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Authors	Nagato Ueda, Eiji Watanabe, Akinori NISHIHARA
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PAPER

Synthesis of 2-Channel IIR Paraunitary Filter Banks by Successive Extraction of 2-Port Lattice Sections

Nagato UEDA[†a)], **Eiji WATANABE**[††], ***Members***, ***and*** **Akinori NISHIHARA**[†], ***Fellow***

SUMMARY This paper proposes a synthesis method of 2-channel IIR paraunitary filter banks by successive extraction of 2-port lattice sections. When a power symmetry transfer function is given, a filter bank is realized as cascade of paraunitary 2-port lattice sections. The method can synthesize both odd- and even-order filters with Butterworth or elliptic characteristics. The number of multiplications per second can also be reduced.

key words: *IIR filters, paraunitary filter banks, extraction, power symmetry*

1. Introduction

Recently, a number of researchers have been studying subband coding by using filter banks for communication, compression, etc. Several methods for designing filter banks have been proposed so far [1], [10], [11].

In general, filter banks have sampling rate alteration. Therefore, output signals may have distortions such as ALD (aliasing distortion), AMD (amplitude distortion) and PHD (phase distortion). If we can eliminate all of these distortions, output signals are perfectly reconstructed. Since human hearing system is lower sensitive to PHD compared with AMD in certain applications such as speech coding [12], [13], it can be regarded that PHD is acceptable in these applications compared with AMD/ALD. Thus, in this paper, we assume that PHD can be disregarded.

As a structure of filter banks, paraunitary filter banks are well known. The merits of those structures are that they have low sensitivity with respect to their coefficients, and a synthesis bank is readily obtained by designing an analysis bank [1]. It is well known that odd-order IIR paraunitary filter banks are constructed by parallel of two real allpass filters [6], [7] and even-order paraunitary IIR filter banks are constructed by complex allpass filters [4]. However, the conventional paraunitary filter banks suffer from constraints in the order of their transfer functions. For odd-order cases, the order of numerator polynomial should be higher than that of denominator by one [1]. For even-order cases, the orders of numerator and denominator should be equal each other [1], [4]. Therefore, we cannot use these structures for other transfer functions to satisfy paraunitiary property.

In this paper, we propose a procedure to synthesize IIR paraunitary filter banks using real allpass filters from any power symmetric transfer functions. We assume that we are given this power symmetric transfer function ($H_0(z)$ in Fig. 1) to be synthesized as a paraunitary structure. At first, we propose a method for deriving a polyphase matrix from a given transfer function. From this matrix and the relation between low pass and high pass filters, we derive that a given transfer function should have power symmetric property so that it constitutes a paraunitary filter bank. When we have such a transfer function, we show that we can synthesize IIR paraunitary cascade connected structure by extracting 2-port lattice sections. We use a WDLF (wave digital lattice filter) as a fundamental section like [3]. Although [3] deals with scalar transfer functions, this paper extends [3] to filter banks. Then, we reconfirm the extraction method in [3]. In [3], when we determine a multiplier coefficient in a zeroth-order section, a value of characteristic function at pole before extraction was arbitrary. But, we confirm the formula to determine the multiplier coefficient in zeroth-order section is different depending on the value of characteristic function at pole. From this procedure, we show that we can synthesize the circuit of IIR paraunitary filter banks without caring the order of the transfer function. If a given transfer function is odd-order, synthesized structure is the same as the conventional structure, and we can also synthesize the even-order circuits with the same procedure. In these circuits, we can reduce the number of multiplication per unit time. In addition, it is shown that we can synthesize circuits even if the order of numerator polynomial is higher than that of denominator by 2 or more.

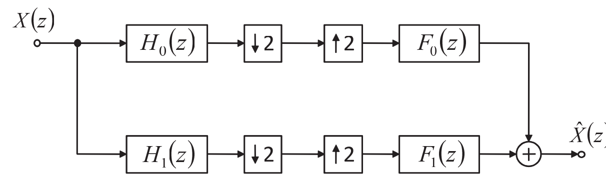

Fig. 1 2-channel filter bank.

2. 2-Channel Parauniray Filter Banks

2.1 Filter Banks

In general, filter banks are composed of filters, decimators and interpolators as shown in Fig. 1. In this filter bank, $X(z)$ and $\hat{X}(z)$ are z-transforms of the input and output signals, respectively. The reconstructed output signal $\hat{X}(z)$ is given

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[†]The authors are with the Department of Communications and Integrated Systems, Tokyo Institute of Technology, Tokyo, 152-8552 Japan.

