PAPER

Linguistic Multi-Criteria Group Decision-Making Method Combining Cloud Model and Evidence Theory

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SUMMARY The linguistic Multi-Criteria Group Decision-Making (MCGDM) problem involves various types of uncertainties. To deal with this problem, a new linguistic MCGDM method combining cloud model and evidence theory is thus proposed. Cloud model is firstly used to handle the fuzziness and randomness of the linguistic concept, by taking both the average level and fluctuation degree of the linguistic concept into consideration. Hence, a method is presented to transform linguistic variables into clouds, and then an asymmetrical weighted synthetic cloud is proposed to aggregate the clouds of decision makers on each criterion. Moreover, evidence theory is used to handle the imprecision and incompleteness of the group assessment, with the belief degree and the ignorance degree. Hence, the conversion from the cloud to the belief degree is investigated, and then the evidential reasoning algorithm is adopted to aggregate the criteria values. Finally, the average utility is applied to rank the alternatives. A numerical example, which is given to confirm the validity and feasibility, also shows that the proposed method can take advantage of cloud model and evidence theory to efficiently deal with the uncertainties caused by both the linguistic concept and group assessment.

key words: *MCGDM, linguistic, uncertainty, cloud model, evidence theory*

1. Introduction

Multi-Criteria Decision-Making (MCDM) is one of the most important decision methodologies, which is widely used in the fields of science [1], business [2], and engineering [3], etc. Due to the inherent vagueness of human preferences and the complexity of decision-making problems, the Decision Makers (DMs) usually feel more confident in providing linguistic assessments than providing exact numerical values in real MCDM cases. This decision-making case is defined as a linguistic MCDM problem, that is, the alternative assessments with regard to criteria are given with linguistic variables [4].

Recently, linguistic MCDM methods have been studied in many fields [5]–[7]. Herrera et al. [8] and Martínez et al. [9] survey that three main types of linguistic MCDM methods have been put forward: 1) The method on the basis of membership functions, which could convert the linguistic concept into some fuzzy numbers by membership functions [10]–[12]. However, in the transformation process, this method could possibly lead to information distortion. 2) The method on the basis of symbols, which could convert the linguistic concept into some real numbers [13]–[15]. But this method absolutely loses the fuzziness of linguistic variables. 3) The method on the basis of 2-tuple linguistic models, which could also transform the linguistic concept into some real numbers and introduce the uncertain decision making into the precise domain [16]–[18]. However, this method may violate the original intention of DMs.

Furthermore, the linguistic concept usually involves various uncertainties, among which the fuzziness and randomness are the two most important. The fuzziness of a concept denotes the uncertainty with regard to the extension range of this concept, and the randomness of a concept indicates that any concept is not isolated [19]. Fuzzy sets theory and probability theory are the main mathematical instruments to respectively settle fuzziness and randomness. Cloud model proposed by Li et al. [20], which can represent the fuzziness of a concept with the normal membership function and the randomness with the normal distribution, is founded on fuzzy sets theory and probability theory. Over the last few decades, cloud model has been successfully applied in risk assessment [21], anomaly detection [22], recommendation system [23] and game problems [24]. Some methods [25]–[27] have successfully applied cloud model to linguistic MCDM, by transforming the linguistic assessment into the cloud. Cloud model describes the linguistic assessment with three numerical characteristics that reflect not only the average levels of the linguistic concept but also the fuzziness and randomness. Meanwhile, some related aggregation methods of the clouds, such as synthetic cloud [28], floating cloud [25], and cloud weighted arithmetic averaging (CWAA) operator [26], have been studied. However, the aggregated clouds using these aggregation methods either fail to cover the domain of all original clouds, or cover the domain of the irrelevant cloud. In this paper, we use cloud model to handle the fuzziness and randomness of the linguistic concept. The linguistic assessments of DMs are transformed into clouds, and then an asymmetrical weighted synthetic cloud is proposed to aggregate these clouds.

In real MCDM cases, the criteria value of alternative is often provided by a group of DMs because an individual may be incapable of providing reliable assessments due to the lack of necessary information or experiences. This decision-making case is called a Multi-Criteria Group Decision-Making (MCGDM) problem. The MCGDM problem is often encountered in information systems, such as crowd sensing [29] and computing task assignment [30]. Nowadays it is more and more common that the solution of

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this problem is achieved by the linguistic MCGDM methods [31]–[33]. Due to the uncertainty caused by aggregation of DMs assessments, the criteria value can be hesitant, imprecise, or even incomplete. Fortunately, evidence theory can provide good solutions to these problems. Evidence theory originated from Dempster's work [34] and further extended by Shafer [35], is a generalization of traditional probability, which allows us to get a better handle on the imprecision and incompleteness. It is particularly useful for dealing with uncertain subjective judgments when multiple pieces of evidence must be simultaneously considered. An Evidential Reasoning (ER) algorithm on the base of both evidence theory and decision theory was proposed by Yang et al. [36], [37] to handle the uncertain MCDM problem in 1990s. So far, evidence theory and its extensions have been widely applied in many areas such as pattern recognition [38], rule-based systems [39], forex market analysis [40], and group decision analysis [41]. In this paper, we suppose that the group assessment consists of the linguistic assessment of each decision maker, just the belief degree of each linguistic assessment is different. And we use evidence theory to handle the imprecision and incompleteness of the group assessment. The ER algorithm is adopted to aggregate the group assessments on each criterion.

We propose a new method to solve the linguistic MCGDM problem in this paper. To handle the fuzziness and randomness of the linguistic concept, cloud model is used to represent and aggregate DMs linguistic assessments. Besides, to cope with the imprecision and incompleteness of the group assessment, evidence theory is used to fuse the assessment of each criterion. The new method takes advantage of these two theories, and the major contributions of this paper are as followings. Firstly, a method for transforming linguistic variables into the corresponding clouds is proposed. Secondly, an asymmetrical weighted synthetic cloud is presented to aggregate the clouds of DMs on each criterion. Thirdly, the conversion from the cloud to the belief degree is developed. Finally, the ER algorithm is adopted to aggregate the criteria values, and then the average utility is used to rank the alternatives. This method is different from other MCGDM methods in that it can deal with uncertainties caused by both the linguistic concept and group assessment.

The remainder of this paper is organized as follows: Section 2 introduces some relative definitions. Section 3 proposes the linguistic MCGDM method combining cloud model and evidential theory. To validate the proposed method, a numerical example is examined in Sect. 4. Section 5 concludes this paper.

2. Preliminaries

This section involves some basic concepts of cloud model and evidence theory.

2.1 Basics of Cloud Model

Li et al. [20] presented a cloud model as a new representation model of uncertainty on the base of fuzzy sets theory and probability theory.

Define a qualitative concept T over a universe of discourse $U = \{u\}$. Then $x \in U$ is defined as a random instantiation of T, and $\mu_T(x) \in [0, 1]$ is the certainty degree that x belongs to T, which corresponds to a random number with a steady trend. Define a cloud that represents the distribution of x in the universe U, where x is known as a cloud drop [19]. $\forall x \in U$, the mapping $\mu_T(x)$, is essentially one-to-many mapping, which means the certainty degree that x belongs to T is a distribution of probability.

Cloud model can describe a qualitative concept with three numerical characteristics which are Expectation Ex, Entropy En, and Hyper entropy He. Ex is the mathematical expectation of the cloud drops that belongs to a qualitative concept in the universe. En is used to refer to the fuzziness measurement of the concept. He reflects the cloud drops dispersion. Suppose C is a cloud with three numerical characteristics, which can be represented as $C(Ex, En, He)$.

The normal cloud model founded on the normal membership function and normal distribution, which we discuss only in this paper, is generally applicable.

Definition 1 [19]. Suppose U is the discourse universe and $\tilde{A}$ is a qualitative concept in U. If $x \in U$ is a random instantiation of the concept $\tilde{A}$, which follows $x \sim N(Ex, En'^2)$, $En' \sim N(En, He^2)$, and the certainty degree that x belongs to the concept $\tilde{A}$ follows $\mu = e^{-(x-Ex)^2/(2(En')^2)}$, then the x distribution in the universe U is called a normal cloud.

