Secure Sets and Defensive Alliances in Graphs: A Faster Algorithm and Improved Bounds

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SUMMARY   Secure sets and defensive alliances in graphs are studied. They are sets of vertices that are *safe* in some senses. In this paper, we first present a fixed-parameter algorithm for finding a small secure set, whose running time is much faster than the previously known one. We then present improved bound on the smallest sizes of defensive alliances and secure sets for hypercubes. These results settle some open problems paused recently.
key words: secure set, defensive alliance, fixed-parameter tractability, hypercube

1. Introduction

The concept of a defensive alliance in a graph is introduced by Kristiansen, Hedetniemi, and Hedetniemi [14]. Intuitively, a defensive alliance is a set of vertices that is *safe* from attacks by its neighborhood. Later the concept of a secure set in a graph is introduced by Brigham, Dutton, and Hedetniemi [1]. Roughly speaking, a secure set is a much safer set of vertices than a defensive alliance. As the readers see from the formal definitions in the next section, any secure set is a defensive alliance. The problem of finding a smallest secure set or a smallest defensive alliance is of interest. These concepts have been intensively studied (see e.g. [1], [4], [5], [10], [14]). Recently, Isaak, Johnson, and Petrie [12], [16] have introduced fractional variants of secure sets.

Jamieson, Hedetniemi, and McRae [13] showed that the problem of deciding whether a graph has a defensive alliance of size at most given k is NP-complete. Fernau and Raible [9] showed that the problem is fixed-parameter tractable when parameterized by the size of a defensive alliance. Enciso and Dutton [6] later presented a fixed-parameter algorithm with a better running time.

To the best of our knowledge, the complexity of finding a smallest secure set is not known. It is known that given a graph and a set of vertices, it is coNP-complete to decide whether the set is a secure set of the graph [3]. Enciso and Dutton [7] presented a fixed-parameter algorithm for solving the problem of deciding whether a given n-vertex graph has a secure set of size at most k in time $O(2^{k \log 2k} n)$.

1.1   Our Results

In Sect. 3, we present a faster fixed-parameter algorithm with a running time of $O(2^{3k} k^2 n)$ for finding a secure set of size at most k. In Sect. 4, we present graph-theoretic contributions. For hypercubes, we first determine exactly the defensive alliance number, and also the defensive alliance partition number as a corollary. We then generalize these results to Hamming graphs. Next we present a significantly improved lower bound of the security number of hypercubes. The previously known lower bound due to Petrie [16] is $2^{\lfloor d/2 \rfloor}$, and our lower bound is more than $2^{0.9d}$ for large enough d.

2. Preliminaries

All graphs in this paper are finite, simple, and undirected. For a graph G, we denote its vertex set and edge set by $V(G)$ and $E(G)$, respectively. The (*open*) *neighborhood* $N(v)$ of a vertex $v \in V(G)$ is the set $\{u \mid \{u, v\} \in E(G)\}$. The *degree* of $v \in V(G)$ is $|N(v)|$ and denoted by $\deg_G(v)$. The *closed neighborhood* $N[v]$ of a vertex $v \in V(G)$ is the set $\{v\} \cup N(v)$. For a subset $S \subseteq V(G)$, we define its *closed neighborhood* as $N[S] = \bigcup_{v \in S} N[v]$. The subgraph induced by $S \subseteq V(G)$ is denoted by $G[S]$. The *distance* between $u, v \in V(G)$, denoted $\mathrm{dist}(u, v)$, is the length of a shortest u–v path in G.

The *Cartesian product* of graphs G and H, denoted $G \square H$, is the graph with the vertex set $V(G) \times V(H)$ and the edge set $\{\{(g, h), (g, h')\} \mid g \in V(G), \{h, h'\} \in E(H)\} \cup \{\{(g, h), (g', h)\} \mid h \in V(H), \{g, g'\} \in E(G)\}$. The *dth Cartesian power of a graph G*, denoted G^d, is defined as $G^1 = G$ and $G^d = G \square G^{d-1}$ for $d \geq 2$. The *Hamming graph* K_k^d is the dth Cartesian power of the complete graph K_k for some d and k. The *d-dimensional hypercube* Q_d is the dth Cartesian power of K_2; that is, $Q_d = K_2^d$.

