LETTER

A Novel Robust Carrier Activation Selection Scheme for OFDM-IM System with Power Allocation*

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SUMMARY In this paper, we investigate the subcarriers combination selection and the subcarriers activation of OFDM-IM system. Firstly, we propose an algorithm to solve the problem of subcarriers combination selection based on the transmission rate and diversity gain. Secondly, we ropose a more concise algorithm to solve the problem of power allocation and carrier combination activation probability under this combination to improve system capacity. Finally, we verify the robustness of the algorithm and the superiority of the system scheme in the block error rate (BLER) and system capacity by numerical results.

key words: *OFDM-IM, algorithm, combination, system capacity*

1. Introduction

Index modulation (IM) is a derivative form of spatial modulation (SM) in MIMO systems [1], [2], which has great attraction in spectrum efficiency and energy efficiency. Literatures [3], [4] proposed the combination of IM and OFDM transmission technology, called OFDM-IM. As a kind of modulation scheme, how to encode the input bits into the modulation symbol and the index of subcarriers becomes the key problem to improve the superiority of the system. M. Wen proposed that the system could generate a certain coding gain by using an equiprobable subcarrier activation (ESA) scheme [7], but without any diversity gain. S. Dang proposed two mapping selection schemes based on on-ff keying (OOK) to provide more flexible mapping relations and frequency diversity gain [8], [9]. However, the varying number of activated subcarriers increases the difficulty of receiver detection. In [11], a novel codebook design scheme for OFDM-IM is proposed, which is proved to be simpler without increasing the block error rate (BLER). However, the scheme lacks a complete subcarriers combination selection algorithm and ignores corresponding system capacity problem. So we propose corresponding algorithms to further improve the system performance.

Manuscript received April 3, 2020.
Manuscript revised September 7, 2020.
Manuscript publicized October 2, 2020.

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*This work was supported in part by the NSF of China # 61661005 and # 61761007 and # 61761006, and the NSF of Guangxi # 2018GXNSFAA138079 and # 2017GXNSFAA198276.

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DOI: 10.1587/transinf.2020EDL8052

2. System Model and Optimization Problems-MaxU

We consider the improved OFDM-IM system with lexicographic order, called LO-OFDM-IM in this letter. The system first sorts the subcarriers based on lexicographic order, and then selects the subcarriers participating in constellation modulation according to index bits '0'(silent) - '1'(active). As shown in Fig. 1, the transmitter contains N subcarriers, corresponding to N points IFFT. Assuming that m bits are input for each frame, including P_i index bits and P_m constellation modulation bits. The first P_i bits are input as the activation vector to determine the number and combination of activated subcarriers based on lexicographic order. And then the remaining P_m bits are modulated into constellation symbols by the activated subcarriers and input to the next level. Without loss of generality, we represent the input of the system as $\mathbf{x}(c, \mathbf{b})$.

For the M-order constellation modulation, the transmission rate B can be expressed as $B = \left\lfloor \log_2 \binom{N}{K} \right\rfloor + K \log_2 M$, where (:) is the binomial coefficient and K represents the number of activated subcarriers. It can prove that B is a non monotonic function. After the index mapping and constellation mapping, the LO-OFDM-IM system converts the data into parallel series, transforms the frequency-domain data into time-domain data through N-points IFFT, and then adds CP to complete the baseband data processing. After sampling, discarding CP and performing N-point FFT, we can express the received OFDM block $\mathbf{y}(c, \mathbf{b})$ as follows

$$\mathbf{y}(c, \mathbf{b}) = \sqrt{\frac{P_t}{K}} \mathbf{H}\mathbf{x}(c, \mathbf{b}) + \mathbf{w}, \tag{1}$$

where P_t is the total transmission power; $\mathbf{w}$ denotes N

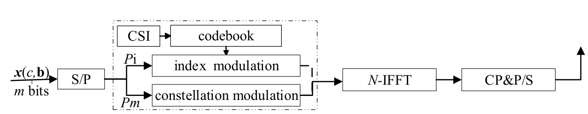

Fig. 1 Schematic diagram of the transmitting part of LO-OFDM-IM.

independently combined additive white Gaussian noise (AWGN) with N_0 noise power density sampled vector on each subcarrier; $\mathbf{H}$ is an $N{\times}N$ diagonal channel state matrix characterizing the channel quality.