2.2 Basics of Evidence Theory

Evidence theory was originally presented by Dempster [34] and later developed by Shafer [35]. Evidence theory was introduced in the uncertain MCDM problem by Yang et al. [36], [37] in the early 1990s.

Definition 2. Define Θ as a set of elements that are collectively exhaustive and mutually exclusive, where an element can be an object, a hypothesis, or in this paper a linguistic variable. Θ is referred to as the frame of discernment; the set which consists of all the subsets of Θ is denoted by $\Omega(\Theta)$ and seen as the power set of Θ.

Definition 3 [42]. A Basic Probability Assignment (BPA) is a function $m : \Omega(\Theta) \to [0, 1]$, which is called a mass function and satisfies the following equation:

$$m(\Phi) = 0, \sum_{A \subseteq \Omega(\Theta)} m(A) = 1 \tag{1}$$

where Φ is empty, and A is any subset of Θ. The assigned probability $m(A)$, which is also named probability mass, evaluates the belief accurately assigned to A and indicates the support degree of the evidence to A. It does not include the belief in any particular subset of A. Each subset $A \subseteq \Theta$ such that $m(A) > 0$ is referred to as a focal element of m. When a mass value is committed to a subset that has more than one element, it explicitly states that there is not enough information for assigning this belief more exactly to each individual element in the subset. Especially when there is no evidence about Θ at all, the total belief is assigned to the whole frame of discernment $m(\Theta)=1$, where $m(\Theta)$ is called the ignorance degree.

3. Linguistic MCGDM Method Combining Cloud Model and Evidential Theory

3.1 Linguistic MCGDM Problem

With respect to a MCGDM problem, suppose there are n alternatives $A = \{a_1, a_2 \cdots a_i \cdots a_n\}$ and m criteria $C = \{c_1, c_2 \cdots c_j \cdots c_m\}$ with weight vector $W = \{w_1, w_2 \cdots w_j \cdots w_m\}$ associated with C, where $w_j \in [0,1]$ and $\sum_{j=1}^{m} w_j = 1$. Suppose there are s DMs $D = \{d_1, d_2 \cdots d_k \cdots d_s\}$ whose corresponding weight vector is $\lambda = \{\lambda_1, \lambda_2 \cdots \lambda_k \cdots \lambda_s\}$, where $\lambda_k \in [0,1]$ and $\sum_{k=1}^{s} \lambda_k = 1$. The evaluation matrix of the kth DM is expressed as $E^k = (h_{ij}^k)_{m\times n}$, where h_{ij}^k is the assessment for alternative a_i corresponding to criterion c_j given by decision maker d_k.

As mentioned in Sect. 1, it is hard for DMs to give their preferences with numerical values. In this paper, to provide individual preferences, DMs use linguistic variables in the predefined linguistic term set, $h_{ij}^k \in H$.

Definition 4. Let $H = \{h_1, h_2 \cdots h_g \cdots h_N\}$ be a finite linguistic term set, in which the variables are required to be collectively exhaustive and mutually exclusive for the assessment. Without loss of generality, h_1 and h_N are assumed to be the worst and the best variables respectively, and h_{g+1} is preferred to h_g.

Suppose, $N = 5$ for example, H may be defined as follows: $H = \{poor(h_1),\ indifferent(h_2),\ average(h_3),\ good(h_4),\ excellent(h_5)\}$.

3.2 Transform the Linguistic Information into the Cloud

Transformation between a qualitative linguistic concept and its quantitative value has always been a barrier in the inguistic MCGDM problem. As a powerful transformation model of qualitative and quantitative, cloud model makes it possible to address the potential fuzziness and randomness inherent in the linguistic concept. A method for transforming linguistic variables into the corresponding clouds is introduced as follows.

Definition 5. Assuming the linguistic term set is $H = \{h_1, h_2 \cdots h_g \cdots h_N\}$, and a valid non-negative universe $[U_{max}, U_{min}]$ is provided. By applying the average method, $H = \{h_1, h_2, \cdots h_g \cdots h_N\} = \{(B_{min}^1, B_{max}^1), (B_{min}^2, B_{max}^2), \cdots (B_{min}^g, B_{max}^g), \cdots (B_{min}^N, B_{max}^N)\}$, where $Length(B_{min}^g, B_{max}^g) = (U_{max} - U_{min})/N$. Then, the cloud $c_{h_g}(Ex_g, En_g, He_g)$ representing h_g can be defined as:

$$\begin{cases} Ex_g = \dfrac{B_{min}^g + B_{max}^g}{2} \\ En_g = \dfrac{B_{max}^g - B_{min}^g}{6} \\ He_g = \theta \end{cases} \tag{2}$$

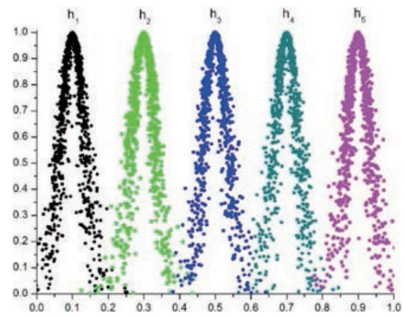

Fig. 1 Clouds of linguistic variables.

Cloud model uses three numerical characteristics to describe a linguistic variable. These characteristics realize the conversion of objective and its interchangeable between a qualitative linguistic concept and its quantitative value. The algorithm for computing three numerical characteristics can be illustrated as shown below. Ex expresses the expectation of the linguistic concept. Thus, it is natural to use the median of the interval, where Interval $[Ex-3En, Ex+3En]$ best describes the qualitative linguistic concept (99.74%, $6En$ rule). Consequently, $6En$ can be adopted to represent the fuzziness and bound of the linguistic concept. He is set to θ which can be adjusted according to actual conditions.

Example 1. Let the universe be [0, 1] and the linguistic assessment set be $H=\{poor(h_1), indifferent(h_2), average(h_3), good(h_4), excellent(h_5)\}$, then $H = \{(B_{min}^1 = 0, B_{max}^1 = 0.2), (B_{min}^2 = 0.2, B_{max}^2 = 0.4), (B_{min}^3 = 0.4, B_{max}^3 = 0.6), (B_{min}^4 = 0.6, B_{max}^4 = 0.8), (B_{min}^5 = 0.8, B_{max}^5 = 1)\}$. Given $He = 0.01$, the following five clouds can be obtained: $C_H = \{c_{h_1}(0.1, 0.033, 0.01),\ c_{h_2}(0.3, 0.033, 0.01),\ c_{h_3}(0.5, 0.033, 0.01),\ c_{h_4}(0.7, 0.033, 0.01),\ c_{h_5}(0.9, 0.033, 0.01)\}$. 1000 cloud drops can be generated for each cloud using the forward cloud generator [19], and the predefined linguistic term set can be explicitly presented by clouds, as shown in Fig. 1.

3.3 Aggregate the Clouds of DMs on Each Criterion

After the linguistic assessment is transformed to the cloud, an aggregation step must be carried out for a collective group assessment. On the basis of synthetic cloud [28], the asymmetrical weighted synthetic cloud aggregation algorithm is proposed in this section to obtain the cloud of the group assessment. The cloud of the group assessment can meet the following requirements: 1) The cloud of the group assessment is required to cover the linguistic assessments of all DMs in group decision-making. 2) The cloud of the group assessment must be limited between the best and worst linguistic assessments and should not be out of the domain. The asymmetrical weighted synthetic cloud can synthesize all DMs assessments on each criterion and reflect more general information coverage.