[††]The author is with the Department of Electronic Information Systems, Shibaura Institute of Technology, Saitama-shi, 337-8570 Japan.

a) E-mail: ueda@nh.cradle.titech.ac.jp

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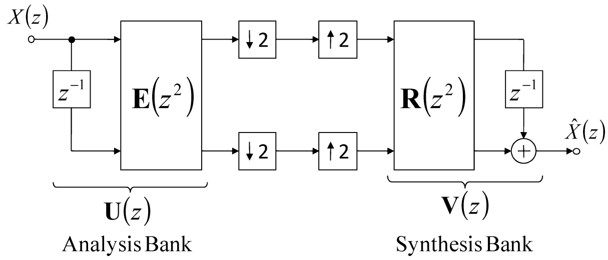

Fig. 2 Polyphase structure of 2-channel filter banks.

by

$$\hat{X}(z) = \frac{1}{2}\{H_0(z)F_0(z) + H_1(z)F_1(z)\}X(z) + \frac{1}{2}\{H_0(-z)F_0(z) + H_1(-z)F_1(z)\}X(-z). \quad (1)$$

If two conditions

$$\frac{1}{2}\{H_0(z)F_0(z) + H_1(z)F_1(z)\} = z^{-k} \quad (2)$$

and

$$\frac{1}{2}\{H_0(-z)F_0(z) + H_1(-z)F_1(z)\} = 0 \quad (3)$$

hold, the output signal $\hat{X}(z)$ is perfectly reconstructed. Now, under a condition where PHD is permitted like in speech applications, we introduce a synthesis method for IIR filter banks to eliminate ALD and AMD.

In Fig. 1, $H_0(z)$ and $H_1(z)$ are decomposed into polyphase components as

$$H_0(z) = E_{00}(z^2) + z^{-1}E_{01}(z^2), \quad (4)$$

$$H_1(z) = E_{10}(z^2) + z^{-1}E_{11}(z^2), \quad (5)$$

and we can get a polyphase matrix

$$\boldsymbol{E}(z^2) = \begin{bmatrix} E_{00}(z^2) & E_{01}(z^2) \\ E_{10}(z^2) & E_{11}(z^2) \end{bmatrix}. \quad (6)$$

Similarly, we can get a polyphase matrix $\boldsymbol{R}(z^2)$ from $F_0(z)$ and $F_1(z)$. Then, Fig. 1 is rewritten as Fig. 2. If there is a relation written as

$$\boldsymbol{R}(z^2)\boldsymbol{E}(z^2) = A(z^2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (7)$$

then, we can get

$$\frac{1}{2}\{H_0(z)F_0(z) + H_1(z)F_1(z)\} = z^{-1}A(z^2), \quad (8)$$

$$\frac{1}{2}\{H_0(-z)F_0(z) + H_1(-z)F_1(z)\} = 0. \quad (9)$$

If $A(z)$ is an allpass transfer function, (8) shows that AMD is eliminated, and (9) shows that ALD is eliminated. As one of solutions to derive (7), we consider the paraunitary structure.

2.2 2-Channel IIR Paraunitary Filter Banks

Let $\boldsymbol{P}(z)$ be a scattering matrix of a 2-port system shown in Fig. 3. In this figure, the input-output relation is written as

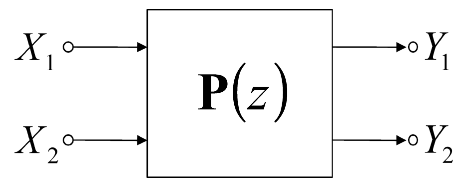

Fig. 3 Digital 2-port network.

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \boldsymbol{P}(z)\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (10)$$

$$= \begin{bmatrix} P_{00}(z) & P_{01}(z) \\ P_{10}(z) & P_{11}(z) \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \quad (11)$$

We define $\boldsymbol{P}_*(z)$ as

$$\boldsymbol{P}_*(z) = \boldsymbol{P}^t(z^{-1}). \quad (12)$$

When $\boldsymbol{P}(z)$ and $\boldsymbol{P}_*(z)$ satisfy

$$\boldsymbol{P}_*(z) \cdot \boldsymbol{P}(z) = \boldsymbol{I}, \quad (13)$$

we call $\boldsymbol{P}(z)$ a paraunitary matrix. If $\boldsymbol{E}(z^2)$ and $\boldsymbol{R}(z^2)$ in Fig. 2 have this property, we call this filter bank a paraunitary filter bank. In addition, if $\boldsymbol{E}(z^2)$ and $\boldsymbol{R}(z^2)$ are constructed by allpass filters, we can easily derive (7).

Now, let the given transfer function $H_0(z)$ be

$$H_0(z) = \frac{\sum_{i=0}^{n} a_i z^{-i}}{\sum_{j=0}^{m/2} b_j z^{-2j}} \qquad (a_n,\ b_{m/2} \neq 0), \quad (14)$$

where m is an even number and $H_0(z)$ is irreducible. From the paraunitary property, $\boldsymbol{E}(z^2)$ is written as

$$\boldsymbol{E}(z^2) = \begin{bmatrix} E_{00}(z^2) & E_{01}(z^2) \\ E_{10}(z^2) & E_{11}(z^2) \end{bmatrix} \quad (15)$$

$$= \frac{1}{D(z^2)}\begin{bmatrix} N_0(z^2) & N_1(z^2) \\ -z^{-2l}N_1(z^{-2}) & z^{-2l}N_0(z^{-2}) \end{bmatrix}, \quad (16)$$

where

$$N_1(z^2)N_1(z^{-2}) = D(z^2)D(z^{-2}) - N_0(z^2)N_0(z^{-2}), \quad (17)$$

$$2l = \begin{cases} n & (n : \text{even}) \\ n-1 & (n : \text{odd}) \end{cases}. \quad (18)$$

