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Time Difference Estimation Based on Blind Beamforming for Wideband Emitter*

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SUMMARY In the localization systems based on time difference of arrival (TDOA), multipath fading and the interference source will deteriorate the localization performance. In response to this situation, TDOA estimation based on blind beamforming is proposed in the frequency domain. An additional constraint condition is designed for blind beamforming based on maximum power collecting (MPC). The relationship between the weight coefficients of the beamformer and TDOA is revealed. According to this relationship, TDOA is estimated by discrete Fourier transform (DFT). The efficiency of the proposed estimator is demonstrated by simulation results.
key words: *time difference of arrival (TDOA), multipath, beamforming*

1. Introduction

The problem of estimating the time difference between two received signals from the same source is an essential topic in many applications such as in radar [1] and sound source localization [2]. In this paper, we focus on baseband time difference of arrival (TDOA) measurement in wideband emitter localization system with multiple spatially separated receivers. Herein, wideband represents the power spectrum of the complex envelope signal of radiation cover most of the baseband bandwidth determined by the baseband sampling ratio. Precise time difference measurement is crucial for the localization of the emitter. The traditional generalized cross-correlation (GCC) methods are usually employed for time difference estimation or direct position determination [3]. However, it is difficult to obtain highly precise time difference measurement in the presence of multipath and interference. In order to suppress the multipath and interference effects in the measurement of time difference, the beamforming concept is employed in this paper.

Beamforming is widely used in radar, sonar and communication systems, etc.[1], [4], [5]. The uniform array and narrowband signal assumptions are often needed in traditional beamforming methods. However, in the emitter localization system with multiple spatially separated receivers, each receiver is usually equipped with a single antenna in order to save costs. Moreover, the receivers are randomly arranged. This configuration is usually employed in the multi-station TDOA localization, which is applied to the illegal signal positioning, passive radar, seismic surveys, and so on [6], [7]. Herein, we use this configuration for developing the illegal signal positioning equipment in the radio spectrum monitoring system. Hence, the condition of uniform array cannot always be satisfied. Furthermore, in the case of a passive scenario, the emitter source signal to be localized is unknown a priori, and may not always be of a narrow band. Hence, the traditional beamforming methods may be insufficient for TDOA localization systems. Recently, the blind beamforming based on maximum power collecting (MPC) has been proposed for signal enhancement in TDOA system [6], where the finite impulse response (FIR) filter is employed in order to implement the delay-sum beamforming in the time domain. When the total output power is maximized, the relationship among the tap coefficients of the FIR filters will reflect the time difference relationship among the received signals [6], [7]. However, this kind of systems usually needs FIR filters with large numbers of taps for implementing subsample delay,** which would lead to increased computational load.

In this paper, TDOA measurement method based on frequency-domain blind beamforming is proposed. In order to avoid the use of subsample delay filters in the time domain, the wideband signal is transformed into frequency domain, then the blind beamforming algorithm is implemented in each frequency bin, i.e., the weight coefficients for beamforming are obtained at each frequency bin. Furthermore, the relationship between the weight coefficients of beamforming and time differences is derived. Using this relationship, TDOA can be estimated through discrete Fourier transform (DFT). Finally, it is verified by simulations that the proposed method outperforms the MPC based on FIR (MPC-FIR) method [6] and the GCC based on phase transform (GCC-PHAT) method [9].

The main contribution of this paper is summarized as follows:

1. A blind beamforming criterion with a new constraint is developed for TDOA systems.

2. The relationship between the beamforming weighting coefficients and TDOA is derived.

3. A time difference estimation method for wideband signals based on blind beamforming is proposed in the fre-

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**Subsample delay filter is also called fractional delay filter,which is usually implemented through FIR interpolater [8].