Let G be a graph. A non-empty set $S \subseteq V(G)$ is a *defensive alliance* if for each $s \in S$, $|N[s] \cap S| \geq |N[s] \setminus S|$. The *defensive alliance number* $\mathsf{da}(G)$ of G is the size of a smallest defensive alliance of G. A non-empty set $S \subseteq V(G)$ is a *secure set* if for each $X \subseteq S$, $|N[X] \cap S| \geq |N[X] \setminus S|$ (see

[1]). The *security number* $\mathsf{s}(G)$ of G is the size of a smallest secure set of G. Clearly, $\mathsf{da}(G) \le \mathsf{s}(G)$ for any graph G.

3. A Faster Fixed-Parameter Algorithm for Secure Sets

Recall that the running time of the algorithm by Enciso and Dutton [7] is $O(2^{k \log 2k} n)$, where n is the number of vertices of G. Here we present an improved $O(2^{3k} k^2 n)$-time algorithm.

Theorem 3.1. *The problem of deciding whether* $\mathsf{s}(G) \le k$ *can be solved in time* $O(2^{3k} k^2 n)$.

Proof. For any $S \subseteq V(G)$, the following claims follows from the definitions (see [7]):

Claim 3.2. *If* $|N[S]| > 2k$, *then there is no secure set* S' *that satisfies* $S \subseteq S'$ *and* $|S'| \le k$.

Claim 3.3. *If* $S' \supsetneq S$ *is a minimal secure set, then there is a vertex* $u \in S' \setminus S$ *such that* $u \in N[S] \setminus S$.

Now we describe our algorithm. See Algorithm 1. We first guess a vertex $v \in V(G)$, and call FIND($\{v\}$, ∅) to find a secure set containing v. For $S \subseteq V(G)$ and $F \subseteq N[S] \setminus S$, the procedure FIND($S$, F) finds a secure set S' of size at most k such that $S \subseteq S'$ and $F \cap S' = \emptyset$. If S itself is secure, then FIND(S, F) outputs S and stops.

Assume S is not secure and there is a minimal secure set $S' \supsetneq S$ with $|S'| \le k$ and $S' \cap F = \emptyset$. Clearly, $|S| < k$. Since $F \subseteq N[S'] \setminus S'$, we have $|F| \le |S'| \le k$. By Claim 3.2, $|N[S]| \le 2k$. Hence, if not all the conditions are satisfied, then there is no such S'. If all the conditions are satisfied, the procedure recursively finds such S'. If S' exists, then there is a vertex $u \in S' \setminus S$ such that $u \in N[S] \setminus (S \cup F)$ by Claim 3.3. Moreover, if $u \in N[S] \setminus (S \cup F)$, then either $u \in S' \setminus S$ or $u \in N[S'] \setminus S'$ holds. We add u into S in the former case and into F in the latter case. To check both cases, we call FIND($S \cup \{u\}$, F) and FIND(S, $F \cup \{u\}$).

Algorithm 1 Find a secure set of G with size at most k.

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 1: for all v ∈ V(G) do
 2:     FIND({v}, ∅)

 3: procedure FIND(S, F)
 4:     if SECURE(S) then
 5:         Output S, and stop.
 6:     if |S| < k and |F| ≤ k and |N[S]| ≤ 2k then
 7:         u := a vertex in N[S] \ (S ∪ F)
 8:         FIND(S ∪ {u}, F)
 9:         FIND(S, F ∪ {u})

10: procedure SECURE(S)
11:     for all X ⊆ S do
12:         if |N[X] ∩ S| < |N[X] \ S| then
13:             return false
14:     return true
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Next we show the running time. We first have n branches corresponding to the calls of FIND($\{v\}$, ∅) for all $v \in V(G)$. Each call of FIND(S, F) has no or two branches that correspond to the calls of FIND($S \cup \{u\}$, F) and FIND(S, $F \cup \{u\}$) if $|S| < k$, $|F| \le k$, and $|N[S]| \le 2k$. Therefore, the search-tree has depth at most $2k$ in which the root has n children, and other inner nodes have two children for each. Thus it has at most $1 + n \sum_{i=0}^{2k-1} 2^i \le 2^{2k} n$ nodes. Since the algorithm takes $O(2^k k^2)$ time for checking whether S is secure for each node of the search-tree, the total running time is $O(2^{3k} k^2 n)$. □

4. Improved Bounds for Hypercubes

4.1 Defensive Alliances in Hypercubes

The *defensive alliance partition number of G*, $\psi_{\mathsf{da}}(G)$, is defined to be the maximum number of sets in a partition of $V(G)$ such that each set is a defensive alliance [14]. Eroh and Gera [8] showed that $\psi_{\mathsf{da}}(Q_d) \ge 2^{\lceil d/2 \rceil}$, and asked whether the bound is tight. In this section, we first answer the question affirmatively. We then generalize the result to show similar results for Hamming graphs.

