE which brings noticeable improvement in time delay estimation accuracy and effectively reduce the influence of sampling rate while has little complexity increment compared to the basic correlation method.

Table 1 Complexity comparison of the basic cross-correlation, quadratic correlation, generalized cross-correlation with weight ML/PHAT/SCOT and STE.

Name	Algorithm complexity
Basic cross-correlation	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + N\right)$
Generalized cross-correlation (SCOT)	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + 6N\right)$
Generalized cross-correlation (PHAT)	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + 4N\right)$
Generalized cross-correlation (ML)	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + 8N\right)$
Quadratic correlation	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + 3N\right)$
The proposed: STE	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + N + 8\Delta + 4\right), \Delta = 3$
Parabolic interpolation	$\mathrm{O}\left(N^2 + \Delta^4 + 2\Delta^3 + 6\right)$
Cosine interpolation	$\mathrm{O}\left(\left(3N\log_2 N\right)/2 + N\right)$
Sinc function convolution	$\alpha\left(N^2 + \Delta^2 m\right)$

7. Conclusion

The traditional algorithms based on the cross-correlation function obtain the time delay estimation which is an integer multiple of the sampling interval, and the MSE can not decrease even when the SNR increases. In this paper, we derive the cross-correlation function between digitized transmitted and received signals. Simultaneously, we proposed the STE algorithm based on the least squares approximation of the cross-correlation function. In the simulation, we conduct 10,000 independent simulation experiments and represent the MSE curve.Simulation results show that the proposed algorithm outperforms a variety of common algorithm approaching the CRB. Therefore, we can conclude that the proposed algorithm which the complexity is comparable to the cross-correlation methods can effectively improve the accuracy of time delay estimation.

Acknowledgments

This work was supported by the Guangdong Basic and Applied Basic Research Foundation (2022A1515010167, 2020A1515010962, 2022A1515011604) and the Science, Technology Planning Project of Guangzhou (202002030251, 202102080352) and the National Natural Science Foundation of China (61871191).

References

[1] C. Knapp and G. Carter, “The generalized correlation method for estimation of time delay,” IEEE Trans. Acoust., Speech, Signal Process., vol.24, no.4, pp.320–327, Aug. 1976.
[2] M. Liang, L. Xi-Hai, Z. Wan-Gang, and L. Dai-Zhi, “The generalized cross-correlation method for time delay estimation of infrasound signal,” Fifth International Conf. Instrumentation and Measurement, Computer, Commun. and Control, pp.1320–1323, 2015.
[3] H. Dou, Q. Lei, J. Cheng, and W. Li, “Research on TDOA parameters estimation for satellite interference localization,” IEEE 11th International Conf. Signal Processing, pp.259–262, 2012.
[4] Q. Zhang and L. Zhang, “An improved delay algorithm based on generalized cross correlation,” IEEE 3rd Information Technol. and Mechatronics Engineering Conf., pp.395–399, 2017.
[5] A. Piersol, “Time delay estimation using phase data,” IEEE Trans. Acoust., Speech, Signal Process., vol.29, no.3, pp.471–477, June

1981.

[6] K.L. Kosbar and J.J. Zaninovich, "Periodic PN sequence delay estimation using phase spectrum data," Proc. GLOBECOM'93. IEEE Global Telecommunications Conf., vol.3, pp.1665–1669, 1993.

[7] J.E. Lindop, G.M. Treece, A.H. Gee, and R.W. Prager, "Phase-based ultrasonic deformation estimation," IEEE Trans. Ultrason., Ferroelectr., Freq. Control, vol.55, no.1, pp.94–111, Jan. 2008.

[8] S.W. Heo and H. Kim, "A novel power spectrum calculation method using phase-compensation and weighted averaging for the estimation of ultrasound attenuation," Ultrason., vol.50, no.6, 592–599, 2010.

[9] S.R. Ara, F. Mohsin, F. Alam, S. Rupa, S. Lee, M.K. Hasan, and R. Awwal, "Phase-based direct average strain estimation for elastography," IEEE Trans. Ultrason., Ferroelectr., Freq. Control, vol.60, no.11, pp.2266–2283, Nov. 2013.

[10] H. Fu and P.-Y. Kam, "Phase-based, time-domain estimation of the frequency and phase of a single sinusoid in AWGN — The role and applications of the additive observation phase noise model," IEEE Trans. Inf. Theory., vol.59, no.5, pp.3175–3188, May 2013.

[11] I. Djurović, P. Wang, M. Simeunović, and P.V. Orlik, "Parameter estimation of coupled polynomial phase and sinusoidal FM signals," Signal Processing, vol.149, pp.1–13, 2018.

[12] J. Wu, X. Tang, Z. Liu, L.I. Baiyu, and F. Wang, "Unbiased interference suppression method based on spectrum compensation," IEICE Trans. Commun., vol.E103-B, no.1, pp.52–59, Jan. 2020.

[13] Y. Zhou, Y.L. Guan, and C.L. Law, "Modified phase-only correlator with kurtosis-based amplified-noise suppression," IEEE Trans. Wireless Commun., vol.9, no.11, pp.3341–3345, Nov. 2010.

[14] Z. Lu, F. Ma, C. Yang, and M. Chang, "A novel method for estimating time of flight of ultrasonic echoes through short-time Fourier transforms," Ultrason., vol.103, 106104, 2020.

[15] H. Zhang, T. Shan, S. Liu, and R. Tao, "Optimized sparse fractional Fourier transform: Principle and performance analysis," Signal Processing, vol.174, 107646, 2020.

[16] C. Yang, Y. Yu, C. Wu, and G. Ning, "A fast time-delay estimator for linear frequency modulation signal based on frFt," IEEE 3rd Inform. Technol., Networking, Elect. Automation Control Conf., pp.2278–2281, 2019.

[17] F. Viola and W.F. Walker, "A spline-based algorithm for continuous time-delay estimation using sampled data," IEEE Trans. Ultrason., Ferroelectr., Freq. Control, vol.52, no.1, pp.80–93, Jan. 2005.

[18] S.V. Porshnev, D.V. Kusaykin, and M.A. Klevakin, "On features of reconstruction of finite-length discrete-time signal with nonzero constant component using sinc interpolation," International Multi-Conf. Eng., Computer and Inform. Sciences, pp.0167–0171, 2019.

[19] M.Z. Hussain, M. Irshad, M. Sarfraz, and N. Zafar, "Interpolation of discrete time signals using cubic spline function," 19th International Conf. Inform. Visualisation, pp.454–459, 2015.

[20] S.-H. Hong, L. Wang, and T.-K. Truong, "An improved approach to the cubic-spline interpolation," 25th IEEE International Conf. Image Processing, pp.1468–1472, 2018.

[21] W.-H. Li, S.-Q. Li, W.-W. Chen, and P.-J. Wang, "Design of a silicon waveguide admittance detecting circuit using quadratic cross-correlation method," 14th IEEE International Conf. Solid-State and Integrated Circuit Technol., pp.1–3, 2018.

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