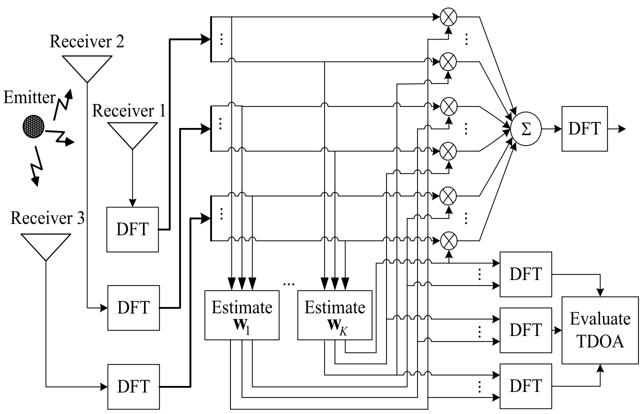

Fig. 1 Block diagram of TDOA estimation

quency domain.

2. System Model

In the emitter localization system based on TDOA, the estimation of the time differences among different receivers are required for determining the position of the emitter. Without loss of generality, it is assumed that three receivers are randomly arranged (see Fig. 1). The different delay versions of the signal from emitter to receivers are given by

$$x_i(t) = s(t - \tau_i) + q_i(t), i = 1, 2, 3 \tag{1}$$

where $x_i(t)$ is the received signal at the ith receiver, $s(t)$ is the noise-free complex envelope stationary random signal, $q_i(t)$ is the additive white Gaussian noise (AWGN), and it is assumed that the noises are independent identically distributed (IID) at different receivers, τ_i denotes the propagation delay from the emitter to the ith receiver. Time difference $\tau_{i,j}$, which is needed for the hyperbolic localization of the emitter, is the unknown parameter to be estimated, and is given by

$$\tau_{i,j} \stackrel{\Delta}{=} \tau_i - \tau_j, i \neq j. \tag{2}$$

In the traditional time difference measurement methods, time difference $\tau_{i,j}$ is estimated using $x_i(t)$ and $x_j(t)$. Herein, $\tau_{i,j}$ is estimated using all received signals.

3. TDOA Estimation Based on Blind Beamforming

It is assumed that the observed signal is partitioned into N non-overlapping blocks in the time domain. According to signal model (1), the discrete Fourier transform (DFT) of the nth block of the observed signal at the ith receiver is given by

$$\tilde{x}_{i,k}(n) = \tilde{s}_k(n)e^{-j\frac{2\pi k}{K}\tau_i} + \tilde{q}_{i,k}(n) \tag{3}$$

where k and n denote the kth frequency bin and the nth block data with $k \in \{1, \cdots, K\}$ and $n \in \{1, 2, \cdots, N\}$, respectively. The superscript $\tilde{}$ of $\tilde{u}$ denotes the Fourier coefficients of u. If we define the vectors

$$\tilde{\mathbf{x}}_k(n) \stackrel{\Delta}{=} [\tilde{x}_{1,k}(n), \tilde{x}_{2,k}(n), \tilde{x}_{3,k}(n)]^T \tag{4}$$

$$\tilde{\mathbf{q}}_k(n) \stackrel{\Delta}{=} [\tilde{q}_{1,k}(n), \tilde{q}_{2,k}(n), \tilde{q}_{3,k}(n)]^T \tag{5}$$

with superscript ^{T} denoting transpose, the frequency domain vector form of signal model could be presented as

$$\tilde{\mathbf{x}}_k(n) = \mathbf{a}_k(\boldsymbol{\tau})\tilde{s}_k(n) + \tilde{\mathbf{q}}_k(n) \tag{6}$$

where

$$\mathbf{a}_k(\boldsymbol{\tau}) = \left[e^{-j\frac{2\pi k}{K}\tau_1}, e^{-j\frac{2\pi k}{K}\tau_2}, e^{-j\frac{2\pi k}{K}\tau_3}\right]^T \tag{7}$$