In this letter, we consider the slow fading channel model, and assume that the optimized codebook and CSI are known. The detection results $\hat{\mathbf{x}}(c, \hat{\mathbf{b}})$ based on ML criterion can be expressed as follows:

$$\hat{\mathbf{x}}(c, \hat{\mathbf{b}}) = \mathop{\arg\min}_{\mathbf{x}(c,\mathbf{b})\in\mathcal{X}(c)} \left\| \dot{\mathbf{y}}(c, \dot{\mathbf{b}}) - \sqrt{\frac{P_t}{K}}\mathbf{H}\mathbf{x}(c, \mathbf{b}) \right\|_F . \tag{2}$$

Obviously, the subcarriers combination number S must be the power of 2, that is $S = 2^{\lfloor \log_2 \binom{N}{K} \rfloor}$. However, mathematically there are $L = \binom{N}{K}$ combinations of subcarriers in total. In other words, $\Delta(N, K)$ combinations need to be filtered, where $\Delta(N, K) = L - S$. It leads to the problem of how to select subcarriers from so many possibilities. In order to quantify the quality of the selected scheme, we introduce the concept of the diversity gain d_o to LO-OFDM-IM system:

$$d_o = \begin{cases} 1, \Delta(N, K) < \Omega(N, K, 1) \\ \nu, \Omega(N, K, \nu - 1) \le \Delta(N, K) < \Omega(N, K, \nu), \end{cases} \tag{3}$$

where $\Omega(N, K, \nu){=}\sum_{\xi=1}^{\nu} \binom{N-\xi}{K-1}$ represents the sum of the number of combinations out of L with ones in the ν most significant bits.

To represent the trade-off between index modulation and constellation modulation, We define the joint gain of the system $U(N, K, M){=}d_o^w B^{1-w}$, where $w \in [0, 1]$ can be given or specified adaptively according to the channel environment and quality of service requirements.

Apparently the function $U(N, K, M)$ is discontinuous. And the relationship between function and its parameters is nonlinear. So, we propose the optimization of the maximum joint gain:

$$\begin{aligned} &[N, K] = \arg\max \left\{ U = d_o^w B^{1-w} \right\}, \\ &s.t. \quad 2 \le N \le N_{max}, 1 \le K \le N - 1, \end{aligned} \tag{4}$$

where N_{max} is the maximum subcarriers number. The problem must meet the requirements of $d_o > \underline{d_o}$ and $B > \underline{B}$, where $\underline{d_o}$ and $\underline{B}$ respectively represent the minimum diversity gain and transmission rate that meet the requirements.

3. Optimization Analysis

3.1 Optimization Algorithm

For $U(N, K, M)$, a large N_{max} or a varied w will bring considerable computational overhead. Genetic algorithm (GA) has the advantage of not being constrained by the derivative and continuity and it has better parallelism and global optimization capability. Because of these advantages, We design a scheme based on GA to solve the problem.

The core steps of GA are selection, crossover and mutation. In order to avoid falling into the local optimum, we select and improve the elite retention strategy. We use the best solution of the parent generation replaces the worst solution, which ensures that the evolution process will not degenerate and genetic diversity.

