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#### Abstract

SUMMARY In this paper, we consider the problem of generating uniformly random mosaic floorplans. We propose a polynomial-time algorithm that generates such floorplans with $f$ faces. Two modified algorithms are created to meet additional criteria.


key words: random generation, algorithm, floorplan, mosaic floorplan, classification tree

## 1. Introduction

It is useful to have a way to uniformly and randomly generate (or sample) objects in a specified class. For instance, uniformly random generators have been proposed for trees [2], triangulations [10], and bipartite permutation graphs [11]. In this paper, we design an algorithm for generating uniformly random mosaic floorplans with $f$ faces.

A mosaic floorplan partitions a rectangle, called the boundary, into smaller rectangles, called faces. Each face includes its boundary. Some examples are shown in Fig. 1. Mosaic floorplans are one of basic models used for very-large-scale integration (VLSI) design [7], [8]. The number of mosaic floorplans with $f$ faces is known [12], and there is a one-to-one correspondence between mosaic floorplans and Baxter permutations [1], [5]. Also, the number of mosaic floorplans with various properties is known [1]: more precisely, there is an exact formula for the number of mosaic floorplans with $f$ faces, $r$ maximal vertical line segments not on the boundary, $n$ n-touch faces and $w$ w-touch faces, where an n-touch face and a w-touch face share a line segment with the uppermost horizontal line segment and the leftmost vertical line segment, respectively.

Our idea for a random generation algorithm is as follows. First, we define a tree $T_{f}$, called the classification tree, in which (1) each leaf in the tree corresponds to a distinct mosaic floorplan, and (2) each vertex $v$ in the tree corresponds to the set of mosaic floorplans corresponding to the leaves in the subtree rooted at $v$; see Fig. 5. The mosaic floorplans corresponding to each vertex are partitioned into subsets, each of which corresponds to a child of the vertex. Thus, each path from the root to a leaf corresponds to

[^0]a distinct mosaic floorplan with $f$ faces. If we can choose such paths uniformly and randomly, then we can generate uniformly random mosaic floorplans. We can make this uniformly random selection as follows. Assume that we are now at vertex $v$ in $T_{f}$, and assume that $v$ has children $c_{1}, c_{2}, \ldots$. Which child should we choose as the next vertex of the path? We first compute the number, say $M(v)$, of leaves in the subtree rooted at $v$, and for each $i$, the number, say $M\left(c_{i}\right)$, of leaves in the subtree rooted at $c_{i}$. Then, with probability $M\left(c_{i}\right) / M(v)$, we choose each child $c_{i}$.

In this paper, we first propose an algorithm that generates a mosaic floorplan with $f$ faces in $\mathrm{O}\left(f^{2}\right)$ time. With a similar idea but with modified classification trees, we present two algorithms that generate mosaic floorplans with additional specified properties.

An algorithm can be designed to generate a uniformly random mosaic floorplan with $f$ faces in linear time; this can be done by generating uniformly random "watermelon structures" [4], which have a bijection into mosaic floorplans [6]. However, by slightly modifying our algorithm, we can also generate mosaic floorplans with various additional properties that are not possible with the linear-time method.

We modify our algorithm to create two additional algorithms that add additional criteria: one generates floorplans that include $f_{N}$ n-touch faces, and the other generates floorplans that include $f_{N}$ n-touch faces and $f_{W}$ w-touch faces. Designating the number of faces that share a line segment with the boundary of a mosaic floorplan arises naturally from VLSI applications. A mosaic floorplan can be used as a model of a circuit, and each face corresponds to a module of the circuit. If a module shares a boundary with the circuit, then it can connect to outside devices. Hence, designating the number of faces sharing line segments with the boundary of a mosaic floorplan corresponds to designating the number of such modules.