Definition 6. Let $c_k(Ex_k, En_k, He_k), k = 1, 2 \cdots s$, be a collection of clouds, and let $\lambda = \{\lambda_1, \lambda_2 \cdots \lambda_k \cdots \lambda_s\}$ be a weight vector with $\lambda_k \in [0,1], \sum_{k=1}^{s} \lambda_k = 1$. Their asymmetrical

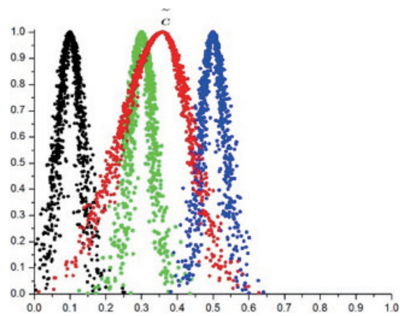

Fig. 2 Asymmetrical weighted synthetic cloud.

weighted synthetic cloud $\tilde{c}(Ex, En^l, En^r, He)$ can be defined as:

$$\begin{cases} Ex = \sum_{k=1}^{s} \lambda_k Ex_k \\ En^l = \dfrac{Ex - \min\limits_k\{Ex_k - 3En_k\}}{3} \\ En^r = \dfrac{(\max\limits_k\{Ex_k + 3En_k\} - Ex)}{3} \\ He_s = \sum_{k=1}^{s} \lambda_k He_k \end{cases} \tag{3}$$

The asymmetrical weighted synthetic cloud $\tilde{c}(Ex, En^l, En^r, He)$ is made up of two half clouds. One is the left half-up cloud $c^l(Ex, En^l, He)$, the other is the right half-down cloud $c^r(Ex, En^r, He)$. The entropy En^l or En^r of the asymmetrical weighted synthetic cloud is larger than the entropy En_k of each individual cloud. And the values of En^l and En^r ensure that the asymmetrical weighted synthetic cloud cover the domain of all original clouds, and not cover the domain of the irrelevant cloud. It can aggregate assessments of DMs and be regarded as the result of the group decision on each criterion.

Example 2. Given three clouds $c_1(0.1, 0.033, 0.01)$, $c_2(0.3, 0.033, 0.01)$, and $c_3(0.5, 0.033, 0.01)$ with an associated weight vector $\lambda = \{0.2, 0.3, 0.5\}$, then $Ex = 0.2 \times 0.1 + 0.3 \times 0.3 + 0.5 \times 0.5 = 0.36$, $En^l = \frac{0.36 - \min\{(0.1-3\times0.033),(0.3-3\times0.033),(0.5-3\times0.033)\}}{3} = 0.12$, $En^r = \frac{\max\{(0.1+3\times0.033),(0.3+3\times0.033),(0.5+3\times0.033)\} - 0.36}{3} = 0.08$, $He = 0.2 \times 0.01 + 0.3 \times 0.01 + 0.5 \times 0.01 = 0.01$. The synthetic cloud $\tilde{c}(Ex = 0.36, En^l = 0.12, En^r = 0.08, He = 0.01)$ can be obtained, as shown in Fig. 2.

Especially, if En^l=En^r, then the asymmetrical weighted synthetic cloud is equal to a normal cloud. Of course, a normal cloud also can be seen as a special asymmetrical weighted synthetic cloud.

3.4 Convert the Cloud into the Belief Degree

Evidence theory describes and handles uncertainties using the concept of the belief degree. In this section, we convert the cloud into the belief degree. A cloud is composed of cloud drops. Given a cloud drop (x, μ), its certainty degree that x belongs to the concept is μ. If $B_{min}^g \le x \le B_{max}^g$, we consider that μ is the certainty degree that h_g belongs to the group assessment. If enough cloud drops can be obtained as samples, the distribution of the belief degree can be obtained according to Monte Carlo method.

Definition 7. Given an asymmetrical weighted synthetic cloud and applied the forward asymmetrical weighted synthetic cloud generator, n_{all} cloud drops can be generated. Suppose there are n_1 cloud drops $\{(x_1, \mu_1), (x_2, \mu_2) \cdots (x_{t_1}, \mu_{t_1}) \cdots (x_{n_1}, \mu_{n_1})$, between $[B_{min}^1, B_{max}^1], \cdots$, n_g cloud drops $\{(x_1, \mu_1), (x_2, \mu_2) \cdots (x_{t_g}, \mu_{t_g}) \cdots (x_{n_g}, \mu_{n_g})$ between $[B_{min}^g, B_{max}^g], \cdots$, n_N cloud drops $\{(x_1, \mu_1)$, $(x_2, \mu_2) \cdots (x_{t_N}, \mu_{t_N}) \cdots (x_{n_N}, \mu_{n_N})$ between $[B_{min}^N, B_{max}^N]$ and $\sum_{g=1}^{N} n_g = n_{all}$. Then the distribution of the belief degree can be calculated as follows:

$$\beta = \begin{cases} \beta(h_1) = \dfrac{\sum_{t_1=1}^{n_1} \mu_{t_1}}{\sum_{t=1}^{n_{all}} \mu_t} & \forall x_{t_1} \in [B_{min}^1, B_{max}^1] \\ \vdots \\ \beta(h_g) = \dfrac{\sum_{t_g=1}^{n_g} \mu_{t_g}}{\sum_{t=1}^{n_{all}} \mu_t} & \forall x_{t_g} \in [B_{min}^g, B_{max}^g] \\ \vdots \\ \beta(h_N) = \dfrac{\sum_{t_N=1}^{n_N} \mu_{t_N}}{\sum_{t=1}^{n_{all}} \mu_t} & \forall x_{t_N} \in [B_{min}^N, B_{max}^N] \end{cases} \tag{4}$$

where $\beta(h_g)$ denotes a belief degree to which the criterion is confirmed to h_g.

The forward asymmetrical weighted synthetic cloud generator based on the forward cloud generator [19] generates cloud drops in accordance with the numerical characteristics (Ex, En^l, En^r, He). Definition 8 details the generating procedure of the algorithm for the asymmetrical weighted synthetic cloud $\tilde{c}$ as follows:

Definition 8. Forward Asymmetrical Weighted Synthetic Cloud Generator $FAWSCG(Ex, En^l, En^r, He, N)$.

Input: Four parameters, Ex, En^l, En^r, and He, and cloud drop number N.

Output: N cloud drops.

Steps:

(1) Generate a random number $En_i^{l\prime}$ which follows normal distribution, with expectation En^l and variance He^2.

(2) Generate a random number x_i which follows normal distribution, with expectation Ex and variance He^2.

(3) If $x_i > Ex$, repeat step 2. Else, calculate $\mu_i = e^{-(x_i - Ex)^2/(2(En_i^{l\prime})^2)}$.

(4) Drop (x_i, μ_i) is a cloud drop of the left half-up cloud.

(5) Repeat steps 1-4 until $\lfloor N/2 \rfloor$ cloud drops of the left half-up cloud have been generated.

(6) Generate a random number $En_t^{r\prime}$ which follows normal distribution, with expectation En^r and variance.

(7) Generate a random number x_t which follows normal distribution, with expectation Ex and variance $(En_t^{r\prime})^2$.

(8) If $x_t < Ex$, repeat step 7. Else, calculate $\mu_t = e^{-(x_t-Ex)^2/2(En_t^{r\prime})^2}$.

(9) Drop (x_t, μ_t) is a cloud drop of the right half-down cloud.

(10) Repeat steps 6-9 until $N - \lfloor N/2 \rfloor$ cloud drops of the right half-down cloud have been generated.

Example 3. Given an asymmetrical weighted synthetic cloud $\tilde{c}(Ex = 0.36, En^l = 0.12, En^r = 0.08, He = 0.01)$, 1000 cloud drops can be generated by FAWSCG as shown in Fig. 2. Then its distribution of the belief degree can be calculated: $\beta = \{\beta(h_1) = 0.03,\ \beta(h_2) = 0.731,\ \beta(h_3) = 0.239,\ \beta(h_4) = 0,\ \beta(h_5) = 0\}$.