Then, analysis bank $\boldsymbol{U}(z)$ is written as

$$\boldsymbol{U}(z) = \begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} \quad (19)$$

$$= \boldsymbol{E}(z^2)\begin{bmatrix} 1 \\ z^{-1} \end{bmatrix}. \quad (20)$$

By substitution of (16) into (20), $H_0(z)$ and $H_1(z)$ are written as

$$H_0(z) = \frac{1}{D(z^2)}\{N_0(z^2) + z^{-1}N_1(z^2)\}, \tag{21}$$

$$H_1(z) = \frac{1}{D(z^2)}\{-z^{-2n}N_0(z^{-2}) + z^{-1} \cdot z^{-2n}N_1(z^{-2})\}. \tag{22}$$

We define $T_0(z)$, $T_1(z)$ as numerator polynomials in $H_0(z)$ and $H_1(z)$. Then, we can get a relation between $T_0(z)$ and $T_1(z)$ as

$$T_1(z) = -z^{-(2n+1)}T_0(-z^{-1}). \tag{23}$$

From (21), it is clear that $N_0(z^2)$ and $N_1(z^2)$ are polyphase components of $H_0(z)$. That is, we can get polyphase matirix $\boldsymbol{E}(z^2)$ from $H_0(z)$. Now we consider what kind of properties $H_0(z)$ should have. In order for $\boldsymbol{E}(z^2)$ to be a paraunitary matrix, $H_0(z)$ and $H_1(z)$ must have power complementary property [1]. That is

$$T_0(z)T_0(z^{-1})+T_1(z)T_1(z^{-1})=D(z^2)D(z^{-2}). \tag{24}$$

Then we substitute (23) into (24), and we get

$$T_0(z)T_0(z^{-1}) + T_0(-z)T_0(-z^{-1}) = D(z^2)D(z^{-2}). \tag{25}$$

This formula is a power symmetry property which is introduced in [1]. It is clear that when a given transfer function has power symmetry property, we can get paraunitary $\boldsymbol{E}(z^2)$ without fail.

Now, we compare power symmetry transfer functions which were realized in the past and the transfer functions which are realized in this paper, and classify these transfer functions in Fig. 4 in terms of the order of transfer functions. Like (14), m and n are the orders of denominator and numerator polynomials of a given transfer function. In the existing structures, it is well known that the given transfer function should have $n - m = 1$ and $T_0(z)/T_0(-z) = -T_0(z^{-1})/T_0(-z^{-1})$, and be decomposed into two allpass filters ((a) in Fig. 4). When $n - m = 0$, this transfer function has been realized by a complex allpass filter ((c) in Fig. 4). When $n - m \geqq 2$ under $m > 0$, no realization has been proposed ((d) in Fig. 4). In the case of FIR transfer functions, $D(z^2) = 1$ in (17). Therefore, since the order of $N_0(z^2)$ is equal to that of $N_1(z^2)$, given FIR transfer function has been odd-order ((e) in Fig. 4). That is to say, it is considered that even-order FIR paraunitary filter banks ((f) in Fig. 4) can not be realized [1]. From the above, we discuss the synthesis method for the structure of (a)–(e) in Fig. 4 by using real allpass filters with the focus on extraction of paraunitary sections.

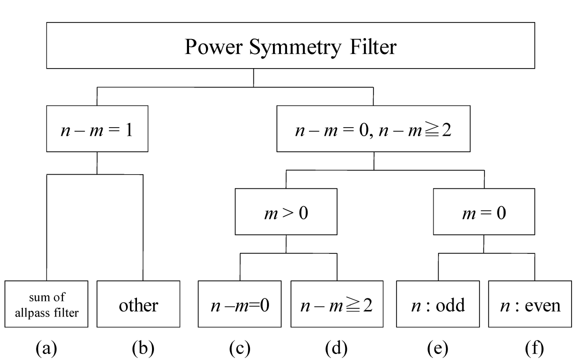

Fig. 4 Classification of power symmetry filters.

3. Extraction of Lattice Sections

3.1 Lattice Sections

In the previous section, we can derive $\boldsymbol{E}(z^2)$ from a given transfer function. It is, however, difficult to directly construct the circuit of analysis bank which have paraunitary property [8]. It is appropriate to synthesize the circuit by connecting low-order sections.

As sections to be extracted and extracting method, we use the technique introduced in [3]. That literature uses a wave digital lattice filter (WDLF) shown in Fig. 5 which has paraunitary property. Since WDLF uses allpass filters, we expect we can eliminate AMD. In addition, because $\boldsymbol{E}(z^2)$ in Fig. 2 is modified to be $\boldsymbol{E}(z)$ by noble identities, allpass filters in Fig. 5 are polynomials of z^{-1}. Then, in the next subsection, we consider extraction of WDLF sections.

3.2 Extraction Method

In order to decompose a polyphase matrix $\boldsymbol{E}(z)$ into

$$\boldsymbol{E}(z) = \prod_{i=1}^{n} \boldsymbol{E_i}(z), \tag{26}$$

we first extract $\boldsymbol{E_1}(z)$ from $\boldsymbol{E}(z)$, that is

$$\boldsymbol{E'}(z) = \boldsymbol{E_1^{-1}}(z) \cdot \boldsymbol{E}(z), \tag{27}$$

where $\boldsymbol{E'}(z)$ is the remainder of the extraction. In (27), the order of $\boldsymbol{E'}(z)$ should be lower than $\boldsymbol{E}(z)$. We classify a first-order WDLF into two types as

- Type 1: $A_0(z) = \dfrac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$ and $A_1(z) = 1$,
- Type 2: $A_0(z) = 1$ and $A_1(z) = \dfrac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$.