The structure of the paper is as follows. Section 2 gives some definitions. Section 3 defines the classification tree. Section 4 presents our first random generation algorithm, and Sect. 5 presents the modified algorithms for floorplans with additional specified properties. Section 6 presents our conclusions. See [13] for a preliminary version of this paper.

## 2. Definitions

In this section we give some definitions.
A mosaic floorplan is a partitioning of a rectangle, called the boundary, into smaller rectangles, called faces.

Some examples are shown in Fig. 1.
Two mosaic floorplans, $M_{1}$ and $M_{2}$, are isomorphic if there exists a one-to-one correspondence between their faces, a one-to-one correspondence between their maximal vertical line segments, and a one-to-one correspondence between their maximal horizontal line segments, such that the set of faces on either side of each vertical line segment and on either side of each horizontal line segment are preserved. For example, the three mosaic floorplans shown in Fig. 1 are isomorphic. Intuitively, mosaic floorplans are isomorphic if and only if they can be converted into each other by sliding some maximal vertical and horizontal line segments, and if doing so preserves the set of faces located on either side of each vertical line segment and on either side of each horizontal line segment.

We assume that no vertices of degree four appear in any mosaic floorplan. A vertex of degree three is w-missing (west missing) if it has line segments to the top, bottom, and right; e-missing (east missing), n-missing (north missing), and $s$-missing (south missing) are defined similarly.

We define a floorplan for each set of isomorphic mosaic floorplans, as follows. A mosaic floorplan is a canonical floorplan if any s-missing vertex appears to the left of any n -missing vertex on any horizontal line segment, and any emissing vertex appears above any w-missing vertex on any vertical line segment. For instance, the mosaic floorplan in Fig. 1(c) is a canonical floorplan.

Let $C$ be a canonical floorplan with $f>1$ faces. The face of $C$ that coincides with the upper-left corner of the boundary is called the first face of $C$. In Figs. 2-4, the first faces are shaded. Let $v$ be the lower-right corner vertex of the first face $F$ of $C$. If $v$ is e-missing, as shown in Fig. 2(a), then by continually shrinking the first face $F$ into the uppermost horizontal line of $C$ while preserving the width of $F$ and enlarging the faces below $F$, as shown in Fig. 3, we can obtain a canonical floorplan with one fewer face. Thus, if $v$ is e-missing, we say that the first face $F$ is upward removable. If $v$ is s-missing, as shown in Fig. 2(b), then by continually shrinking the first face $F$ into the leftmost verti-


Fig. 1 Examples of mosaic floorplans.
cal line of $C$, while preserving the height of $F$ and enlarging the faces located to the right of $F$, we can obtain a canonical floorplan with one fewer face. Thus, if $v$ is s-missing, we say that $F$ is leftward removable. So, if $f>1$, then $F$ is either upward removable or leftward removable. In either case, let $P(C)$ be the floorplan derived from $C$ by removing the first face of $C$, as above. Note that $P(C)$ is also a canonical floorplan.

Given a canonical floorplan $C$, by repeatedly removing the first face of the derived canonical floorplan, we create a sequence $C, P(C), P(P(C)), \ldots$ of canonical floorplans that eventually converge to a canonical floorplan with exactly one face. An example of this is shown in Fig. 4. We call this the removing sequence of $C$. Note that each canonical floorplan has a unique removing sequence.

Let $R S=\left(C_{f}=C, C_{f-1}, \ldots, C_{1}\right)$ be the removing sequence of a canonical floorplan $C$. We define a label $L\left(C_{i}\right)$ for each $C_{i}$ with $i>1$ in $R S$, such that $L\left(C_{i}\right)$ explains how the first face of $C_{i}$ is removed to create $C_{i-1}$. Let $F_{i}$ be the first face of $C_{i}$. If $F_{i}$ is upward removable, and there are $s$ faces located to the south of $F_{i}$, then we define $L\left(C_{i}\right)=(\mathrm{U}, s)$. Otherwise, $F_{i}$ is leftward removable, and if there are $e$ faces located to the east of $F_{i}$, then we define $L\left(C_{i}\right)=(\mathrm{L}, e)$. We call $L\left(C_{i}\right)$ the removing label of $C_{i}$. The first $k$ labels of $C$ are the sequence of $k$ removing labels $\left(L\left(C_{f}\right), L\left(C_{f-1}\right), \ldots, L\left(C_{f-k+1}\right)\right)$. For example, the first five labels of the leftmost canonical floorplan $C$ in Fig. 4 are ((L, 3), (U, 2), (L, 1), (U, 1), (L, 2)). For each canonical floorplan, the set of the first $f-1$ labels is unique.