is the steering vector, and $\boldsymbol{\tau} \stackrel{\Delta}{=} [\tau_1, \tau_2, \tau_3]^T$. Note that, model (6) is similar to that of narrowband array signal [10]. Due to the unknown of parameters $\boldsymbol{\tau}$, the blind beamforming based on MPC [6] may be formulated as a maximization with a constraint, i.e.,

$$\max_{\mathbf{w}_k} \left\{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k\right\}, \ s.t. \ \|\mathbf{w}_k\| = 1 \tag{8}$$

where $\mathbf{R}_k \stackrel{\Delta}{=} E\{\tilde{\mathbf{x}}_k(n)\tilde{\mathbf{x}}_k^H(n)\}$ with superscript ^{H} denoting Hermitian transpose. $E\{\cdot\}$ denotes taking expectation, $\|\cdot\|$ denotes 2-norm and $\mathbf{w}_k$ is the unknown beamforming weight vector at the kth frequency bin. Since $\mathbf{R}_k$ is not related with n due to the stationary random assumption, n is omitted in the expectation operations of the following derivation.

In order to estimate the time difference $\tau_{i,j}$, the jth received signal is considered as reference signal. With the introduction of a further constraint $\Im\{[\mathbf{w}_k]_j\} = 0$, the optimization problem of (8) may be given by

$$\max_{\mathbf{w}_k} \left\{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k\right\}, \ s.t. \ \left\{ \begin{array}{l} \|\mathbf{w}_k\| = 1 \\ \Im\{[\mathbf{w}_k]_j\} = 0 \end{array} \right. \tag{9}$$

where $\Im\{\cdot\}$ denotes taking imaginary-part.

In order to reveal the relationship between the weight coefficients of the beamformer and the time differences, we first consider rewriting the unconstrained objective function as

$$\begin{aligned} \mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k &= \mathbf{w}_k^H E\{\tilde{\mathbf{x}}_k \tilde{\mathbf{x}}_k^H\} \mathbf{w}_k \\ &= \mathbf{w}_k^H \mathbf{a}_k \sigma_{\tilde{s}_k}^2 \mathbf{a}_k^H \mathbf{w}_k + E\{\tilde{\mathbf{q}}_k \tilde{\mathbf{q}}_k^H\} \|\mathbf{w}_k\|^2 \end{aligned} \tag{10}$$

where $\sigma_{\tilde{s}_k}^2 \stackrel{\Delta}{=} E\{\tilde{s}_k \tilde{s}_k^*\}$ is the power of signal at the k frequency bin. Herein, superscript * denotes conjugate. Under the IID assumption of AWGN, $E\{\tilde{\mathbf{q}}_k \tilde{\mathbf{q}}_k^H\}$ can be given as $\sigma_q^2 \mathbf{I}$ where $\mathbf{I}$ denotes identity matrix. Aware that both $\sigma_{\tilde{s}_k}^2$ and σ_q^2 are constant, we could further simplify the optimization as

$$\max_{\mathbf{w}_k} \left\{\mathbf{w}_k^H \mathbf{a}_k \mathbf{a}_k^H \mathbf{w}_k\right\}, \ s.t. \ \left\{ \begin{array}{l} \|\mathbf{w}_k\| = 1 \\ \Im\{[\mathbf{w}_k]_j\} = 0 \end{array} \right. . \tag{11}$$

According to the Cauchy-Schwartz inequality, we can get

$$\mathbf{w}_k^H \mathbf{a}_k \mathbf{a}_k^H \mathbf{w}_k \leq \|\mathbf{w}_k\|^2 \|\mathbf{a}_k\|^2 \tag{12}$$

and the equality holds when

$$\mathbf{w}_k = c_k \cdot \mathbf{a}_k \tag{13}$$

where c_k is a complex constant. Furthermore, according to the constraint condition, c_k can be given by

$$c_k = \frac{[\mathbf{a}_k^H]_j}{\|\mathbf{a}_k\|} = \frac{1}{\sqrt{3}} e^{j\frac{2\pi k}{K}\tau_j}. \tag{14}$$