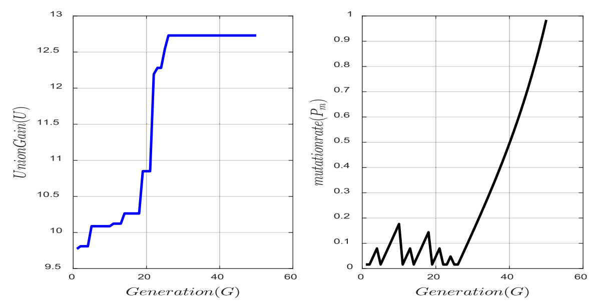

Fig. 2 Relation among U and G and Relation among P_m and G

Obviously, the solution $[N, K]$ of the optimization problem corresponds to binary encoding, where the first half represent N and the last represent K. If it satisfies the probability P_c, the paternal genes of these two parts perform crossover operations at random. And the number of N and K genes involved in crossover are given randomly.

Finally, we consider the mutation operator. We designed mutation probability P_m to be a function rather than a fixed value. P_m is equal to $tan[(0.0202NOP - 0.01)\pi/2]$, where NOP represents the algebra of keeping the optimal value unchanged. The relationship between P_m and evolution generation is shown in Fig. 2. It can be seen that the P_m keeps a low level but P_m keeps rising to improve the probability of jumping out when it may fall into the local optimum. Under the premise of P_m, the system performs one or two reverse operations on the N and K genes respectively. After defining the critical operations, we can give the basic steps as Algorithm 1 in the next page.

3.2 System Capacity and BLER

The system capacity consists of two parts, the modulated symbol space X_S for the M-ary constellation, and the index modulation space X_{IM}. The capacity corresponding to the active index combinations symbols selected probability vector $\mathbf{p}$ can be expressed as [10]–[12]:

$$\begin{aligned} C(\mathbf{P}) = &\sum_{i}^{S} P(\tau_i) \sum_{n\in\tau_i} \log_2 \left(1 + \gamma\rho|h(n)|^2\right) \\ &+ \sum_{i}^{S} P(\tau_i) \int_{\mathbf{y}} P(\mathbf{y}|X_{IM} = \tau_i) \log_2 \left(\frac{P(\mathbf{y}|X_{IM} = \tau_i)}{p(\mathbf{y})} \right) d\mathbf{y}, \end{aligned} \tag{5}$$

where τ_i represents the *i-th* active index combinations; $P(\tau_i)$ is the corresponding activation probability; γ is the average power of subcarriers. The signal-to-noise ratio (SNR) for LO-OFDM-IM is defined as $\rho{=}Eb/N_0$, and $Eb{=}(N{+}N_{cp})/m$ [joules/bit] denotes the average transmitted energy per bit. The posterior probability of the received signal and the likelihood probability are denoted as $P(\mathbf{y})$ and $P(\mathbf{y}|X_{IM} = \tau_i)$ respectively. We can use $h_{i,n}$ to represent $\rho|h(n)|^2, n \in \tau_i$.

Algorithm 1 : Optimization Algorithm for Problem MaxU

Initialize: $N_{max}, M, NOP, w,$ and w, the Maximum Generation G_{max}, P_c, the population $V_p, \underline{B}, \underline{d_o}$ are set under the QOS. Initialize population genes $[N_i, K_i]$ of the $\overline{i_{th}}$ individual, which is randomly selected in the range of $2 \le N_i \le N_{max}$ and $1 \le K_i \le \lfloor N_i/2 \rfloor$.
For $(G \le G_{max})$**:**
calculate the fitness of each individual and return the smallest one;
Replace the worst chromosomes with the best of the parents;
Return the current optimal chromosomes. If the optimal fitness value keep the same, $NOP++$, otherwise $NOP = 1$;
Transform the chromosomes into the binary ones;
Chromosomes crossing under probability P_c. Randomly matched individuals, give the crossing positions of N and K at random, and replace the corresponding positions;
$P_m \longleftarrow tan[(0.0202NOP - 0.01)\pi/2]$;
Chromosomes variation under probability P_m. Mutate one (probability of 2/5) or two (probability of 3/5) genes of N and K;
Transform the chromosomes into the real ones and legalize;
$G \longleftarrow G + 1$.
End: The convergence criterion is satisfied.