Let $f_{U}$ and $f_{L}$, respectively, be the number of upwardremovable faces and the number of leftward-removable faces in the removing sequence of $C$. Let $e_{v}$ and $e_{h}$, respec-

(a)

(b)

Fig. 2 (a) An upward-removable face, and (b) a leftward-removable face.


Fig. 3 Removing the first face.


Fig. 4 The removing sequence.


Fig. 5 The classification tree $T_{4}$
tively, be the number of maximal vertical line segments and the number of maximal horizontal line segments, excluding the contour of the boundary of $C$. Then $f_{U}=e_{h}$ and $f_{L}=e_{v}$ hold, and so does $e_{v}+e_{h}=f-1$.

## 3. Classification Tree

In this section, we define a tree $T_{f}$, called the classification tree, that is related to the canonical floorplans with $f$ faces. In the next section, we will present our main algorithm, which is based on this tree.

Each leaf in the classification tree corresponds to a distinct canonical floorplan, and each vertex $v$ with depth $d$ in the classification tree corresponds to the set of canonical floorplans that correspond to the leaves in the subtree rooted at $v$ and sharing the first $d$ labels. The classification tree $T_{4}$ is shown in Fig. 5. . The root of the classification tree corresponds to the set of all canonical floorplans that have $f$ faces and share the first 0 label.

Now we explain how to compute the number of leaves in the subtree rooted at a given vertex.

A face $F$ of a floorplan $C$ is said to be $n$-touch if $F$ shares a line segment with the uppermost horizontal line segment of $C$. Similarly, a face $F$ of a floorplan $C$ is said to be $w$-touch if $F$ shares a line segment with the leftmost vertical line of $C$.

Lemma 1 ([1]): Let $C(f, r)$ be the set of canonical floorplans with $f$ faces and $r$ maximal vertical line segments not on the boundary, and let $C(f, n, w, r)$ be the set of canonical floorplans with $f$ faces, $r$ maximal vertical line segments not on the boundary, $n$ n-touch faces, and $w$ w-touch faces. Then the following equations hold.

$$
|C(f, r)|=\frac{\binom{f+1}{r}\binom{f+1}{r+1}\binom{f+1}{r+2}}{\binom{f+1}{1}\binom{f+1}{2}}
$$



Fig. 6 A canonical floorplan $C$ with the first three labels ((U, 3), (U, 2), (L, 4)) and the first four floorplans in its removing sequence.

$$
\begin{array}{r}
|C(f, n, w, r)|=\binom{f+1}{r+1} \frac{w n}{f(f+1)}\left(\binom{f-n-1}{f-r-2}\binom{f-w-1}{r-1}\right. \\
\left.-\binom{f-n-1}{f-r-1}\binom{f-w-1}{r}\right)
\end{array}
$$