Considering the hesitance and imprecision caused by aggregation of DMs′ assessments, the discounting coefficient is used as a reliability coefficient. The more linguistic variables the synthetic cloud covers, the less reliable the information source is. We use the discounting coefficient to modify the belief degree.

Definition 9. Given the discounting coefficient α and a belief degree $\beta(h_g)$, the discounted belief degree can be obtained as $\beta'(h_g) = \alpha\beta(h_g)$. If the distribution of the belief degree for alternative a_i on criterion c_j is denoted as β_{ij}, the distribution of the discounted belief degree can be obtained as $\beta'_{ij} = \alpha\beta_{ij}$, where $\alpha = (l-2N+1)/(2-2N), 0.5 \le \alpha \le 1$. l is the number of h_g covered by the synthetic cloud and $l = \max\{g|\beta(h_g) \neq 0\} - \min\{g|\beta(h_g) \neq 0\} + 1$. $\sum_{g=1}^{N} \beta'(h_g) \le 1$, that is, the group assessment may be incomplete.

Example 4. Suppose $\beta_{ij} = \{\beta_{ij}(h_1) = 0.03,\ \beta_{ij}(h_2) = 0.731,\ \beta_{ij}(h_3) = 0.239,\ \beta_{ij}(h_4) = 0,\ \beta_{ij}(h_5) = 0\}$, by applying Definition 9, the following can be obtained: $\alpha = (3-2\times5+1)/(2-2\times5) = 0.75$ and $\beta'_{ij} = \alpha\beta_{ij} = \{\beta'_{ij}(h_1) = 0.00225,\ \beta'_{ij}(h_2) = 0.5483,\ \beta'_{ij}(h_3) = 0.1793,\ \beta'_{ij}(h_4) = 0,\ \beta_{ij}(h_5) = 0\}$

3.5 Aggregate the Criteria Values of Each Alternative

Based on evidence theory, an ER algorithm have been developed for MCDM with the imprecise and incomplete information [42], [43]. We use the ER algorithm to aggregate the criteria values. The group assessment on each criterion and the linguistic assessment set H are respectively treated as the evidence and the frame of discernment Θ. The assessment of each criterion could be fused to obtain the overall evaluation.

Firstly, the belief degree is transformed into the BPA.

Definition 10. Using the following equations, the distribution of the discounted belief degree for alternative a_i on criterion c_j is transformed into the BPA through multiplying the given discounted degree of belief $\beta'_{ij}(h_g)$ by the relative weight of the criterion w_j.

$$m_{i,j} = \begin{cases} m_{i,j}(h_g) = w_j\beta'_{ij}(h_g) \\ \bar{m}_{i,j}(H) = 1 - w_j \\ \tilde{m}_{i,j}(H) = w_j(1 - \sum\limits_{g=1}^{N} \beta'_{ij}(h_g)) \end{cases} \tag{5}$$

where $m_{i,j}(H) = \bar{m}_{i,j}(H) + \tilde{m}_{i,j}(H)$ and $\sum_{j=1}^{m} w_j = 1$. As the ignorance degree, $m_{i,j}(H)$ is divided into two parts: $\bar{m}_{i,j}(H)$ and $\tilde{m}_{i,j}(H)$, where $\bar{m}_{i,j}(H)$ is determined by the relative importance of criterion c_j and $\tilde{m}_{i,j}(H)$ is determined by the imprecision and incompleteness of the group assessment for a_i on c_j.

Next, the BPAs on m criteria are aggregated into the combined probability assignment.

Definition 11. Suppose $m_{i,I(j)}(H)$, $\bar{m}_{i,I(j)}(H)$ and $\tilde{m}_{i,I(j)}(H)$ are the combined probability masses obtained by aggregating the first j criteria for a_i. The following recursive formulas are then proposed to aggregate the first j criteria with the $(j+1)$th criterion.

$$\begin{cases} m_{i,I(j+1)}(h_g) = K_{i,I(j+1)}[m_{i,I(j)}(h_g)m_{i,j+1}(h_g) \\ \qquad +m_{i,I(j)}(H)m_{i,j+1}(h_g)+m_{i,I(j)}(h_g) \\ \qquad m_{i,j+1}(H)] \\ \bar{m}_{i,I(j+1)}(H) = K_{i,I(j+1)}[\bar{m}_{i,I(j)}(H)\bar{m}_{i,j+1}(H)] \\ \tilde{m}_{i,I(j+1)}(H) = K_{i,I(j+1)}[\tilde{m}_{i,I(j)}(H)\tilde{m}_{i,j+1}(H) \\ \qquad + \bar{m}_{i,I(j)}(H)\tilde{m}_{i,j+1}(H)+\tilde{m}_{i,I(j)}(H) \\ \qquad \bar{m}_{i,j+1}(H)] \end{cases} \tag{6}$$

where $K_{i,I(j+1)} = [1 - \sum_{g=1}^{N}\sum_{\substack{t=1 \\ t\neq g}}^{N} m_{i,I(j)}(h_g)m_{i,j+1}(h_t)]^{-1}$, $m_{i,I(j)} = \bar{m}(H)_{i,I(j)} + \tilde{m}_{i,I(j)}(H)$, $j = 1, 2 \cdots m-1$. $m_{i,I(m)}(h_g)$ is a probability mass defined as the degree of support of all the criteria to the hypothesis that a_i is assessed to h_g.

Finally, normalize the combined probability assignment into the overall belief degrees.

Definition 12. Let $\beta_i(h_g)$ and $\beta_i(H)$ denote the overall belief degrees of the integration all assessments for alternative a_i, which are assigned to h_g and H, respectively. The distribution of the overall belief degrees of alternative a_i can be calculated as follows:

$$\beta(a_i) \begin{cases} \beta_i(h_g) = \dfrac{m_{i,I(m)}(h_g)}{1 - \bar{m}_{i,I(m)}(H)} \\ \beta_i(H) = \dfrac{\tilde{m}_{i,I(m)}(H)}{1 - \bar{m}_{i,I(m)}(H)} \end{cases} \tag{7}$$

$\beta_i(H)$ is the unassigned belief degree denoting the extent of imprecision and incompleteness of the overall assessment for alternative a_i.

Example 5. Assume there is an alternative a_i and two criteria c_1, c_2 with weight vector $w_1 = 0.4$, $w_2 = 0.6$. Suppose the distribution of the discounted belief degree for alternative a_1 on criterion c_1 and c_2 are $\beta'_{11} = \{\beta'_{11}(h_1) = 0.0225,\ \beta'_{11}(h_2) = 0.5483,\ \beta'_{11}(h_3) = 0.1793,\ \beta'_{11}(h_4) = 0,\ \beta'_{11}(h_5) = 0\}$, $\beta'_{12} = \{\beta'_{12}(h_1) = 0,\ \beta'_{12}(h_2) = 0.4,\ \beta'_{12}(h_3) = 0.3,\ \beta'_{12}(h_4) = 0.2,\ \beta'_{12}(h_5) = 0\}$, respectively. Using Definition 10, the BPAs can be obtained: $m_{1,1} = \{m_{1,1}(h_1) = 0.009,\ m_{1,1}(h_2) = 0.2193,\ m_{1,1}(h_3) = 0.0717,\ m_{1,1}(h_4) = 0,\ m_{1,1}(h_5) = 0,\ \bar{m}_{1,1}(H) = 0.6,\ \tilde{m}_{1,1}(H) = 0.1\}$, $m_{1,2} = \{m_{1,2}(h_1) = 0,\ m_{1,2}(h_2) = 0.24,\ m_{1,2}(h_3) = 0.18,\ m_{1,2}(h_4) = 0.12,\ m_{1,2}(h_5) = 0,\ \bar{m}_{1,2}(H) = 0.4,$