Any WDLF with real poles can be realized by the cascade of Type 1's and Type 2's according to (27). Then, to unify these formula, we define

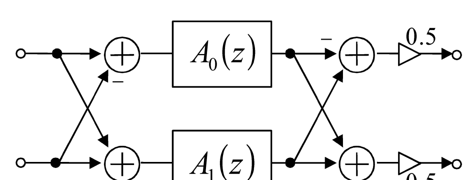

Fig. 5 WDLF section.

$$I(z) = 1 - \alpha z^{-1} \tag{28}$$
$$J(z) = z^{-1} - \alpha, \tag{29}$$

in Type 1, and also

$$I(z) = z^{-1} - \alpha \tag{30}$$
$$J(z) = 1 - \alpha z^{-1}, \tag{31}$$

in Type 2. Thus each section's matrix for WDLF can be written as

$$\begin{aligned} \boldsymbol{E_1}(z) &= \frac{1}{2}\begin{bmatrix} A_1(z)-A_0(z) & A_1(z)+A_0(z) \\ A_1(z)+A_0(z) & A_1(z)-A_0(z) \end{bmatrix} \\ &= \frac{1}{2(1-\alpha z^{-1})}\begin{bmatrix} I(z)-J(z) & I(z)+J(z) \\ I(z)+J(z) & I(z)-J(z) \end{bmatrix}. \end{aligned} \tag{32}$$

Then, in Type 1, extracted section is

$$\begin{aligned} \begin{bmatrix} E'_{00}(z) \\ E'_{10}(z) \end{bmatrix} &= \frac{-1}{2I(z)J(z)} \\ &\cdot \begin{bmatrix} I(z)\{E_{00}(z)-E_{10}(z)\} \\ \quad -J(z)\{E_{00}(z)+E_{10}(z)\} \\ I(z)\{E_{10}(z)-E_{00}(z)\} \\ \quad -J(z)\{E_{00}(z)+E_{10}(z)\} \end{bmatrix}, \end{aligned} \tag{33}$$

where

$$\boldsymbol{E'}(z) = \begin{bmatrix} E'_{00}(z) & E'_{01}(z) \\ E'_{10}(z) & E'_{11}(z) \end{bmatrix}. \tag{34}$$

The reason to derive only $E'_{00}(z)$ and $E'_{10}(z)$ is that $E'_{11}(z)$ depends on $E'_{00}(z)$ and $E'_{01}(z)$ depends on $E'_{10}(z)$. In (33), the orders of $E'_{00}(z)$ and $E'_{10}(z)$ need to be reduced by one. Therefore by the factor theorem, it is necessary that

$$\begin{aligned} E_{00}(z) + E_{10}(z) &= (1 - \alpha z^{-1}) \\ &\quad \cdot(\text{polynomial in } z^{-1}) \end{aligned} \tag{35}$$

and

$$\begin{aligned} E_{00}(z) - E_{10}(z) &= (z^{-1} - \alpha) \\ &\quad \cdot(\text{polynomial in } z^{-1}) \end{aligned} \tag{36}$$

hold. To find the conditions for these formulas, we define the characteristic function written as

$$P_1(z) = \frac{E_{10}(z)}{E_{00}(z)}. \tag{37}$$

Then, substituting pole α into (37), we get

$$P_1(\alpha) = -1. \tag{38}$$

In Type 2, (33) should be

$$\begin{aligned} E_{00}(z) + E_{10}(z) &= (z^{-1} - \alpha) \\ &\quad \cdot(\text{polynomial in } z^{-1}) \end{aligned} \tag{39}$$

$$\begin{aligned} E_{00}(z) - E_{10}(z) &= (1 - \alpha z^{-1}) \\ &\quad \cdot(\text{polynomial in } z^{-1}). \end{aligned} \tag{40}$$

So, we can also get

$$P_1(\alpha) = 1. \tag{41}$$

As a result, we can select the type of WDLF by the value of the characteristic function at the pole.

3.3 Extraction of Zeroth-Order Sections

The condition to extract a WDLF section is that the value of characteristic function is either +1 or −1. However, characteristic functions do not always satisfy that condition. We, then, extract a zeroth-order section (Fig. 6) in advance when the characteristic function is not ±1. The zeroth-order paraunitary section is described by

$$\begin{bmatrix} k & k' \\ k' & -k \end{bmatrix}, \tag{42}$$

where

$$k' = \sqrt{1-k^2}. \tag{43}$$

Now, let $P_2(z)$ be the characteristic function after extraction of a zeroth-order section. By extraction, we can get

$$P_2(z) = \frac{-kP_1(z) + \sqrt{1-k^2}}{\sqrt{1-k^2}P_1(z) + k}. \tag{44}$$

Then, we consider the condition to derive two types of WDLF sections as follows:

- $P_2(\alpha) = -1$ (Type 1)
  From (44),
  $$\sqrt{1-k^2}(1 + P_1(\alpha)) = -k(1 - P_1(\alpha)). \tag{45}$$
  To satisfy this equation, if $k < 0$, $P_1(\alpha)$ must be $-1 < P_1(\alpha) < 1$. So, we can get
  $$k = -\sqrt{\frac{1}{2} + \frac{1}{P_1(\alpha) + 1/P_1(\alpha)}}. \tag{46}$$
  If $k > 0$, $P_1(\alpha)$ must be either $P_1(\alpha) < -1$ or $1 < P_1(\alpha)$. So we can get
  $$k = \sqrt{\frac{1}{2} + \frac{1}{P_1(\alpha) + 1/P_1(\alpha)}}. \tag{47}$$

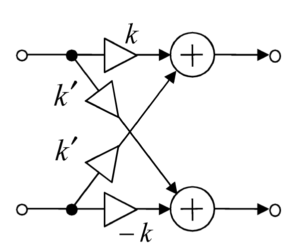

Fig. 6 Zeroth-order paraunitary matrix.