Given a vertex $v$ in the classification tree, now we can calculate the number of corresponding canonical floorplans (sharing some first labels), as follows. We start with an example. Let $f=20$, and let $v$ at depth three correspond to the set of canonical floorplans sharing the first three labels $((\mathrm{U}, 3),(\mathrm{U}, 2),(\mathrm{L}, 4))$. Then, each canonical floorplan shares the same graph structure around the upper-left corner, as shown in Fig. 6, and removing the first three faces, as in the removing sequence, results in a distinct canonical floorplan with 17 faces, including at least three $n$-touch faces and at least four w-touch faces. Note that if the resulting canonical floorplan has two or fewer n-touch faces, then the first three labels will never be ((U, 3), (U, 2), (L, 4)). Conversely, for each canonical floorplan with 17 faces that includes at least three n-touch faces and at least four w-touch faces, if three faces are added in the reverse of the removing sequence, then this results in a distinct canonical floorplan with 20 faces sharing the first three labels ( $(\mathrm{U}, 3),(\mathrm{U}, 2),(\mathrm{L}, 4))$. We can generalize this example. Let $S\left(f, L_{k}\right)$ be the set of canonical floorplans with $f$ faces sharing the first $k$ labels $L_{k}$. Let $n\left(f, L_{k}\right)$ be the minimum number of n -touch faces in the canonical floorplans derived by removing the first $k$ faces from a canonical floorplan in $S\left(f, L_{k}\right)$. We define $w\left(f, L_{k}\right)$
similarly. Note that if a floorplan has $x$ faces, then the maximum number of maximal vertical line segments not on the boundary is $x-1$. We have the following equation.

$$
\begin{equation*}
\left|S\left(f, L_{k}\right)\right|=\sum_{r=0}^{f-k-1} \sum_{n=n\left(f, L_{k}\right)}^{f-k} \sum_{w=w\left(f, L_{k}\right)}^{f-k}|C(f-k, n, w, r)| \tag{1}
\end{equation*}
$$

## 4. Algorithm

In this section, we present the first of our random generation algorithms for mosaic floorplans. We compute a path from the root to a leaf in the classification tree, without constructing the entire tree. We repeatedly choose the next vertex of the path from among the children of the current vertex, so that each leaf has an equal chance of being reached. In this way, we can generate uniformly random mosaic floorplans.

Our algorithm is shown as Algorithm 1. The next vertex of the path is randomly chosen from among the children of the current vertex. $S(f, \varepsilon)$, which corresponds to the root of the classification tree, is the argument for the algorithm; the next random vertex is found recursively.

```
Algorithm 1: Find-Child \(\left(S\left(f, L_{k}\right)\right)\)
    begin
        \(S\left(f, L_{k}\right)\) is the set of canonical floorplans with \(f\) faces
        sharing the first \(k\) labels \(L_{k}\)
        if \(k=f-1\) then
            return \(S\left(f, L_{k}\right) \quad / * S\left(f, L_{k}\right)\) has exactly one
            canonical floorplan. */
        else
            Let \(S\left(f, L_{k+1}^{1}\right), S\left(f, L_{k+1}^{2}\right), \ldots, S\left(f, L_{k+1}^{d}\right)\) be a partition
            of \(S\left(f, L_{k}\right)\), where \(L_{k}\) is the common prefix of
            \(L_{k+1}^{1}, L_{k+1}^{2}, \ldots, L_{k+1}^{d}\).
            Uniformly and randomly generate an integer \(x\) in
            \(\left[1,\left|S\left(f, L_{k}\right)\right|\right]\).
            Choose the minimum \(j\) such that \(x \leq \sum_{i=1}^{j}\left|S\left(f, L_{k+1}^{i}\right)\right|\)
            \(\operatorname{Find}-\operatorname{Child}\left(S\left(f, L_{k+1}^{j}\right)\right)\)
```