The first result is an almost direct consequence of the following beautiful proposition.

Proposition 4.1 (Chung, Füredi, Graham, and Seymour [2]). *If G is a subgraph of a hypercube with average degree s, then $|V(G)| \ge 2^s$.*

Theorem 4.2. *For any d, $\mathsf{da}(Q_d) = 2^{\lfloor d/2 \rfloor}$.*

Proof. Eroh and Gera [8] showed that $\mathsf{da}(Q_d) \le 2^{\lfloor d/2 \rfloor}$. Thus we only show the lower bound. Let S be a defensive alliance of Q_d and $v \in S$. Since $|N[v] \cap S| \ge |N[v] \setminus S|$ and $d = |N[v] \cap S| + |N[v] \setminus S| - 1$, it follows that $\deg_{Q_d[S]}(v) = |N[v] \cap S| - 1 \ge \lceil (d-1)/2 \rceil = \lfloor d/2 \rfloor$. Hence the average degree of $Q_d[S]$ is at least $\lfloor d/2 \rfloor$. Now, by Proposition 4.1, $|S| \ge 2^{\lfloor d/2 \rfloor}$. □

The theorem above implies that $\psi_{\mathsf{da}}(Q_d) \le 2^d / 2^{\lfloor d/2 \rfloor} = 2^{\lceil d/2 \rceil}$. Combining this fact with the lower bound of Eroh and Gera [8], we have the following answer.

Corollary 4.3. *For any d, $\psi_{\mathsf{da}}(Q_d) = 2^{\lceil d/2 \rceil}$.*

Now we generalize the result. For Hamming graphs, the following fact is known.

Proposition 4.4 (Squier, Torrence, and Vogt [17]). *If G is a subgraph of K_k^d with n vertices and m edges, then $2m \le (k-1)n \log_k n$.*

The proposition above can be seen as a generalization of Proposition 4.1.

Corollary 4.5. *If G is a subgraph of K_k^d with average degree s, then $|V(G)| \ge k^{s/(k-1)}$.*

Proof. Assume that G has n vertices and m edges. Then $s = 2m/n$. By Proposition 4.4, $2m \le (k-1)n \log_k n$. Dividing each side by $(k-1)n$, we obtain $s/(k-1) \le \log_k n$. This implies $k^{s/(k-1)} \le n$. □

Theorem 4.6. *For any d and k, $k^{\lfloor d/2 \rfloor} \le \mathsf{da}(K_k^d) \le k^{\lceil d/2 \rceil}$.*

Proof. First observe that K_k^d is $(k-1)d$-regular. To show the upper bound, take a subgraph isomorphic to $K_k^{\lceil d/2 \rceil}$. Each vertex in the subgraph has $(k-1)\lceil d/2 \rceil$ neighbors in the subgraph, and $(k-1)\lfloor d/2 \rfloor$ neighbors outside of the subgraph. Thus the vertex set of the subgraph, which is of size $k^{\lceil d/2 \rceil}$, is a defensive alliance.

Let S be a defensive alliance of K_k^d. By the same way as the one in the proof of Theorem 4.2, we can show that the degree of $v \in S$ is at least $\lfloor (k-1)d/2 \rfloor$. By Corollary 4.5, $|S| \ge k^{\lfloor (k-1)d/2 \rfloor/(k-1)} \ge k^{\lfloor d/2 \rfloor}$. □

Corollary 4.7. *For any d and k, $k^{\lfloor d/2 \rfloor} \le \psi_{\mathsf{da}}(K_k^d) \le k^{\lceil d/2 \rceil}$.*

Proof. The upper bound $\psi_{\mathsf{da}}(K_k^d) \le k^{\lceil d/2 \rceil}$ immediately follows from the lower bound $\mathsf{da}(K_k^d) \ge k^{\lfloor d/2 \rfloor}$ in Theorem 4.6, because $\psi_{\mathsf{da}}(G) \le |V(G)|/\mathsf{da}(G)$ for any graph G.