Algorithm 2 : Algorithm for Activation Probability Vector **P**

Initialize: Total activation combinations S, initialize $P(\tau_1) \in [1/S, 1/2]$.
For(i≤S):
Update the $P_{re} = 1 - \sum_{n=1}^{i-1} P(\tau_i)$;
Get all the legal values of $P(P\tau_i) \in [P_{re}/S, \min\{P(\tau_{i-1}), P_{re}\}]$;
Update the order number $i \leftarrow i + 1$
End: The convergence criterion is satisfied.

Algorithm 3 : Algorithm for Problem Power Allocation

Initialize: Index bits P_i, $h_{i,n}$, the average power distribution ratio $(1-\theta) \in [0,1]$ which is set under the QOS.
For:
Sort the activated subcarriers in ascending order;
$P_{i,n}^* \leftarrow \frac{1}{K-n+1}\left(\theta P_t - \frac{K-n+1}{h_{i,n}} + \sum_{m=n}^{K} \frac{1}{h_{i,n}}\right)$, increase by n until $P_{i,n}^* \ge 0$ firstly, which is recorded as $P_{i,m}^*$;
$P_{i,n}^* \leftarrow 0, \forall n \in [1, m-1]$;
$P_{i,n}^* = P_{i,m}^* + \frac{1}{h_{i,n}} - \frac{1}{h_{i,m}}\ \forall n \in [m, K]$;
$P_{i,n}^* = P_{i,n}^* + (1-\theta)P_t/K, \forall n \in [1, K]$;
End: The convergence criterion is satisfied.

Obviously, $P(\mathbf{y})$is equal to $\sum_{i=1}^{S} P(\tau_i)p(\mathbf{y}|X_{IM} = \tau_i)$.

$$P(\mathbf{y}|X_{IM} = \tau_i) = \frac{1}{(\pi\sigma^2)^S} \prod_{n\in\tau_i} \frac{1}{1+\gamma h_{i,n}} e^{\frac{-|y(n)|^2}{\sigma^2(1+\gamma h_{i,n})}} \prod_{n\notin\tau_i} e^{\frac{-|y(n)|^2}{\sigma^2}} \tag{6}$$

The literatures [11], [12] presents an iterative algorithm to find the optimal activation probability vector $\mathbf{p}$. Considering that in the binary system, $P(\tau_i) \in 1/2^m$ and $\sum_{m=1}^{S-1} + 1/2^{S-1} = 1$, we an deduce that there are only limited number of legal $\mathbf{p}$. In practice, the legal $\mathbf{p}$ can be obtained by Algorithm 2. After the solution space of $\mathbf{p}$ is obtained, the optimal solution $\mathbf{p}_{opt}$ can be achieved by simple comparison operation. That is, we can get the best $\mathbf{p}$ by simple comparison, and avoid the complex iterative process and invalid calculation of the original algorithm. Now we can further consider the power distribution problem. Considering the constraints of $\sum_{n=1}^{K} P_{i,n} = P_t$ and $P_{i,n} \ge P_{min}$, where P_{min} is the lowest transmission power of every activated subcarrier, we can design the Algorithm 3 referring the literature [12]. The proposed waterfilling algorithm avoids negative value and complex iterative process and improves the decision efficiency.

Then let's move on to the problem of block error rate (BLER), whose probability under the channel $\mathbf{H}$ could be

$$\begin{aligned} &P_e(\dot{\mathbf{x}}(c, \dot{\mathbf{b}}) \to \hat{\mathbf{x}}(c, \hat{\mathbf{b}})|\mathbf{H}) \\ &= \frac{1}{2} erfc\left(\sqrt{\frac{P_t}{2N_0K}} \left\| \mathbf{H}\left(\dot{\mathbf{x}}(c, \dot{\mathbf{b}}) - \hat{\mathbf{x}}(c, \hat{\mathbf{b}})\right) \right\|_F^2\right), \end{aligned} \tag{7}$$