Assume that we are now at a vertex $v$ in the classification tree, and assume that $v$ corresponds to the canonical floorplans $S\left(f, L_{k}\right)$ that share the first $k$ labels $L_{k}$. Assume also that after removing $k$ faces from the canonical floorplans in $S\left(f, L_{k}\right)$, the minimum number of n-touch faces is $n\left(f, L_{k}\right)$, and the minimum number of w-touch faces is $w\left(f, L_{k}\right)$. Each child $c_{i}, i=1,2, \ldots, d$, of $v$ in the classification tree corresponds to the set of canonical floorplans $S\left(f, L_{k+1}^{i}\right)$ for some $L_{k+1}^{i}$ that shares the first $k+1$ labels $L_{k+1}^{i}$. Since $L_{k}$ is the common prefix of $L_{k+1}^{1}, L_{k+1}^{2}, \ldots, L_{k+1}^{d}$, then $S\left(f, L_{k+1}^{1}\right), S\left(f, L_{k+1}^{2}\right), \ldots, S\left(f, L_{k+1}^{d}\right)$ is a partition of $S\left(f, L_{k}\right)$. Algorithm 1 computes a uniformly random value, say $x$, in $\left[1,\left|S\left(f, L_{k}\right)\right|\right]$, and it chooses the minimum $j$ with $x \leq \sum_{i=1}^{j}\left|S\left(f, L_{k+1}^{i}\right)\right|$. It then recursively computes $S\left(f, L_{k+1}^{j}\right)$. To randomly compute a path, we need to compute $\left|S\left(f, L_{k}\right)\right|$ and $\left|S\left(f, L_{k+1}^{1}\right)\right|,\left|S\left(f, L_{k+1}^{2}\right)\right|, \ldots,\left|S\left(f, L_{k+1}^{d}\right)\right|$.

For the root, we can use Lemma 1 to compute $\left|S\left(f, L_{0}\right)\right|=\sum_{r=0}^{f-1}|C(f, r)|$. Now, $n\left(f, L_{0}\right)=$ $w\left(f, L_{0}\right)=1$ holds. If we assume that we know $\left|S\left(f, L_{k}\right)\right|, \quad n\left(f, L_{k}\right)$, and $w\left(f, L_{k}\right)$, then we compute $\left|S\left(f, L_{k+1}^{1}\right)\right|,\left|S\left(f, L_{k+1}^{2}\right)\right|, \ldots,\left|S\left(f, L_{k+1}^{d}\right)\right|$ as shown below. Note that we only need to know $n\left(f, L_{k+1}^{i}\right)$ and $w\left(f, L_{k+1}^{i}\right)$, since if we have them, then we can use Eq. (1) to compute $\left|S\left(f, L_{k+1}^{i}\right)\right|$. Assume that the $(k+1)$-th label of $L_{k+1}^{i}$ is $\left(\ell_{1}, \ell_{2}\right)$. We have the following two cases.

Case 1: $\ell_{1}=\mathrm{U}$
Let $F_{k+1}$ be the first face of the canonical floorplan $C_{f-k}$ that is derived from some canonical floorplan in $S\left(f, L_{k+1}^{i}\right)$ by removing $k$ faces. Since $\ell_{1}=\mathrm{U}, F_{k+1}$ is upward removable. Hence, the minimum number $n\left(f, L_{k+1}^{i}\right)$ of n-touch faces of a canonical floorplan in $S\left(f, L_{k+1}^{i}\right)$ is $n\left(f, L_{k}\right)+\ell_{2}-1$. Thus, we have $n\left(f, L_{k+1}^{i}\right)=n\left(f, L_{k}\right)+\ell_{2}-1$. Also, if $w\left(f, L_{k}\right)>1$, then $w\left(f, L_{k+1}^{i}\right)=w\left(f, L_{k}\right)-1$ holds; otherwise, $w\left(f, L_{k}\right)=1$, and $w\left(f, L_{k+1}^{i}\right)=1$ holds.

Case 2: $\ell_{1}=\mathrm{L}$
Similarly, we have $w\left(f, L_{k+1}^{i}\right)=w\left(f, L_{k}\right)+\ell_{2}-1$. Also, if $n\left(f, L_{k}\right)>1$, then $n\left(f, L_{k+1}^{i}\right)=n\left(f, L_{k}\right)-1$ holds; otherwise, $n\left(f, L_{k}\right)=1$, and $n\left(f, L_{k+1}^{i}\right)=1$ holds.