Table 1 Sign in k.

$P_1(\alpha)$	Type 1	Type 2
$-1 < P_1(\alpha) < 1$	$t = -1$ $u = +1$	$t = +1$ $u = -1$
$P_1(\alpha) < -1,\ 1 < P_1(\alpha)$	$t = +1$ $u = +1$	$t = -1$ $u = -1$

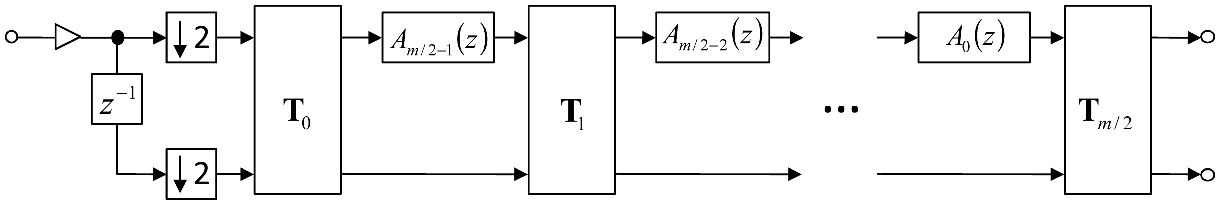

Fig. 7 Structure after extraction (analysis bank).

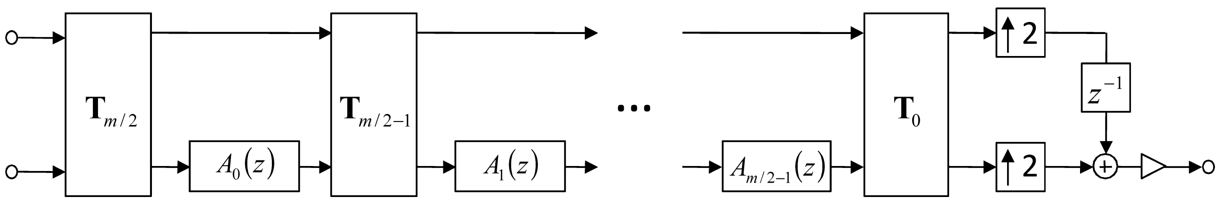

Fig. 8 Structure after extraction (synthesis bank).

- $P_2(\alpha) = 1$ (Type 2)
  From (44),

$$\sqrt{1-k^2}(1 + P_1(\alpha)) = k(1 - P_1(\alpha)). \tag{48}$$

To satisfy this equation, if $k < 0$, $P_1(\alpha)$ must be either $P_1(\alpha) < -1$ or $1 < P_1(\alpha)$. So, we can get

$$k = -\sqrt{\frac{1}{2} - \frac{1}{P_1(\alpha) + 1/P_1(\alpha)}}. \tag{49}$$

If $k > 0$, $P_1(\alpha)$ must be $-1 < P_1(\alpha) < 1$. So we can get

$$k = \sqrt{\frac{1}{2} - \frac{1}{P_1(\alpha) + 1/P_1(\alpha)}}. \tag{50}$$

From the above, k is expressed to be

$$k = t \cdot \sqrt{\frac{1}{2} + \frac{u}{P_1(\alpha) + 1/P_1(\alpha)}}, \tag{51}$$

where t and u take value of ± 1, and these two values are determined by the value of characteristic function at pole and the next section's type of WDLF. We summarize this in Table 1. Therefore, we can select the types of WDLF by $P_1(\alpha)$, t and u. The use of these parameters t and u is the extension of [3]. The choice of Type 1 or Type 2 is arbitrary. For example, it is possible that only Type 1 WDLF sections are used in analysis bank, and only Type 2 WDLF sections are used in synthesis bank.

3.4 Analysis Bank and Synthesis Bank

We can extract low order sections from a high order section by the above procedure. One of the merits of this operation is that we can easily construct synthesis bank from each extracted section in analysis bank. In this subsection, we show the construction method of synthesis bank from extracted sections in analysis bank.

