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# LETTER Accurate Target Motion Analysis from a Small Measurement Set Using RANSAC

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**SUMMARY** Most conventional research on target motion analysis (TMA) based on least squares (LS) has focused on performing asymptotically unbiased estimation with inaccurate measurements. However, such research may often yield inaccurate estimation results when only a small set of measurement data is used. In this paper, we propose an accurate TMA method even with a small set of bearing measurements. First, a subset of measurements is selected by a random sample consensus (RANSAC) algorithm. Then, LS is applied to the selected subset to estimate target motion. Finally, to increase accuracy, the target motion estimation is refined through a bias compensation algorithm. Simulated results verify the effectiveness of the proposed method.

key words: bearing only target motion analysis, RANSAC, least squares

#### 1. Introduction

Target motion analysis is a critical issue for the adoption of acoustic sensor based surveillance systems. Especially for moving platform scenarios, such as the passive sensors installed on ships, bearings are the only reliable form of acquirable information. Considering this constraint, bearing only target motion analysis (BOTMA), which estimates the trajectory of targets based on bearing measurements, has been actively researched over the last few decades [1]–[12].

Early research focused on recursive state estimation based on the Kalman filter (KF) which is limited to linear state/measurement models [1], [2]. To deal with nonlinear models, approaches such as extended KF (EFK) and unscented KF (UKF) have been developed [3]. However, due to the strong dependency of performance on initial motion settings, estimation results often diverge when scenarios are inaccurately initialized. Therefore, to avoid this divergence problem, a particle filter (PF) was employed [4]. Although the particle filter was capable of representing the uncertain initial motion with multiple particles, the computational load caused difficulties in achieving real time operation.

To improve the applicability of the method to real life situations, some studies have focused on accurately obtaining initial target motion. These studies find the best target motion parameter, and assume that the target moves constantly over a short period. Some approaches attempt to find

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the best parameter by using maximum likelihood (ML) estimators [5]. However, such methods often converge at local minima when implemented by iterative numerical search algorithms.

For scenarios applying the white-Gaussian noise assumption, the ML estimator can be simplified to a Least Square (LS) estimator. One advantage of the LS algorithm is that it has a closed form solution and is free of the previously mentioned convergence problem. However, in real noisy environments, LS estimators are vulnerable to a biased estimation problem and to overcome this problem, many new methods have recently been proposed [6]-[12]. In order to suppress the estimation bias, approaches based on an instrumental variable (IV) matrix have been proposed [7]-[9]. In addition, asymptotically unbiased estimation algorithms based on total LS (TLS) have also been proposed [10]–[12]. All the aforementioned methods achieve asymptotically unbiased estimation if a large set of measurements is obtained. However, in practice the demand for an immediate response in many maritime surveillance applications forces surveillance systems to make quick analyses of targets before sufficient measurements have been obtained.

This paper proposes a method of improving the performance of LS based estimations given the limitation of a small measurement dataset. First, a RANSAC algorithm is incorporated into the BOTMA application in order to determine a set of useful measurements. Then, target motion is estimated using the selected subset of measurements. Finally, to improve the accuracy of the estimation results, the estimated parameters are refined via an instrumental variable algorithm.

The rest of this paper is organized as follows: Sect. 2 includes details of BOTMA, while Sect. 3 describes the details of the proposed algorithm. Section 4 provides the experimental results obtained in various scenarios. Finally, conclusions are drawn in Sect. 5.

## 2. BOTMA Formulation

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As shown in Fig. 1, the considered acoustic sensor based surveillance system is composed of a moving passive sensor which obtains consecutive target bearing measurements.

At time k, a bearing measurement can be obtained by the following equation:

$$\theta_k = \tan^{-1} \left( \frac{p_{y,k} - r_{y,k}}{p_{x,k} - r_{x,k}} \right) + n_k \tag{1}$$

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Fig. 1 BOTMA geometry of an acoustic surveillance system.

where  $\theta_k$ ,  $\mathbf{p}_k = [p_{x,k}, p_{y,k}]$ ,  $\mathbf{r}_k = [r_{x,k}, r_{y,k}]$ ,  $n_k$  are the bearing measurement, target location, sensor location, and measurement noise, respectively, at time k. It is assumed that the sensor location is constantly obtained using a Global Positioning System (GPS). In addition, it is assumed that a target maneuvers according to a motion model with constant parameters. Position at time k is given by:

$$\mathbf{p}_k = \mathbf{F}_k \mathbf{x}_0 \tag{2}$$

where  $\mathbf{x}_0 = [\mathbf{p}_0, \mathbf{v}_0, \mathbf{a}_0]$  is a set of model parameters representing the initial position, velocity and acceleration of the target and  $\mathbf{F}_k$  follows the constant acceleration model as:

$$\mathbf{F}_{k} = \begin{bmatrix} 1 & 0 & kT & 0 & 0.5 \times (kT)^{2} & 0 \\ 0 & 1 & 0 & kT & 0 & 0.5 \times (kT)^{2} \end{bmatrix}$$
(3)

where T is the bearing measurement sample interval.