On the other hand, due to (9), the optimum weight vector $\mathbf{w}_k$ could be obtained by

$$\hat{\mathbf{w}}_k = \frac{\mathbf{w}_k^o / [\mathbf{w}_k^o]_j}{\left\| \mathbf{w}_k^o / [\mathbf{w}_k^o]_j \right\|} \tag{15}$$

where $\mathbf{w}_k^o$ is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_k$. In practice, $\mathbf{R}_k$ is usually replaced by the time-averaged counterpart $\hat{\mathbf{R}}_k$ which could be calculated by

$$\hat{\mathbf{R}}_k = \frac{1}{N} \sum_{n=1}^{N} \tilde{\mathbf{x}}_k(n) \tilde{\mathbf{x}}_k^H(n). \tag{16}$$

In model (13), the estimated parameter is implicit in the equation's right, thus we consider $\mathbf{w}_k$ as the observed data for the time difference estimation. Moreover, by considering the estimation error of $\mathbf{w}_k$, model (13) is rewritten as

$$\hat{\mathbf{w}}_k = c_k \cdot \mathbf{a}_k + \mathbf{v}_k \tag{17}$$

where $\hat{\mathbf{w}}_k$ is obtained by (15), and $\mathbf{v}_k$ is the model error vector. Rearrange the weight vector as

$$\mathbf{y}_i \triangleq [[\hat{\mathbf{w}}_1]_i, \cdots, [\hat{\mathbf{w}}_k]_i, \cdots, [\hat{\mathbf{w}}_K]_i]^T. \tag{18}$$

According to (17) and (18), the kth element of $\mathbf{y}_i$ is

$$y_{i,k} = \frac{1}{\sqrt{3}} e^{\frac{-j2\pi k \tau_{i,j}}{K}} + u_{i,k} \tag{19}$$

where $u_{i,k}$ denotes the kth element of the rearranged error vector $\mathbf{u}_i \triangleq [[\hat{\mathbf{v}}_1]_i, \cdots, [\hat{\mathbf{v}}_k]_i, \cdots, [\hat{\mathbf{v}}_K]_i]^T$. Thus the time difference can be given by location corresponding to the peak of the frequency spectrum of $\mathbf{y}_i$, i.e.,

$$\hat{\tau}_{i,j} = \arg\max_{\tau} \{ |\mathbf{f}^H(\tau)\mathbf{y}_i| \} \tag{20}$$

where

$$\mathbf{f}(\tau) = \left[e^{-j\frac{2\pi}{K}\tau}, \cdots, e^{-j\frac{2\pi k}{K}\tau}, e^{-j\frac{2\pi K}{K}\tau} \right]^T \tag{21}$$

and $|\cdot|$ denotes complex module operation. Through (20), we could find that the estimated time difference corresponding to the peak coordinate of the frequency spectrum of $\mathbf{y}_i$. The diagram of TDOA estimation based on blind beamforming is given in Fig. 1. The proposed algorithm is summarized in Algorithm 1.

Algorithm 1 Algorithm steps

1: Partition the received signal into N blocks, then use discrete Fourier transform to get $\tilde{x}_{i,k}(n)$ for $n = 1, 2, \cdots, N$.
2: For $k = 1, 2, \cdots, K$, compute $\hat{\mathbf{R}}_k$ according to (16), then implement the eigenvalue decomposition of $\hat{\mathbf{R}}_k$, and calculate $\hat{\mathbf{w}}_k$ according to (15).
3: Rearrange the weight vector according to (18).
4: Compute time difference $\hat{\tau}_{i,1}$ according to (20).