$\tilde{m}_{1,2}(H) = 0.06\}$ Then, using Definition 11, the following can be obtained: $K_{1,I(2)} = [1 - m_{1,1}(h_1)(m_{1,2}(h_2) + m_{1,2}(h_3)+m_{1,2}(h_4)+m_{1,2}(h_5))-m_{1,1}(h_2)(m_{1,2}(h_1)+m_{1,2}(h_3)+m_{1,2}(h_4)+m_{1,2}(h_5))-m_{1,1}(h_3)(m_{1,2}(h_1)+m_{1,2}(h_3)+m_{1,2}(h_4)+m_{1,2}(h_5))-m_{1,1}(h_4)(m_{1,2}(h_1)+m_{1,2}(h_3)+m_{1,2}(h_4)+m_{1,2}(h_5))-m_{1,1}(h_5)(m_{1,2}(h_1) + m_{1,2}(h_3) + m_{1,2}(h_4) + m_{1,2}(h_5))]^{-1} = 1.10676$

$$\begin{cases} m_{1,I(2)}(h_1) = K_{1,I(2)}[m_{1,1}(h_1)m_{1,2}(h_1)+m_{1,1}(H)m_{1,2}(h_1) \\ \qquad +m_{1,1}(h_1)m_{1,2}(H)]=0.0046 \\ m_{1,I(2)}(h_2) = K_{1,I(2)}[m_{1,1}(h_2)m_{1,2}(h_2)+m_{1,1}(H)m_{1,2}(h_2) \\ \qquad +m_{1,1}(h_2)m_{1,2}(H)]=0.3558 \\ m_{1,I(2)}(h_3) = K_{1,I(2)}[m_{1,1}(h_3)m_{1,2}(h_3)+m_{1,1}(H)m_{1,2}(h_3) \\ \qquad +m_{1,1}(h_3)m_{1,2}(H)]=0.1902 \\ m_{1,I(2)}(h_4) = K_{1,I(2)}[m_{1,1}(h_4)m_{1,2}(h_4)+m_{1,1}(H)m_{1,2}(h_4) \\ \qquad +m_{1,1}(h_4)m_{1,2}(H)]=0.093 \\ m_{1,I(2)}(h_5) = K_{1,I(2)}[m_{1,1}(h_5)m_{1,2}(h_5)+m_{1,1}(H)m_{1,2}(h_5) \\ \qquad +m_{1,1}(h_5)m_{1,2}(H)]=0 \\ \bar{m}_{1,I(2)}(H) = K_{1,I(2)}[\bar{m}_{1,1}(H)\bar{m}_{1,2}(H)]=0.2656 \\ \tilde{m}_{1,I(2)}(H) = K_{1,I(2)}[\tilde{m}_{1,1}(H)\tilde{m}_{1,2}(H)+\bar{m}_{1,1}(H)\tilde{m}_{1,2}(H) \\ \qquad +\tilde{m}_{1,1}(H)\bar{m}_{1,2}(H)]=0.0908 \end{cases}$$

Finally, using Definition 12, the distribution of the overall belief degrees can be calculated: $\beta(a_1) = \{\beta_1(h_1) = \frac{m_{1,I(2)}(h_1)}{1-\bar{m}_{1,I(2)}(H)} = 0.0062$, $\beta_1(h_2) = \frac{m_{1,I(2)}(h_2)}{1-\bar{m}_{1,I(2)}(H)} = 0.4845$, $\beta_1(h_3) = \frac{m_{1,I(2)}(h_3)}{1-\bar{m}_{1,I(2)}(H)} = 0.2591$, $\beta_1(h_4) = \frac{m_{1,I(2)}(h_4)}{1-\bar{m}_{1,I(2)}(H)} = 0.1266$, $\beta_1(h_5) = \frac{m_{1,I(2)}(h_5)}{1-\bar{m}_{1,I(2)}(H)} = 0.0$, $\beta_1(H) = \frac{\tilde{m}_{1,I(2)}(H)}{1-\bar{m}_{1,I(2)}(H)} = 0.0062\}$

3.6 Rank Alternatives

The average, minimum and maximum utilities are introduced to rank N alternatives [43]. The least preferred linguistic variable with the lowest utility is supposed as h_1, and the most preferred linguistic variable with the highest utility is supposed as h_N.

Definition 13. Suppose the utility of a linguistic variable h_g is $u(h_g)$, then the average, minimum and maximum utilities of the alternative a_i are defined as follows:

$$\begin{cases} u_{avg}(a_i) = \dfrac{u_{min}(a_i)+u_{max}(a_i)}{2} \\ u_{min}(a_i) = \sum_{g=2}^{N} \beta_i(h_g)u(h_g)+(\beta_i(h_1)+\beta_i(H))u(h_1) \\ u_{max}(a_i) = \sum_{g=1}^{N-1} \beta_i(h_g)u(h_g)+(\beta_i(h_N)+\beta_i(H))u(h_N) \end{cases} \tag{8}$$

where $\beta_i(h_g)$ and $\beta_i(H)$ are the overall belief degrees of the alternative a_i. It is obvious that if $\beta_i(H) = 0$ then $u_{avg}(a_i) = u_{max}(a_i) = u_{min}(a_i)$.

In this paper, we rank the alternatives according to their average utilities. If $u_{avg}(a_i) > u_{avg}(a_t)$, a_i is said to be preferred to a_t.

Example 6. Suppose the utility of linguistic variables are $u(h_1) = 0$, $u(h_2) = 0.25$, $u(h_3) = 0.5$, $u(h_4) = 0.75$, $u(h_5) = 1$, If $\beta(a_1) = \{\beta_1(h_1) = 0.0062$, $\beta_1(h_2) = 0.4845$, $\beta_1(h_3) = 0.2591$, $\beta_1(h_4) = 0.1266$, $\beta_1(h_5) = 0$, $\beta_1(H) = 0.1236\}$, then the minimum, maximum and average utilities can be calculated: $u_{min}(a_1) = 0.4845\times0.25+0.2591\times0.5+0.1266\times 0.75 + 0 \times 1 + (0.0062 + 0.1236) \times 0 = 0.3456$, $u_{max}(a_1) = 0.0062\times0+0.4845\times0.25+0.2591\times0.5+0.1266\times0.75+(0+0.1236)\times1 = 0.4692$, $u_{avg}(a_1) = \frac{0.4692+0.3456}{2} = 0.4074$.

3.7 Procedure for Linguistic MCGDM

As a result of the discussion above, we are now in a position to describe a procedure for solving the linguistic MCGDM problem. The procedure comprises the following steps.

Step 1: Transform the linguistic variable into the cloud $(h_{ij}^k)_{m\times n} \rightarrow (c_{ij}^k(Ex_g, En_g, He_g))_{m\times n}$.

Transform the linguistic variables in the linguistic term set into the corresponding clouds using Definition 5 in Sect. 3.2. Then, the evaluation matrix of the kth DM is obtained as $(c_{ij}^k(Ex_g, En_g, He_g))_{m\times n}$.

Step 2: Aggregate the clouds of DMs on each criterion $\underset{k=1}{\overset{s}{aggreate}}(h_{ij}^k)_{m\times n} \rightarrow (\tilde{c}_{ij}(Ex, En^l, En^r, He))_{m\times n}$.

Considering the weight vector of DMs $\lambda = \{\lambda_1, \lambda_2 \cdots \lambda_k \cdots \lambda_s\}$, aggregate the clouds of s DMs on each criterion using Definition 6 in Sect. 3.3. Then, the group evaluation matrix is obtained as $(\tilde{c}_{ij}(Ex, En^l, En^r, He))_{m\times n}$.

Step 3: Convert the cloud into the belief degree $(\tilde{c}_{ij}(Ex, En^l, En^r, He))_{m\times n} \rightarrow (\beta'_{ij})_{m\times n}$.

First, convert the asymmetrical weighted synthetic cloud into the belief degree using Definition 7 in Sect. 3.4, then use the discounting coefficient to modify the belief degree according to Definition 9 in the same section. The distribution of the discounted belief degree for alternative a_i on criterion c_j is denoted as β'_{ij}.

Step 4: Aggregate the criteria values of each alternative $\underset{j=1}{\overset{m}{aggreate}}\beta'_{ij} \rightarrow \beta(a_i)$.