# 3. Proposed Method

#### 3.1 Selection of Measurements Based on RANSAC

RANSAC is a popular algorithm in the field of computer vision and typically applied to estimate the parameters of geometric transformations between images [13]. It allows the outliers of the transformation matrices to be discarded by iteratively searching for faulty matched points between two images. Likewise, in the BOTMA scenario, measurements with large errors are also regarded as outliers of the target motion model. Therefore, the purpose of applying RANSAC to BOTMA is to reduce outliers and enhance the performance of target motion estimation.

Figure 2 presents a flowchart of the proposed measurement selection method. The method is composed of a hypothesis generation process and an update process. The hypothesis generation process creates a hypothesis representing a possible target trajectory. In other words, at time k, m measurements are randomly selected from the acquired measurements  $\theta_1, \theta_2, \ldots, \theta_k$  that compose the trajectory hypothesis. Then, using those selected measurements, the model parameters of the hypothesis are calculated by least squares estimation. From these model parameters, a trajectory hypothesis can be constructed using Eq. (2).



Fig. 2 Flowchart of the proposed method

In the update process, the generated hypothesis is updated by comparing its inlier costs to those of the best hypothesis estimated by the previous iteration process. The set of inliers,  $S_{inlier}$ , is selected by the following equation:

$$S_{inlier} = \{\theta_i : ||\theta_i - \hat{\theta}_i|| < \theta_{th}\}$$
(4)

where  $\theta_{th}$  is a predefined constant for determining the inliers and  $\hat{\theta}_i$  is the re-estimated bearing of the generated trajectory at time *i*. The costs of the selected measurements are simply determined using the following equation:

$$C_{inlier} = |\mathbf{S}_{inliner}| \tag{5}$$

where  $|\mathbf{S}_{inliner}|$  represents the number elements in the set of inliers. If the total cost of the inliers of a generated hypothesis exceeds the current maximum cost, the set  $(\mathbf{S}_{best})$  and cost  $(C_{best})$  of inliers are updated. This hypothesis generation and update process is repeated until the number of iterations reaches a maximum iteration number *D* defined as:

$$D = \frac{\log(1 - p_{inlier})}{\log(1 - \alpha^{m_k})}$$
(6)

where  $\alpha$  is the ratio of the number of inliers to the number of acquired bearing measurements and  $p_{inlier}$  is the probability of only selecting actual inlier measurement at least once in the random generation process [13], [14].

# 3.2 Refinement with Selected Measurements Based on Instrumental Variables

Instrumental Variable (IV) method is applied as shown

in Fig. 2 to improve the performance of the LS estimator using the selected subset of measurements ( $S_{best} = \{\theta_{s_1}, \theta_{s_2}, \dots, \theta_{s_L}\}$ ). The estimation based on the IV method is constructed as follows [7]:

$$\hat{\mathbf{x}}_0 = \left(\mathbf{G}^T \mathbf{A}\right)^{-1} \mathbf{G}^T \mathbf{b}$$
(7)

where **G** is the matrix of instrumental variables defined as the following:

$$\mathbf{G} = \begin{bmatrix} \hat{\mathbf{v}}_{s_1}^T \mathbf{F}_{s_1} \\ \hat{\mathbf{v}}_{s_2}^T \mathbf{F}_{s_2} \\ \vdots \\ \hat{\mathbf{v}}_{s_L}^T \mathbf{F}_{s_L} \end{bmatrix}, \quad \hat{\mathbf{v}}_{s_i} = \begin{bmatrix} \sin \hat{\theta}_{s_i} \\ -\cos \hat{\theta}_{s_i} \end{bmatrix}$$
(8)

where  $\hat{\theta}_{s_i}$  is the bearing angle re-estimated by the bias compensated LS estimator using the selected subset of measurements at time  $s_i$ . Then, **A** and **b** can be constructed by consecutive bearing measurements and the positions of the sensor as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{v}_{s_1}^T \mathbf{F}_{s_1} \\ \mathbf{v}_{s_2}^T \mathbf{F}_{s_2} \\ \vdots \\ \mathbf{v}_{s_L}^T \mathbf{F}_{s_L} \end{bmatrix}, \quad \mathbf{v}_{s_i} = \begin{bmatrix} \sin \theta_{s_i} \\ -\cos \theta_{s_i} \end{bmatrix}$$
(9)
$$\mathbf{b} = \begin{bmatrix} \mathbf{v}_{s_1}^T \mathbf{r}_{s_1} \\ \mathbf{v}_{s_2}^T \mathbf{r}_{s_2} \\ \vdots \\ \mathbf{v}_{s_L}^T \mathbf{r}_{s_L} \end{bmatrix}$$
(10)