(1) WDLF section

The scattering matrix of WDLF section is

$$\boldsymbol{S}(z) = \frac{1}{2}\begin{bmatrix} A_1(z)-A_0(z) & A_1(z)+A_0(z) \\ A_1(z)+A_0(z) & A_1(z)-A_0(z) \end{bmatrix}. \tag{52}$$

Now, we define $\boldsymbol{S'}(z)$ as

$$\boldsymbol{S'}(z) = \frac{1}{2}\begin{bmatrix} A_0(z)-A_1(z) & A_0(z)+A_1(z) \\ A_0(z)+A_1(z) & A_0(z)-A_1(z) \end{bmatrix}. \tag{53}$$

This matrix is obtained by exchanging $A_0(z)$ and $A_1(z)$ in $\boldsymbol{S}(z)$. When we multiply these two matrices, we get

$$\boldsymbol{S'}(z) \cdot \boldsymbol{S}(z) = A_0(z)A_1(z)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{54}$$

(2) Zeroth-order section

From (42), zeroth-order section is expressed as

$$\boldsymbol{T} = \begin{bmatrix} k & \sqrt{1-k^2} \\ \sqrt{1-k^2} & -k \end{bmatrix}. \tag{55}$$

From this matrix, we have

$$\boldsymbol{T} \cdot \boldsymbol{T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{56}$$

Assuming that we have analysis bank written as

$$\boldsymbol{E}(z) = \boldsymbol{T_L}(z)\boldsymbol{S_{L-1}}(z)\cdots\boldsymbol{S_0}(z)\boldsymbol{T_0}(z), \tag{57}$$

where $\boldsymbol{S_i}$ is a WDLF section and $\boldsymbol{T_i}$ is a zeroth-order section. From $\boldsymbol{E}(z)$, we construct synthesis bank $\boldsymbol{R}(z)$ written as

$$\boldsymbol{R}(z) = \boldsymbol{T_0}(z)\boldsymbol{S'_0}(z)\cdots\boldsymbol{S'_{L-1}}(z)\boldsymbol{T_L}(z). \tag{58}$$

We can also derived $\boldsymbol{R}(z)$ from $\boldsymbol{E}(z)$ by "time reversal transpose" method [14]. But, if we use "time reversal transpose", time delay from input to output will become very large. We consider this spoils the merit of using IIR filters especially for speech applications. By allowing PHD, we can use IIR filters without time reversal so that the time delay is small. Therefore we use the above method. Then, by using the above relations (54), (56), we can get a formula which is the same as (7). Therefore, we can readily get the structure of synthesis bank by extraction while keeping the amplitude distortion free property.

From (57), (58) a structure of paraunitary filter banks is shown as Fig. 7 and Fig. 8. Now, we consider $\boldsymbol{T_i}$ in analysis and synthesis banks. In these two figures, $\boldsymbol{T_i}$ is (a) in Fig. 9, and (a) can be modified as (b). In (b), b' is

$$b' = \frac{b}{a}. \tag{59}$$

Since the a in Fig. 9(b) influences only the gain level, we can remove a from zeroth-order sections, and move the multipliers to input (or output). This movement of mulitipliers is applied when signal scaling is not critical.

4. Comparison of the Number of Multiplications

In the previous section, we can synthesize filter banks by extraction. Since these filter banks have polyphase structure, noble identity applies to polyphase matrix. Therefore, the sampling rate of the input signal is reduced, and we can reduce the number of multiplications. This reduction has been restricted to the cases where the order of given transfer function is odd for IIR filter banks.

If the order is even, the structure in Fig. 10 introduced in [5] is used, and complex allpass filter applies to this structure for eliminating amplitude distortion [4]. In Fig. 11, however, we cannot move decimators to input nor inerpolators to output. Let f_s be the sampling frequency, and let m be the order of the denominator polynomial of the given transfer function. Then we need $m/2$ first-order complex allpass filters, and each filter has four real multiplications. Therefore, the total number of multiplications per second is

$$\frac{m}{2} \cdot 4 \cdot f_s = 2mf_s. \tag{60}$$

In contrast, when we use the proposed method, the decimators can be moved as shown in Fig. 7. In Fig. 7, there are $m/2$ allpass sections and $(m/2 + 1)$ zeroth-order sections. In addition, considering the input multiplier, we have the total number of multiplications as

$$\frac{1}{2} f_s \left\{ \frac{m}{2} + 2\left(\frac{m}{2} + 1\right) \right\} + f_s \tag{61}$$

$$= \left(\frac{3}{4}m + 2\right) f_s. \tag{62}$$

From (60) and (62), we can reduce the number of multiplications per second in the proposed structure.

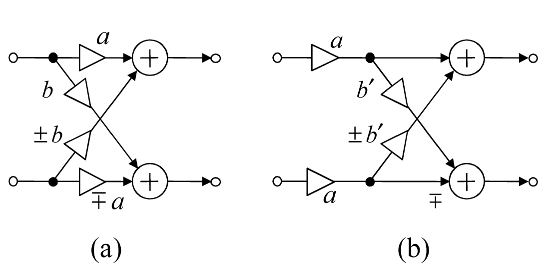

Fig. 9 Transposision of multipliers.

5. Examples

In this section, we synthesize some circuits with the proposed synthesis procedure. To begin with, assuming that the order of numerator is n and that of denominator is m, we focus on the order of transfer function. With both odd and even of n, we show that we can synthesize the circuit using real allpass filters.

Moreover, we show we can also synthesize the transfer functions in which the difference of n and m is more than 2.