The criteria values are aggregated and the overall belief degree of alternative a_i is obtained by the ER algorithm given in Definition 10–12 in Sect. 3.5.

Step 5: Rank alternatives $\beta(a_i) \rightarrow u_{avg}(a_i)$.

With the use of Definition 13 in Sect. 3.6, calculate the average utility of each alternative. Then, rank the alternatives according to the average utility $u_{avg}(a_i)$.

4. Illustrative Example

This section demonstrates the implementation process of the proposed method through a numerical example. Consider an emergency planning evaluation problem for water pollution incidents on six criteria [44]. It is necessary to make a decision of five emergency plans $A = \{a_1, a_2.a_3, a_4, a_5\}$ according to the following six criteria $C = \{c_1, c_2, c_3, c_4, c_5, c_6\}$: Effectiveness c_1, Operability c_2, Completeness c_3, Rapidity c_4, Flexibility c_5, Rationality c_6, with the associated weight vector $W = \{0.12, 0.15, 0.18, 0.25, 0.2, 0.1\}$. In addition, there are three DMs $D = \{d_1, d_2, d_3\}$ whose weight vector is $\lambda = \{0.35, 0.4, 0.25\}$, given the assessment with the linguistic term set $H = \{h_1 = \textit{very poor}, h_2 = \textit{poor}, h_3 = \textit{medium poor}, h_4 = \textit{fair}, h_5 = \textit{medium good}, h_6 = \textit{good}, h_7 = \textit{very good}\}$ Suppose the utilities of linguistic variables are $U = \{u(h_1) = 0, u(h_2) = 0.167, u(h_3) = 0.333, u(h_4) = 0.5, u(h_5) = 0.667, u(h_6) = 0.833, u(h_7) = 1,\}$. The linguistic evaluation matrices of these three DMs are shown in Tables 1–3.

4.1 Procedure for Linguistic MCGDM

Step 1: Transform the linguistic variable into the cloud.

Given the universe $[U_{max}, U_{min}] = [0, 1]$ and $He = 0.01$, using Definition 5, the linguistic variables in the linguistic term set can be transformed into 7 clouds as $c_{h_1}(0.071, 0.024, 0.01)$, $c_{h_2}(0.214, 0.024, 0.01)$, $c_{h_3}(0.375, 0.024, 0.01)$, $c_{h_4}(0.5, 0.024, 0.01)$, $c_{h_5}(0.643, 0.024, 0.01)$, $c_{h_6}(0.786, 0.024, 0.01)$, $c_{h_7}(0.929, 0.024, 0.01)$, Then, the assessment information can be represented by these 7 clouds.

Step 2: Aggregate the clouds of DMs on each criterion.

Table 1 Evaluation matrix of d_1

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	h_3	h_4	h_2	h_2	h_1	h_3
a_2	h_2	h_1	h_5	h_5	h_2	h_4
a_3	h_6	h_5	h_6	h_7	h_4	h_6
a_4	h_3	h_2	h_4	h_5	h_2	h_3
a_5	h_3	h_2	h_4	h_2	h_3	h_6

Table 2 Evaluation matrix of d_2

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	h_4	h_2	h_3	h_3	h_4	h_2
a_2	h_3	h_2	h_4	h_3	h_6	h_4
a_3	h_5	h_7	h_5	h_7	h_6	h_4
a_4	h_3	h_2	h_5	h_3	h_5	h_4
a_5	h_7	h_1	h_2	h_4	h_3	h_1

Table 3 Evaluation matrix of d_3

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	h_5	h_3	h_2	h_4	h_1	h_1
a_2	h_2	h_5	h_3	h_5	h_4	h_4
a_3	h_6	h_7	h_5	h_7	h_5	h_6
a_4	h_2	h_6	h_3	h_4	h_5	h_3
a_5	h_4	h_2	h_4	h_2	h_3	h_1

For each criteria, using Definition 6, the asymmetrical weighted synthetic cloud can be obtained by aggregating the clouds of d_1, d_2, d_3. Then, the group evaluation matrix is shown in Table 4.

Step 3: Convert the cloud into the belief degree.

For each asymmetrical weighted synthetic cloud, use Definition 7 and Definition 9 to obtain the distribution of the discounted belief degree. The result is shown in Table 5.

Step 4: Aggregate the criteria values of each alternative.

Firstly, using Definition 10, the distribution of the discounted belief degree of each criterion is transformed into the BPA, as shown in Table 6. Next, the BPAs of all six criteria are aggregated into the combined probability assignment for each alternative using Definition 11. The result is shown in Table 7. Finally, using Definition 12, normalize the combined probability assignments into the overall belief degree, as described in Table 8.

Step 5: Rank alternatives.

The average utility of each alternative can be acquired using Definition 13 as follows: $u_{avg}(a_1) = 0.3169$, $u_{avg}(a_2) = 0.4572$, $u_{avg}(a_3) = 0.7958$, $u_{avg}(a_4) = 0.4530$, $u_{avg}(a_5) = 0.3454$. According to the average utility, the order of alternatives can be determined, and they are as follows: $a_3 > a_2 > a_4 > a_5 > a_1$.

4.2 Comparative Analysis and Discussion

For validation of the feasibility of the proposed method, a comparative analysis is executed by using another four linguistic MCDM methods. This analysis uses the same numerical example as described in Sect. 4.1. The results are shown in Table 9.

As Table 9 shows, the ranking of alternatives in Sect. 4.1 is identical to the results of the methods in [10] and [16], but is different from the result of the method in [25] and [26]. A uniform granular linguistic assessment scale is adopted in [10] and [16], but a multi-granular linguistic assessment scale is applied in [25] and [26]. The linguistic concept involves uncertainties due to the inherent subjective nature. Different individuals hold different understanding about the linguistic concept and even the same individual may hold different opinions at different times. Not only the average levels of evaluation information but also the

Table 4 Group evaluation matrix.

	c_1	c_2	c_3
a_1	$\tilde{c}_{11}(0.4857, 0.0669, 0.0764, 0.01)$	$\tilde{c}_{12}(0.3498, 0.0693, 0.0741, 0.01)$	$\tilde{c}_{13}(0.2712, 0.0431, 0.0526, 0.01)$
a_2	$\tilde{c}_{21}(0.2712, 0.0431, 0.0526, 0.01)$	$\tilde{c}_{22}(0.2712, 0.0907, 0.1479, 0.01)$	$\tilde{c}_{23}(0.5143, 0.0764, 0.0669, 0.01)$
a_3	$\tilde{c}_{31}(0.7288, 0.0526, 0.0431, 0.01)$	$\tilde{c}_{32}(0.8289, 0.086, 0.574, 0.01)$	$\tilde{c}_{33}(0.693, 0.0407, 0.055, 0.01)$
a_4	$\tilde{c}_{41}(0.3212, 0.0597, 0.0359, 0.01)$	$\tilde{c}_{42}(0.357, 0.0717, 0.167, 0.01)$	$\tilde{c}_{43}(0.5214, 0.0788, 0.0645, 0.01)$
a_5	$\tilde{c}_{51}(0.6215, 0.1122, 0.1265, 0.01)$	$\tilde{c}_{52}(0.1568, 0.0526, 0.0431, 0.01)$	$\tilde{c}_{53}(0.3856, 0.0812, 0.0621, 0.01)$
	c_4	c_5	c_6
a_1	$\tilde{c}_{14}(0.3427, 0.0669, 0.0764, 0.01)$	$\tilde{c}_{15}(0.2426, 0.0812, 0.1098, 0.01)$	$\tilde{c}_{16}(0.2283, 0.0764, 0.0669, 0.01)$
a_2	$\tilde{c}_{24}(0.5286, 0.0812, 0.0621, 0.01)$	$\tilde{c}_{25}(0.5143, 0.1241, 0.1146, 0.01)$	$\tilde{c}_{26}(0.5, 0.024, 0.024, 0.01)$
a_3	$\tilde{c}_{34}(0.929, 0.024, 0.024, 0.01)$	$\tilde{c}_{35}(0.5929, 0.044, 0.407, 0.01)$	$\tilde{c}_{36}(0.786, 0.024, 0.024, 0.01)$
a_4	$\tilde{c}_{44}(0.4929, 0.0693, 0.0741, 0.01)$	$\tilde{c}_{45}(0.4929, 0.1169, 0.0741, 0.01)$	$\tilde{c}_{46}(0.4142, 0.0431, 0.0526, 0.01)$
a_5	$\tilde{c}_{54}(0.3284, 0.0621, 0.0812, 0.01)$	$\tilde{c}_{55}(0.357, 0.024, 0.024, 0.01)$	$\tilde{c}_{56}(0.3212, 0.1074, 0.1789, 0.01)$