# 4. Experimental Results

#### 4.1 Experimental Environment

As displayed in Fig. 3, the considered scenario is composed of a moving target and an observer. It is assumed that the target moves in a curved line  $\mathbf{p}_0 = [-160, 1550]$ ,  $\mathbf{v}_0 = [-0.8, -1.3]$ ,  $\mathbf{a}_0 = [0.005, -0.005]$ . In addition, the moving observer collects bearing measurements of the target featuring i.i.d. zero mean Gaussian noise and a sampling interval T = 1. The predefined parameters of the measurement selection algorithm are summarized in Table 1.

The effectiveness of the proposed algorithm is evaluated in terms of the root mean squared error (RMSE), which is defined by the following equation:

$$\mathbf{RMSE} = \sqrt{\frac{1}{N_s} \sum_{i=1}^{N_s} \|\mathbf{x}_0 - \hat{\mathbf{x}}_{0,i}\|}$$
(11)

Table 1 Detail parameters for RANSAC in the experiments

Parameter	Value
$\theta_{th}$	2°
$p_{\it inlier}$	0.99
α	0.7

where  $N_S$  is the total number of Monte Carlo runs,  $\mathbf{x}_0$  is the actual model parameter of the target and  $\hat{\mathbf{x}}_{0,i}$  is the model parameter of the target as estimated by the *i*-th iteration. Simulations were conducted using MATLAB with the Monte-Carlo simulation over 500 iterations.

#### 4.2 Discussion of Experimental Results

Figure 4 shows the time averaged RMSE of position against the number of randomly selected measurements (m) in the hypothesis generation process. The bearing noise standard deviation is set to 3 degrees. According to the results, setting a large number for *m* increases randomness and decreases performance of the proposed algorithm. On the other hand, setting an insufficient value for *m* generates inaccurate estimations for the hypothesis trajectory. Since the tipping point is m = 10, we set the value of *m* to 10 in all experiments hereafter.

Figure 5 shows the RMSE of the estimated initial



Fig. 3 Simulated surveillance scenario for BOTMA



Fig. 4 Time averaged RMSE of the estimated position against the number of randomly selected measurement



Fig. 5 RMSE of estimated model parameter, (a) position (b) velocity, over time



**Fig. 6** Time averaged RMSE of the estimated model parameter against standard deviations of noise, (a) position (b) velocity

position and velocity of the target. The bearing noise standard deviation is also set to 3 degrees. The red line represents the RMSE of the LS estimator, the green line the RMSE of the TLS estimator [11], the blue line the RMSE of the IV estimator [8] and the black line the RMSE of the proposed algorithm based on measurement selection. According to the results, the performance of the LS estimator is degraded due to severe bearing measurement noise. However, utilizing the reduced set of measurements selected by the RANSAC based method drastically reduces the RMSE of the position and velocity. It is particularly noted that the proposed algorithm outperforms conventional estimators when small measurement sets are available and that the proposed algorithm converges faster than conventional estimators do.

Figure 6 shows the time averaged RMSE of position and velocity for varying standard deviations of noise. According to the results, the proposed measurement selection algorithm has no effect on estimation performance in relatively low-noise environments. However, they demonstrate that as the standard deviation of bearing noise increases the proposed measurement selection method slightly increases performance. Therefore, it is possible to say that the proposed measurement selection algorithm applied in BOTMA is effective when given a small set of noisy measurements.

#### 5. Conclusions

This paper investigated the BOTMA problem when an only small set of measurements is available. Because the prompt generation of target information is critical to many maritime surveillance applications, this paper attempted to provide more precise motion information when using a small set of measurements. The proposed method consisted of the following two procedures. First, a measurement selection process based on RANSAC reduced the influence of noisy measurements. Second, an efficient estimation method based on the selected measurements helped to determine target motion precisely. According to the simulation results, compared to conventional methods, the proposed method was better at quickly estimating precise motion in highnoise conditon.

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