5.1 Odd-Order IIR Filter ($n - m = 1$)

We use a 5th-order elliptic lowpass filter. This filter's transfer function is used in [2]. We extract WDLF and zerothoder sections successively from this transfer function. Then we get one Type 1 section and one Type 2 WDLF section. So, organizing these sections, we get the circuit shown in Fig. 12. It is clear that this structure in Fig. 12 is the same structure which is introduced in [1]. The transfer functions of two allpass filters in Fig. 12 are

$$A_0(z) = \frac{z^{-1} + 0.2368041466}{1 + 0.2368041466z^{-1}}, \tag{63}$$

$$A_1(z) = \frac{z^{-1} + 0.7149039978}{1 + 0.7149039978z^{-1}}. \tag{64}$$

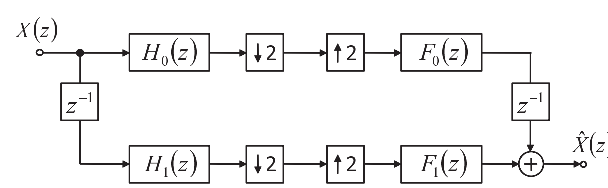

Fig. 10 Prototype structure for even-order filter banks.

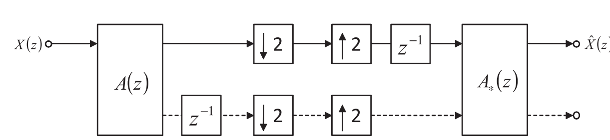

Fig. 11 Conventional even-order structure using complex allpass filter.

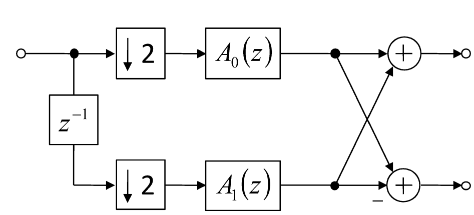

Fig. 12 Synthesized circuit for odd order IIR filter bank.

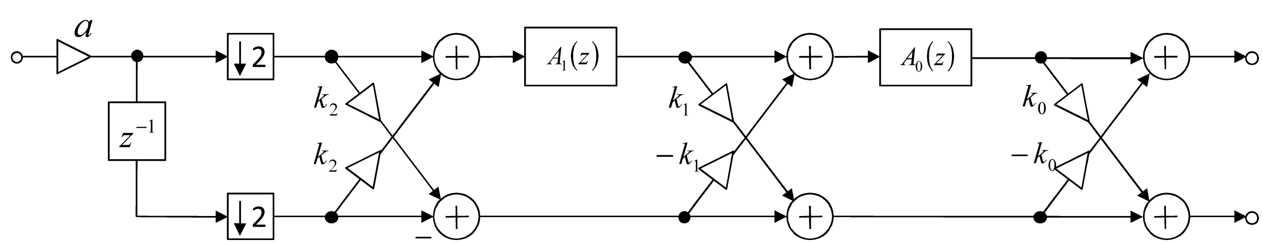

Fig. 13 Synthesized circuit for 4th-order IIR filter (analysis bank).

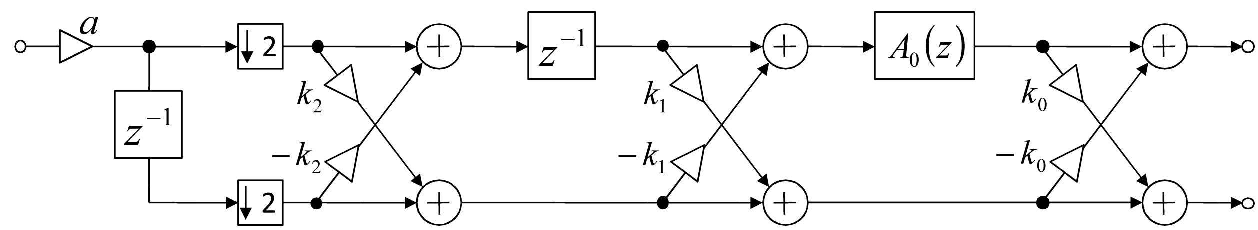

Fig. 14 IIR-FIR hybrid structure (analysis bank).

Table 2 Values of coefficients and transfer functions in Fig. 13.

a	−0.009332229256
k_0	5.027339492
k_1	−2.414213562
k_2	0.6681786379
$A_0(z)$	$\frac{0.03956612990 + z^{-1}}{1 + 0.03956612990z^{-1}}$
$A_1(z)$	$\frac{0.4464626922 + z^{-1}}{1 + 0.4464626922z^{-1}}$

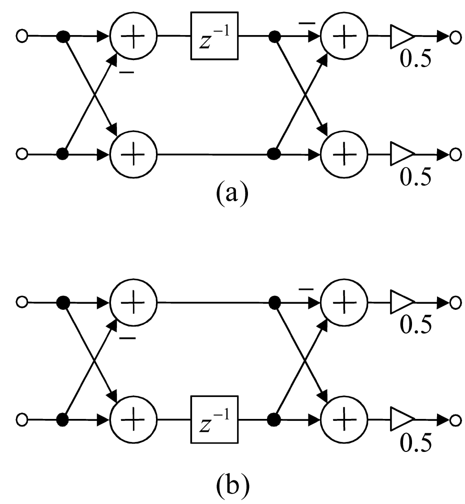

Fig. 15 (a) FIR WDLF section type 1, (b) FIR WDLF section type 2.

5.2 Even-Order IIR Filter ($n - m = 0$)

In this subsection, we use a 4th-order Butterworth lowpass filter. To this transfer function, we do the same operations as odd transfer functions. Then, we get a circuit shown in Fig. 13. The coefficients and transfer functions in Fig. 13 are shown in Table 2.