Table 5 Distribution of the discounted belief degree.

	c_1	c_2	c_3
a_1	$\tilde{\beta}'_{11}\{0, 0, 0.1005, 0.6827, 0.0501, 0.0001, 0\}$	$\tilde{\beta}'_{12}\{0, 0.0828, 0.6883, 0.0622, 0.0001, 0, 0\}$	$\tilde{\beta}'_{13}\{0.0001, 0.6042, 0.312, 0.0004, 0, 0, 0\}$
a_2	$\tilde{\beta}'_{21}\{0.0001, 0.6042, 0.312, 0.0004, 0, 0, 0\}$	$\tilde{\beta}'_{22}\{0.0157, 0.3537, 0.2494, 0.0464, 0.0015, 0, 0\}$	$\tilde{\beta}'_{23}\{0, 0, 0.0508, 0.6845, 0.098, 0.0001, 0\}$
a_3	$\tilde{\beta}'_{31}\{0, 0, 0, 0, 0.3406, 0.5757, 0.004\}$	$\tilde{\beta}'_{32}\{0, 0, 0, 0, 0.0256, 0.6065, 0.2013\}$	$\tilde{\beta}'_{33}\{0, 0, 0, 0, 0.0006, 0.6606, 0.2554, 0.001\}$
a_4	$\tilde{\beta}'_{41}\{0.0002, 0.1967, 0.7188, 0.001, 0, 0, 0\}$	$\tilde{\beta}'_{42}\{0, 0.0596, 0.4236, 0.1574, 0.0246, 0.0014, 0\}$	$\tilde{\beta}'_{43}\{0, 0, 0, 0.6606, 0.2554, 0.0001\}$
a_5	$\tilde{\beta}'_{51}\{0, 0, 0.0054, 0.1743, 0.3845, 0.098, 0.0044\}$	$\tilde{\beta}'_{52}\{0.3234, 0.05929, 0.0004, 0, 0, 0, 0\}$	$\tilde{\beta}'_{53}\{0, 0.0336, 0.6639, 0.1357, 0.0001, 0, 0\}$
	c_4	c_5	c_6
a_1	$\tilde{\beta}'_{14}\{0.0001, 0.6042, 0.312, 0.0004, 0, 0, 0\}$	$\tilde{\beta}'_{15}\{0.0338, 0.4977, 0.2127, 0.0058, 0.0001, 0, 0\}$	$\tilde{\beta}'_{16}\{0.0508, 0.6845, 0.098, 0.0001, 0, 0, 0\}$
a_2	$\tilde{\beta}'_{24}\{0, 0, 0.0357, 0.6592, 0.1365, 0.0001, 0\}$	$\tilde{\beta}'_{25}\{0, 0.0028, 0.1063, 0.402, 0.1512, 0.0043, 0\}$	$\tilde{\beta}'_{26}\{0, 0, 0.0005, 0.9978, 0.0017, 0, 0\}$
a_3	$\tilde{\beta}'_{34}\{0, 0, 0, 0, 0, 0.0012, 0.9988\}$	$\tilde{\beta}'_{35}\{0, 0, 0.0002, 0.2654, 0.6507, 0.0004, 0\}$	$\tilde{\beta}'_{36}\{0, 0, 0, 0, 0.0011, 0.9968, 0.0021\}$
a_4	$\tilde{\beta}'_{44}\{0, 0, 0.0877, 0.6926, 0.0529, 0.0001\}$	$\tilde{\beta}'_{45}\{0, 0.0048, 0.171, 0.5265, 0.0476, 0.0001, 0\}$	$\tilde{\beta}'_{46}\{0, 0.0006, 0.6086, 0.3074, 0.0001, 0, 0\}$
a_5	$\tilde{\beta}'_{54}\{0.0001, 0.1503, 0.6444, 0.0385, 0.0001, 0, 0\}$	$\tilde{\beta}'_{55}\{0, 0.0011, 0.9968, 0.0021, 0, 0, 0\}$	$\tilde{\beta}'_{56}\{0.0052, 0.1861, 0.274, 0.1015, 0.0157, 0.0007, 0\}$

Table 6 Basic probability assignment.

	c_1	c_2	c_3
a_1	$m_{1,1}\{0, 0, 0.0121, 0.0819, 0.006, 0, 0, 0.88, 0.02\}$	$m_{1,2}\{0, 0.0124, 0.1033, 0.0093, 0, 0, 0, 0.85, 0.025\}$	$m_{1,3}\{0, 0.1088, 0.0562, 0, 0, 0, 0, 0.82, 0.015\}$
a_2	$m_{2,1}\{0, 0.0725, 0.0374, 0.0001, 0, 0, 0, 0.88, 0.02\}$	$m_{2,2}\{0.0024, 0.053, 0.0374, 0.007, 0.0002, 0, 0, 0.85, 0.05\}$	$m_{2,3}\{0, 0, 0.0091, 0.1232, 0.0176, 0, 0, 0.82, 0.03\}$
a_3	$m_{3,1}\{0, 0, 0, 0, 0.0409, 0.0691, 0, 0.88, 0.01\}$	$m_{3,2}\{0, 0, 0, 0, 0.0038, 0.091, 0.0302, 0.85, 0.025\}$	$m_{3,3}\{0, 0, 0, 0.0001, 0.1189, 0.046, 0, 0.82, 0.03\}$
a_4	$m_{4,1}\{0, 0.0236, 0.0863, 0.0001, 0, 0, 0, 0.88, 0.01\}$	$m_{4,2}\{0, 0.009, 0.0635, 0.0236, 0.0037, 0.0002, 0, 0.85, 0.05\}$	$m_{4,3}\{0, 0, 0.0079, 0.1213, 0.0208, 0, 0, 0.82, 0.03\}$
a_5	$m_{5,1}\{0, 0, 0.0007, 0.0209, 0.0461, 0.0118, 0.0005, 0.88, 0.04\}$	$m_{5,2}\{0.0485, 0.0889, 0.0001, 0, 0, 0, 0, 0.85, 0.0125\}$	$m_{5,3}\{0, 0.0061, 0.1195, 0.0244, 0, 0, 0, 0.82, 0.03\}$
	c_4	c_5	c_6
a_1	$m_{1,4}\{0, 0.0251, 0.1707, 0.0125, 0, 0, 0, 0.75, 0.0417\}$	$m_{1,5}\{0.0068, 0.0995, 0.0425, 0.0012, 0, 0, 0, 0.8, 0.05\}$	$m_{1,6}\{0, 0, 0, 0.0998, 0.0002, 0, 0, 0.9, 0\}$
a_2	$m_{2,4}\{0, 0, 0.0094, 0.1648, 0.0341, 0, 0, 0.75, 0.0417\}$	$m_{2,5}\{0, 0.0006, 0.0212, 0.0804, 0.0302, 0.0009, 0, 0.8, 0.0667\}$	$m_{2,6}\{0, 0, 0, 0.1, 0, 0, 0, 0.9, 0\}$
a_3	$m_{3,4}\{0, 0, 0, 0, 0, 0.0003, 0.2497, 0.75, 0\}$	$m_{3,5}\{0, 0, 0, 0.0531, 0.1301, 0.0001, 0, 0.8, 0.0167\}$	$m_{3,6}\{0, 0, 0, 0, 0.0001, 0.0997, 0.0002, 0.9, 0\}$
a_4	$m_{4,4}\{0, 0, 0.0219, 0.1732, 0.0132, 0, 0, 0.75, 0.0417\}$	$m_{4,5}\{0, 0.001, 0.0342, 0.1053, 0.0095, 0, 0, 0.8, 0.05\}$	$m_{4,6}\{0, 0.0001, 0.0609, 0.0307, 0, 0, 0, 0.9, 0.0083\}$
a_5	$m_{5,4}\{0, 0.0376, 0.1611, 0.0096, 0, 0, 0, 0.75, 0.0417\}$	$m_{5,5}\{0, 0.0002, 0.1994, 0.0004, 0, 0, 0, 0.8, 0\}$	$m_{5,6}\{0.0005, 0.0186, 0.0274, 0.0102, 0.0016, 0, 0, 0.9, 0.0417\}$

Table 7 Combined probability assignment.