Thus, we can compute $n\left(f, L_{k+1}^{i}\right)$ and $w\left(f, L_{k+1}^{i}\right)$ in constant time. Also note that $\ell_{2}<f$ holds. Thus, the maximum number of children is at most $2 f-2$.

We have the following theorem.
Theorem 1: After preprocessing in polynomial time, our algorithm generates each uniformly random mosaic floorplan in $\mathrm{O}\left(f^{2}\right)$ time.
Proof. For the preprocessing phase, we compute all possible values of $\left|S\left(f, L_{k}\right)\right|$. Specifically, we compute the righthand side of Eq. (1) for all $f^{\prime}\left(1 \leq f^{\prime} \leq f\right), k^{\prime}\left(1 \leq k^{\prime} \leq f^{\prime}\right)$ $n^{\prime}\left(f, L_{k}\right)\left(1 \leq n^{\prime}\left(f, L_{k}\right) \leq f-k\right), w^{\prime}\left(f, L_{k}\right)\left(1 \leq w^{\prime}\left(f, L_{k}\right) \leq\right.$ $f-k)$, and $r^{\prime}\left(1 \leq r^{\prime} \leq f\right)$. This computation can be done in polynomial time. By using this table, we can obtain $\left|S\left(f, L_{k}\right)\right|$ in $\mathrm{O}(1)$ time. To choose a child in Algorithm 1, since the number of the children is at most $2 f$, we need to look up a value in the table at most $\mathrm{O}(f)$ times. We repeatedly choose a child $f-1$ times, so we need $\mathrm{O}\left(f^{2}\right)$ time for the entire algorithm.
Q.E.D.

## 5. Random Generation of Mosaic Floorplans with Particular Properties

We propose two algorithms for generating uniformly random mosaic floorplans with various particular properties.

The first algorithm generates mosaic floorplans with $f$ total faces, including exactly $f_{N}$ n-touch faces.

Similar to what we did in Sect. 3, we can define a classification tree $T_{f}^{f_{N}}$ that is related to the canonical floorplans for this set.

Each leaf in the classification tree corresponds to a distinct canonical floorplan, and each vertex $v$ with depth $d$
in the classification tree corresponds to the set of canonical floorplans that correspond to the leaves in the subtree rooted at $v$ and that share the first $d$ labels. For example, $T_{4}^{2}$ is shown in Fig. 7.

Let $S\left(f, f_{N}, L_{k}\right)$ be the set of canonical floorplans with $f$ total faces, including exactly $f_{N}$ n-touch faces and with the first $k$ labels $L_{k}$. Let $n\left(f, f_{N}, L_{k}\right)$ be the number of n -touch faces in a canonical floorplan derived by removing the first $k$ faces from a canonical floorplan in $S\left(f, f_{N}, L_{k}\right)$. Note that every such floorplan has exactly $n\left(f, f_{N}, L_{k}\right)$ n-touch faces. Let $w\left(f, f_{N}, L_{k}\right)$ be the number of w-touch faces in such floorplans. We can compute $n\left(f, f_{N}, L_{k}\right)$ and $w\left(f, f_{N}, L_{k}\right)$ in a manner similar to what we did in Sect. 4, but with a different initialization: $n\left(f, f_{N}, L_{k}\right)=f_{N}$. Now $\left|S\left(f, f_{N}, L_{k}\right)\right|$ can be calculated by using the following equation.

$$
\begin{align*}
& \left|S\left(f, f_{N}, L_{k}\right)\right|= \\
& \sum_{r=0}^{f-k-1} \sum_{w=w\left(f, f_{N}, L_{k}\right)}^{f-k}\left|C\left(f-k, n\left(f, f_{N}, L_{k}\right), w, r\right)\right| \tag{2}
\end{align*}
$$

Similar to what was done in the algorithm in Sect. 4, we can compute a uniformly random path in the classification tree $T_{f}^{f_{N}}$.