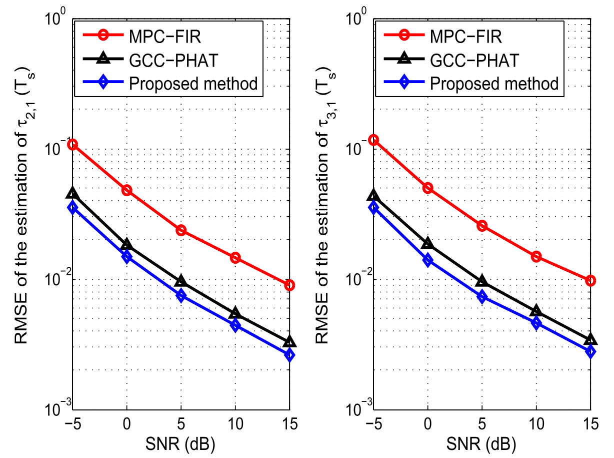

Fig. 2 The accuracy of TDOA estimation under single path environment

4. Simulation Results

In the following experiments, the observed signal is produced by mixing and sampling of BPSK signal, where the BPSK symbols are selected at random. The observed signals are sampled at the Nyquist rate. In addition, we assume that the digital baseband noise is white Gaussian. These assumptions are identical in all simulations. In Monte Carlo simulations, the root mean square error (RMSE) is obtained by 500 independent trials. The signal to noise ratio (SNR) is defined as the ratio between the power of the direct wave of signal and the power of additional white Gaussian noise. In order to save space, only the estimation performance of $\tau_{2,1}$ and $\tau_{3,1}$ are shown in this section. We consider some specific cases to illustrate the behavior of the proposed method.

Case A: In single-path environment, the proposed algorithm's estimation accuracy of time difference is shown in Fig. 2. And it is compared with the MPC-FIR [6] and GCC-PHAT [9]. In this experiment, the propagation delays are set as $20T_s$,$22T_s$,$23T_s$ for the three direct wave signals respectively, where T_s denotes the sampling period. We assume that the corresponding amplitude coefficients are inversely proportional to the propagation delays. It's obvious that the proposed method outperforms both MPC-FIR and GCC-PHAT. GCC-PHAT method is considered as the optimal method for the time difference estimation using reference signal and auxiliary signal. However, all the received signals are used for the time difference estimation in the proposed method. For example, in order to estimate $\tau_{2,1}$, $x_1(t)$