	$m_{i,I(6)}(h_1)$	$m_{i,I(6)}(h_2)$	$m_{i,I(6)}(h_3)$	$m_{i,I(6)}(h_4)$	$m_{i,I(6)}(h_5)$	$m_{i,I(6)}(h_6)$	$m_{i,I(6)}(h_7)$	$\bar{m}_{i,I(6)}(H)$	$\tilde{m}_{i,I(6)}(H)$
a_1	0.0064	0.1978	0.2639	0.0574	0.0032	0	0	0.3849	0.0864
a_2	0.0013	0.0701	0.066	0.3326	0.0492	0.0005	0	0.3784	0.1019
a_3	0	0	0	0.0292	0.1755	0.177	0.17	0.4142	0.0341
a_4	0	0.018	0.1669	0.3134	0.0273	0.0002	0	0.3774	0.0968
a_5	0.0264	0.0869	0.3559	0.0358	0.0241	0.006	0.0003	0.3826	0.082

Table 8 Distribution of the overall belief degree.

	$\beta_i(h_1)$	$\beta_i(h_2)$	$\beta_i(h_3)$	$\beta_i(h_4)$	$\beta_i(h_5)$	$\beta_i(h_6)$	$\beta_i(h_7)$	$\beta_i(H)$
a_1	0.0104	0.3217	0.429	0.0933	0.0051	0	0	0.1405
a_2	0.002	0.1128	0.1062	0.5351	0.0791	0.0008	0	0.164
a_3	0	0	0	0.0498	0.2966	0.3021	0.2903	0.0582
a_4	0	0.0289	0.2681	0.5034	0.0438	0.0003	0	0.1555
a_5	0.0428	0.1408	0.5764	0.0579	0.0391	0.0097	0.0004	0.1329

fluctuation is considered in cloud model using En and He. FAWSCG realizes the conversion from a qualitative linguistic concept to its quantitative value According to FAWSCG, a cloud drop is generated by two random numbers. Thus, the corresponding cloud drops generated by FAWSCG each time are not the same, though they follow the same distribution. Obviously, the belief degrees calculated from the cloud drops are not the same neither. We suppose that the group assessment consists of the linguistic assessment of each decision maker, just the belief degree of each linguistic assessment is different. The imprecision and incompleteness of the group assessment are caused by aggregation of DMs assessments. Evidence theory can model the imprecision and incompleteness with the ignorance degree which is the probability mass assigned to the whole set. The ER algorithm is adopted to aggregate the group assessments on each criterion. By comparison, the aggregation results of the existing methods which use the traditional aggregation operators such as the ordered weighted geometric averaging (OWGA) operator and the ordered weighted averaging (OWA) operator, cannot reflect the uncertainty of the group assessment. Because evidence theory can model the ignorance degree in the belief structure, which is neglected by the traditional aggregation operators, the proposed method exhibits a higher performance than the methods mentioned above. We repeat the proposed method 5000 times, and the results are shown in Table 10.

The frequency of $a_3 > a_2 > a_4 > a_5 > a_1$ is much higher

Table 9 Comparison with different methods.

Methods	*Ranking results*
the method based on membership functions [10]	$a_3 > a_2 > a_4 > a_5 > a_1$
the method based on 2 − *tuple linguistic model* [16]	$a_3 > a_2 > a_4 > a_5 > a_1$
the method based on a cloud mode [25]	$a_3 > a_4 > a_2 > a_5 > a_1$
the method based on cloud aggregation operators [26]	$a_3 > a_4 > a_2 > a_5 > a_1$

Table 10 Results of repeating the proposed method 5000 times.

Ranking results	*Frequency*
$a_3 > a_2 > a_4 > a_5 > a_1$	4986
$a_3 > a_4 > a_2 > a_5 > a_1$	14

Table 11 Maximum average utility and minimum average utility

	min	*max*
$u_{avg}(a_1)$	0.3138	0.3200
$u_{avg}(a_2)$	0.4541	0.4595
$u_{avg}(a_3)$	0.7942	0.7985
$u_{avg}(a_4)$	0.4514	0.4565
$u_{avg}(a_5)$	0.3438	0.3474

than $a_3 > a_4 > a_2 > a_5 > a_1$. The former appeared 4986 times, and the latter appeared 14 times. The difference lies in the sequence of a_2 and a_4. The maximum average utility and the minimum average utility of each alternative in the 5000 times are shown in Table 11.

It is clear from Table 11 that the average utilities of a_2 and a_4 are close, and the minimum average utility of a_2 is not larger than the maximum utilities of a_4. So it is possible that a_4 is preferred to a_2, though the average level of a_2 is higher than a_4.

The result of the proposed method prefers to $a_3 > a_2 > a_4 > a_5 > a_1$ and includes $a_3 > a_4 > a_2 > a_5 > a_1$ at a lower frequency at the same time, which is the result of multi-granular linguistic assessment scale. Traditional methods totally abandon the uncertainty of the qualitative linguistic concept and the uncertainty caused by aggregation of DMs assessments. However, in the proposed method, cloud model can handle the fuzziness and randomness of the linguistic concept, and evidence theory can handle the imprecision and incompleteness of the group assessment. So the proposed method can get the results of different understanding for the same linguistic concept, and prefer the more common one. Take the factors mentioned previously into consideration, the proposed method performs more reliable and precise than traditional methods.

5. Conclusion

The linguistic MCGDM problem is widespread in the areas such as science, business, engineering, and military applications. Most real linguistic MCGDM problems involve various types of uncertainties. However, the existing methods are not robust enough to completely reflect the uncertainties caused by both the linguistic concept and group assessment. In this paper, we propose a new linguistic MCGDM method combining cloud model and evidence theory. The new method can take advantage of cloud model and evidence theory to deal with the fuzziness and randomness of the linguistic concept, as well as the imprecision and incompleteness of the group assessment. The contributions of this paper are fourfold. First, a method for transforming linguistic variables into the corresponding clouds is proposed. Second, an asymmetrical weighted synthetic cloud is presented to aggregate the clouds of different DMs. Third, a method for converting the cloud into the belief degree is developed. Last but not the least, the ER algorithm and the average utility are adopted to aggregate different criteria values and rank the alternatives respectively. Moreover, a numerical example together with the corresponding comparison analysis with other methods is used to demonstrate the feasibility and validity of the proposed method. The results show that the new method can get the results of different understanding for the same linguistic concept, and prefer the more common one. Thus, this method can deal with uncertainties caused by both the linguistic concept and group assessment, which neutralize most of the information distortion. In this paper, different DMs use the same linguistic term set which has the same domain and utility. In future research, we will apply our method in resolution of the multigranular MCGDM problem.

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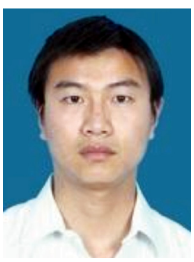

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