The lower bound $\psi_{\mathsf{da}}(K_k^d) \ge k^{\lceil d/2 \rceil}$ follows from the construction of size $k^{\lceil d/2 \rceil}$ defensive alliances in the proof of Theorem 4.6. The vertices with fixed first $\lfloor d/2 \rfloor$ coordinates induce $K_k^{\lceil d/2 \rceil}$. Thus we can partition K_k^d into $k^{\lfloor d/2 \rfloor}$ isomorphic copies of $K_k^{\lceil d/2 \rceil}$. □

4.2 Secure Sets of Hypercubes

Petrie [16] showed that $2^{\lfloor d/2 \rfloor} \le \mathsf{s}(Q_d) \le 2^{d-1}$, and asked whether the bounds can be improved. In this section we present an improved lower bound on $\mathsf{s}(Q_d)$ which is roughly $2^{0.9d}$. We only use the simple fact that S is not secure if $|S| < |N[S] \setminus S|$. Harper [11] showed that the breadth-first search ordering gives sets S that minimize $|N[S] \setminus S|$. Using this result, we show that $|S| < |N[S] \setminus S|$ for any $S \subseteq V(Q_d)$ with $1 \le |S| \le \sum_{i=0}^{\lfloor (d-2)/3 \rfloor} \binom{d}{i}$. The following proposition follows from Program and Theorem 1 in Harper's paper [11].

Proposition 4.8 (Harper [11]). *For any positive integer $k \le 2^d$, there exist a set $S \subseteq V(Q_d)$, a vertex $u_0 \in V(Q_d)$, and an integer r, such that $|S| = k$, $|N[S] \setminus S| = \min_{T \subseteq V(Q_d),|T|=k} |N[T] \setminus T|$, and $\{v \mid \mathrm{dist}(u_0, v) \le r\} \subseteq S \subset \{v \mid \mathrm{dist}(u_0, v) \le r+1\}$.*

Theorem 4.9. *For $d \ge 2$, it holds that $\mathsf{s}(Q_d) > \sum_{i=0}^{\lfloor (d-2)/3 \rfloor} \binom{d}{i}$.*

Proof. First, we show a property of a partial sum over binomial coefficients.

Claim 4.10. *For $d \ge 2$, $\sum_{i=0}^{r} \binom{d}{i} < \binom{d}{r+1}$ for $r \le \lfloor (d-2)/3 \rfloor$.*

We prove the claim by induction on r. The case of $r = 0$ clearly holds. Assume $\sum_{i=0}^{r-1} \binom{d}{i} < \binom{d}{r}$ for some r with $1 \le r \le \lfloor (d-2)/3 \rfloor$. The assumption $r \le \lfloor (d-2)/3 \rfloor$ implies $r + 1 \le d - 2r - 1$. Therefore, it holds that

$$\frac{\sum_{i=0}^{r-1} \binom{d}{i}}{\binom{d}{r}} < 1 \le \frac{d - 2r - 1}{r+1} = \frac{d-r}{r+1} - 1,$$

which implies that

$$\sum_{i=0}^{r-1} \binom{d}{i} < \binom{d}{r}\left(\frac{d-r}{r+1} - 1\right) = \binom{d}{r+1} - \binom{d}{r}.$$

Thus we have $\sum_{i=0}^{r} \binom{d}{i} < \binom{d}{r+1}$.

Next, by using Proposition 4.8 and Claim 4.10, we can show the following.

Claim 4.11. *For any subset $S \subseteq V(Q_d)$, if $|S| \le \sum_{i=0}^{\lfloor (d-2)/3 \rfloor} \binom{d}{i}$, then $|N[S] \setminus S| > |S|$.*

To prove the claim, let $k \le \sum_{i=0}^{\lfloor (d-2)/3 \rfloor} \binom{d}{i}$. Let S, u_0, and r be the set, the vertex, and the integer in Proposition 4.8, respectively. Recall that $|\{v \mid \mathrm{dist}(u_0, v) = i\}| = \binom{d}{i}$. Obviously, $r \le \lfloor (d-2)/3 \rfloor$. Hence, by Claim 4.10, we have $\sum_{i=0}^{r} \binom{d}{i} < \binom{d}{r+1}$. If $k = \sum_{i=0}^{r} \binom{d}{i}$, then $S = \{v \mid \mathrm{dist}(u_0, v) \le r\}$ and $N[S] \setminus S = \{v \mid \mathrm{dist}(u_0, v) = r+1\}$. Thus, the claim holds in this case. In the following, we will focus on the case where $k > \sum_{i=0}^{r} \binom{d}{i}$. In this case, it holds that $r \le \lfloor (d-2)/3 \rfloor - 1 \le (d-5)/3$. Let $S_i = \{v \mid v \in S, \mathrm{dist}(u_0, v) = i\}$. Clearly,

$$|S| = |S_{r+1}| + \sum_{i=0}^{r} \binom{d}{i} < |S_{r+1}| + \binom{d}{r+1}. \tag{1}$$