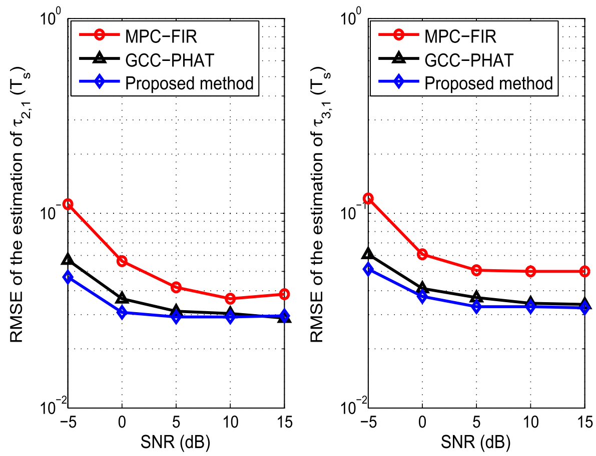

Fig. 3 The accuracy of TDOA estimation under multipath environment

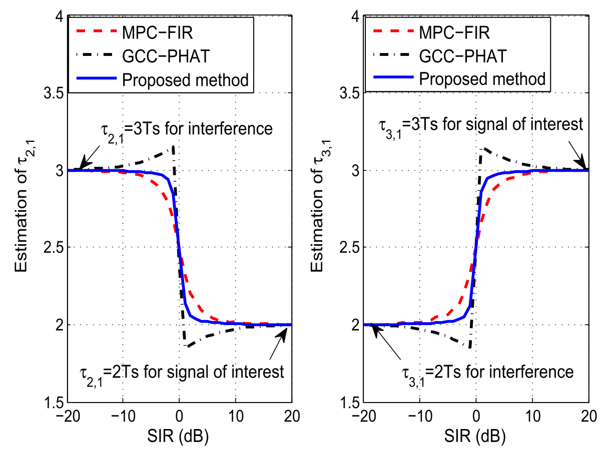

Fig. 5 Mean of TDOA estimation vs. SIR

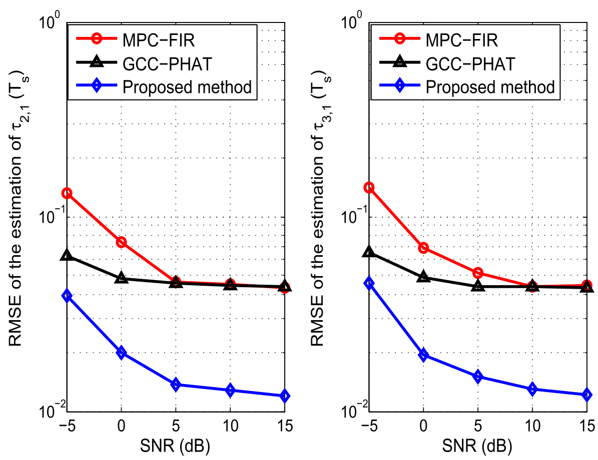

Fig. 4 The accuracy of TDOA estimation under interference environment

and $x_2(t)$ are used in GCC-PATH, yet $x_1(t)$, $x_2(t)$ and $x_3(t)$ are used in the proposed method. Although $\tau_{2,1}$ is not related to $x_3(t)$, the SNR can be improved when $x_3(t)$ is used in the beamforming. Therefore, the proposed method is better than GCC-PHAT.

Case B: In multipath environment, the estimation accuracy of the time difference of the proposed method is shown in Fig. 3. In this experiment, it is assumed that the 1st observed signal contains only direct wave with delay $20T_s$, and the 2nd observed signal contains two propagation paths with corresponding delays $[22, 28]T_s$, and the 3rd observed signal contains three propagation paths with corresponding delays $[23, 29, 35]T_s$. The corresponding amplitude coefficients are 1, $[1, 0.6]$, $[1, 0.6, 0.3]$ for all propagation paths of the three observed signals respectively. The simulation result shows that the proposed algorithm exhibits obviously better performance than MPC-FIR over all SNRs, and outperforms GCC-PHAT in low SNRs.

Case C: In interference environment, the estimation accuracy of the time difference of the proposed method is shown in Fig. 4. The interference signal is selected to be BPSK signal, and the power spectral features of the interference signal is identical to those of the interested signal, but the BPSK symbols are different from each other. It is assumed that the interesting and interference signals propagate in direct path, and the delays from emitter to three receivers are $20T_s$, $22T_s$, $23T_s$, respectively. The delays from interference source to three receivers are $28T_s$, $31T_s$, $30T_s$. The received signals are identical amplitude for three receivers. And the ratio of the amplitude of the interference to signal is 0.4. The proposed algorithm improves the time difference estimation performance due to significant interference suppression.

Case D: Fig. 5 shows the mean of the estimated TDOA as a function of the signal power to interference power ratio (SIR), where the prorogation delays are the same as *Case C*. Herein, SIR is defined as the ratio between the power of the interested signal and the power of the interference. When the SIR is less than 0dB, the interference's power is stronger than the interested signal's, therefore the interference would be mistaken as interested signal, and corresponding time differences would still been calculated. The simulation result shows that the estimated TDOA uncertainty region of the proposed method is obviously narrower than those of the other algorithms. Furthermore, the performance of the proposed method in strong interference environments is also better than the other two.

5. Conclusion

The TDOA estimator based on blind beamforming is developed for the wideband emitter localization. And the relationship between the weight coefficients of blind beamforming and the time differences is revealed. Finally, simulation results verify that the proposed estimator behaves well under either multipath propagation or interference environments.

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