We have the following theorem.
Theorem 2: After a preprocessing phase that can be completed in polynomial time, our algorithm generates a uniformly random mosaic floorplan with $f$ total faces, including exactly $f_{N}$ n-touch faces, in $\mathrm{O}\left(f^{2}\right)$ time.

The second algorithm generates mosaic floorplans with $f$ total faces, including exactly $f_{N}$ n-touch faces and exactly $f_{W}$ w-touch faces.

Similar to what we did in Sect. 3, we can define a classification tree $T_{f}^{f_{N}, f_{W}}$ that is related to this set of canonical floorplans.

Each leaf in the classification tree corresponds to a distinct canonical floorplan, and each vertex $v$ with depth $d$ in the classification tree corresponds to the set of canonical floorplans that correspond to the leaves in the subtree rooted at $v$ and that share the first $d$ labels.

Let $S\left(f, f_{N}, f_{W}, L_{k}\right)$ be the set of canonical floorplans with $f$ total faces, including exactly $f_{N}$ n-touch faces and exactly $f_{W}$ w-touch faces, and with the first $k$ labels $L_{k}$.


Fig. 7 The classification tree $T_{4}^{2}$.

Let $n\left(f, f_{N}, f_{W}, L_{k}\right)$ be the number of n-touch faces in a canonical floorplan derived by removing the first $k$ faces from a canonical floorplan in $S\left(f, f_{N}, f_{W}, L_{k}\right)$. We define $w\left(f, f_{N}, f_{W}, L_{k}\right)$ similarly. We can compute $n\left(f, f_{N}, f_{W}, L_{k}\right)$ and $w\left(f, f_{N}, f_{W}, L_{k}\right)$ in a manner similar to what we did in Sect. 4, but with different initializations: $n\left(f, f_{N}, f_{W}, L_{k}\right)=$ $f_{N}$ and $w\left(f, f_{N}, f_{W}, L_{k}\right)=f_{W}$.

Now we can compute $\left|S\left(f, f_{N}, f_{W}, L_{k}\right)\right|$ by the following equation.

$$
\begin{align*}
& \left|S\left(f, f_{N}, f_{W}, L_{k}\right)\right| \\
& \quad=\sum_{r=0}^{f-k}\left|C\left(f-k, n\left(f, f_{N}, f_{W}, L_{k}\right), w\left(f, f_{N}, f_{W}, L_{k}\right), r\right)\right| \tag{3}
\end{align*}
$$

By using a method similar to the algorithm in Sect. 4, we can compute a uniformly random path in the classification tree $T_{f}^{f_{N}, f_{W}}$.

We have the following theorem.
Theorem 3: After a preprocessing phase that can be completed in polynomial time, our algorithm generates a mosaic floorplan with $f$ total faces, including exactly $f_{N}$ n-touch faces and exactly $f_{W}$ w-touch faces, in $\mathrm{O}\left(f^{2}\right)$ time.

## 6. Conclusions

We have designed an algorithm that generates a uniformly random mosaic floorplan in polynomial time. We propose two additional algorithms that generate a uniformly random mosaic floorplans with additional specified properties.

A rectangular drawing is a drawn graph in which every face is a rectangle. The two drawings in Fig. 1(b) and (c) are isomorphic as mosaic floorplans, but they are distinct as rectangular drawings. Can we generate uniformly random rectangular drawings?

Our random generation algorithms are related to enumeration algorithms. For example, if we traverse the classification tree in a depth-first manner instead of choosing uniformly random children, we obtain an enumeration algorithm for mosaic floorplans. Based on a similar idea, we have designed enumeration algorithms for floorplans [9]. Such enumeration algorithms based on tree traversing, called reverse search [3], can be modified to obtain random generation algorithms, if the number of descendants of each vertex of the tree can be calculated.

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