Let $\partial_i = (N[S] \setminus S) \cap \{v \mid \mathrm{dist}(u_0, v) = i\}$. Then $|N[S] \setminus S| = |\partial_{r+1}| + |\partial_{r+2}|$. Since S_r is exactly the set $\{v \mid \mathrm{dist}(u_0, v) = r\}$, it follows $|\partial_{r+1}| = \binom{d}{r+1} - |S_{r+1}|$. Therefore, by (1),

$$|S| < |\partial_{r+1}| + 2|S_{r+1}|.$$

Now it suffices to show that $2|S_{r+1}| \le |\partial_{r+2}|$. For any $v \in S_{r+1}$, $|N[v] \cap \partial_{r+2}| = d - r - 1$. On the other hand, for any $v \in \partial_{r+2}$, $|N[v] \cap S_{r+1}| \le r + 2$. Thus, if F is the set of edges between S_{r+1} and ∂_{r+2}, then

$$(d - r - 1)|S_{r+1}| = |F| \le (r+2)|\partial_{r+2}|,$$

which implies $((d - r - 1)/(r+2))|S_{r+1}| \le |\partial_{r+2}|$. Since $r \le (d-5)/3$ means $2 \le (d-r-1)/(r+2)$, we have $2|S_{r+1}| \le |\partial_{r+2}|$.

Claim 4.11 implies that there is no secure set of size at most $\sum_{i=0}^{\lfloor (d-2)/3 \rfloor} \binom{d}{i}$. Therefore, the theorem follows. □

To see that the lower bound above is a significant improvement, we now show that $\sum_{i=0}^{\lfloor (d-2)/3 \rfloor} \binom{d}{i} > 2^{0.9d}$ for large enough d. (An explicit lower bound will be given in the proof.) To this end, we need the following lower bound of binomial coefficients.

Proposition 4.12 (Mitzenmacher and Upfal [15, Corollary 9.3]). *For $0 \le q \le 1/2$,*

$$\binom{d}{\lceil d \cdot q \rceil} \ge \frac{2^{d \cdot H(q)}}{d+1},$$

where $H(q)$ is the binary entropy function $H(q) = -q \lg q - (1-q) \lg(1-q)$.

Corollary 4.13. $\mathsf{s}(Q_d) > 2^{0.9d}$ *for large enough d.*

Proof. We prove the statement for $d \geq 652$. By Theorem 4.9, it suffices to show that $\binom{d}{\lfloor(d-2)/3\rfloor} > 2^{0.9d}$ for $d \geq 652$. Observed that $(d-5)/3 > 0.33d$ since $d > 500$, and hence

$$\binom{d}{\lfloor(d-2)/3\rfloor} \geq \binom{d}{\lceil(d-5)/3\rceil} \geq \binom{d}{\lceil 0.33d \rceil} > \frac{2^{0.9149d}}{d+1},$$

where the last inequality follows from Proposition 4.12 and the fact that $H(0.33) > 0.9149$. Thus

$$\binom{d}{\lfloor(d-2)/3\rfloor} \geq 2^{0.9d} \cdot \frac{2^{0.0149d}}{d+1} > 2^{0.9d} \cdot \frac{1.01^d}{d+1}.$$

It is easy to verify that $1.01^d > d+1$ for $d \geq 652$. Hence $\binom{d}{\lfloor(d-2)/3\rfloor} > 2^{0.9d}$. $\square$

In a similar way, one can show that for any $\epsilon > 0$, there is an integer d_0 such that for $d \geq d_0$, $\mathsf{s}(Q_d) > 2^{(H(1/3)-\epsilon)d}$. Note that we need different argument to show a significantly better lower bound since $H(1/3) = 0.918\ldots < 0.92$.

Acknowledgments

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