5.3 IIR-FIR Hybrid Type ($n - m \geqq 2$)

Let us now consider a transfer function with $n - m \geqq 2$. Such a transfer function has been derived so far [9], but the structure which can eliminate AMD has not been synthesized from the transfer function. In this subsection, we consider the synthesis method for a transfer function with $n - m \geqq 2$. In 5.1 and 5.2, since $n - m$ is either 0 or 1, we have only to focus on the poles of the transfer function. However, if $n - m \geqq 2$, FIR sections remain even if we extract IIR sections. Whereat, we consider to extract FIR sections. From this, we can treat IIR-FIR hybrid type.

As a section to be extracted, we use FIR sections (Fig. 15) because these sections have paraunitary and FIR properties. In the same way as Sect. 3.2, we derive the condition to extract these sections as

Table 3 Value of coefficients and transfer function in Fig. 14.

a	−0.604583247
k_0	−1.518993591
k_1	−2.039507733
k_2	−1.696140478
$A_0(z)$	$\frac{0.4202041029 + z^{-1}}{1 + 0.4202041029z^{-1}}$

$$P(0) = \begin{cases} +1 & \text{(Type 1)} \\ -1 & \text{(Type 2)} \end{cases}, \tag{65}$$

where $P(z)$ is the characteristic function. In addition, if $P(0) \neq \pm 1$, it is necessary to extract zeroth-order section.

Now, as an example of the proposed structure, we realize the transfer function

$$H_0(z) = \frac{(1 + z^{-1})^4 (1 - 0.127016653793z^{-1})}{1 - 0.420204102887z^{-2}}. \tag{66}$$

From this transfer function, we extract some paraunitary sections. As a result, we get a circuit shown in Fig. 14. In this structure, coefficients and transfer functions are written in Table 3.

6. Conclusion

In this paper, we propose a procedure to design the circuit of IIR paraunitary filter banks. We derive that a given transfer function should have power symmetry property. From a power symmetric transfer function, we can get a polyphase matrix $\boldsymbol{E}(z^2)$ shown in Fig. 2, and by applying the proposed extraction method to this polyphase matrix, we can get the structure of 2-channel paraunitary filter banks. In extraction operation, we can derive the condition of extraction procedure from the value of the characteristic function $P(z)$. Focusing the order of the given transfer function, we can synthesize even-order filter banks without using complex allpass filters, and we can reduce the number of multiplications per unit time. In addition, we can synthesize circuits from the transfer function whose order of numerator polynomial is higher than that of denominator polynomial by more than 2.

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Nagato Ueda received the B.E. in Computer Science from Tokyo Institute of Technology in 2008. He is currently the candidate for the M.E. degree at Tokyo Institute Technology. His reseach intersts are in design of filter banks.

Eiji Watanabe was born in Ehime Prefecture, Japan on August 20, 1958. He received the B.E. and M.E. degrees in radio communications from the University of Electro-Communications in Chofu-shi, Japan in 1981 and 1983, respectively. He received the Dr.E. degree in physical electronics from Tokyo Institute of Technology in Tokyo, Japan in 1986. From 1986 to 1991 he was a research associate in the Department of Information Processing at Tokyo Institute of Technology in Yokohama-shi, Japan. From 1991 to 1995 he was a lecturer in the Department of Electronic Information Systems at Shibaura Institute of Technology in Saitama-shi, Japan. From 1995 to 2000 he was an associate professor and currently is a full professor in the same department. His research interests are in circuit theory and signal processing. Prof. Watanabe is a senior member of IEEE.

Akinori Nishihara received the B.E., M.E. and Dr.Eng. degrees in electronics from Tokyo Institute of Technology in 1973, 1975 and 1978, respectively. Since 1978 he has been with Tokyo Institute of Technology, where he is now Professor of the Center for Research and Development of Educational Technology. His main research interests are in one- and multi-dimensional signal processing, and its application to educational technology. He has published more than 200 technical papers in international journals and conferences. He served as an Associate Editor of the IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences from 1990 to 1994, and then an Associate Editor of the Transactions of IEICE Part A (in Japanese) from 1994 to 1998. He was an Associate Editor of the IEEE Transactions on Circuits and Systems II from 1996 to 1997 and Editor-in-Chief of Transactions of IEICE Part A (in Japanese) from 1998 to 2000. He has been serving in IEEE Region 10 Executive Committee in various positions, and Executive Committee Member of IEEE Tokyo Section and IEEE Japan Council for about 10 years. He was a member of the Board of Governors, IEEE Circuits and Systems Society (2004–2005), and was Chair of IEEE Circuits and System Society Japan Chapter (2006–2007). He served for IEICE in various positions; He was Director, Conferences, Student Activities and Education (2007–2009) and has been a member of Strategic Planning Office since 2001. He received Best Paper Awards of the IEEE Asia Pacific Conference on Circuits and Systems in 1994 and 2000, a Best Paper Award of the IEICE in 1999, IEEE Third Millennium Medal in 2000, 4th LSI IP Design Award in 2002, and Distinguished Service Award for IEEE Student Activities in 2006. Prof. Nishihara is a Fellow of IEEE, and a member of EURASIP, Japan Society for Educational Technology, Association for the Advancement of Computing in Education, and Asia Pacific Signal and Information Processing Association.