where $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-\eta^2} d_\eta$ is the complementary error function, which can be writed as $erfc(x) \approx \frac{1}{6}e^{-x^2} + \frac{1}{2}e^{-\frac{4}{3}x^2}$. And the channel gain probability density (PDF) is an exponential distribution with μ as the coefficient, recorded as $g(x) = \frac{1}{\mu}\exp(-\frac{x}{\mu})$. Then, Eq. (9) is equal to

$$\begin{aligned} &P_e(\dot{\mathbf{x}}(c, \dot{\mathbf{b}}) \to \hat{\mathbf{x}}(c, \hat{\mathbf{b}})|\mathbf{H}) \\ &\approx \sum_{i=1}^{2} \lambda_i \prod_{m_r=1}^{N} \exp(-\eta_i \frac{P_t}{2N_0K} \delta(r, m_r)|G(m_r)|), \end{aligned} \tag{8}$$

where $\delta(r, m_r)$ is equal to $|\dot{x}(r, \dot{m}_r) - \hat{x}(\hat{r}, m_r)|^2$; $G(x)$ is the allocation function corresponding to $g(x)$; $\lambda = \{\frac{1}{12}, \frac{1}{4}\}$ and $\eta = \{\frac{1}{2}, \frac{2}{3}\}$. Because we introduce lexicographic order in subcarriers selection, PDF of the new $\xi_{(k)}$ order changes to $\phi_{\xi_{(k)}}(x) = k\binom{N}{k}[G(x)]^{k-1}[1-G(x)]^{N-k}g(x)$. Let ε be the adjustment coefficient corresponding to $erfc(x)$, with a value of 1.02. Then the Eq. (11) can be expressed as

$$\begin{aligned} &P_e(\dot{\mathbf{x}}(c, \dot{\mathbf{b}}) \to \hat{\mathbf{x}}(c, \hat{\mathbf{b}})) \\ &= \sum_{i=1}^{2} \lambda_i \prod_{m_r=1}^{N} \varepsilon \frac{N!}{(N-m_r)!} \frac{{\alpha_{i,m_r}}^{\alpha_{i,m_r}-\frac{1}{2}}}{{\beta_{i,m_r}}^{\beta_{i,m_r}-\frac{1}{2}}} \end{aligned} \tag{9}$$

where, the value of β_{i,m_r} is: $\beta_{i,m_r} = N + 1 + \eta_i \frac{\mu P_t}{N_0 K}\delta(m_r)$; and α_{i,m_r} is equal to $\beta_{i,m_r} - m_r$.

4. Numerical Results

In this section, we verify the performance superiority of the proposed scheme by numerical results. To avoid accidental results, all experiments are repeated more than 10^6 trials.

4.1 Optimization Algorithm Performance

Consider the impact parameters P_c, V_p, N_{max}, w, M and the approximate solution ratio *index*, we use the control variable method to draw the correlation curve, including Fig. 3 and Fig. 4, under the conditions of the highest evolution algebra G_{max}=100, $\underline{d_o}$= 2 and $\underline{B}$=10.

On the whole, the algorithm achieve convergence of the state in 50 generations. A appropriate higher crossing rate achieve a faster convergence rate in Fig. 3, because crossing ensures gene diversity. In Fig. 4, it displays the larger

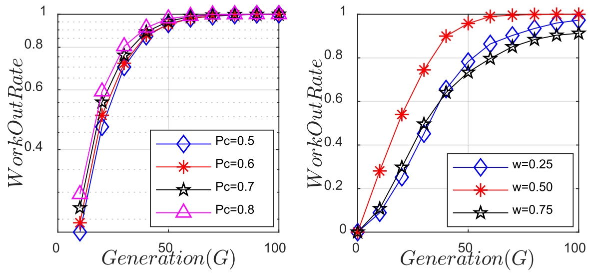

Fig. 3 Relation among *WorkOutRate* and G with P_c=0.5, 0.6, 0.7, 0.8 (left) and relation among *WorkOutRate* and G with w= 0.25, 0.50, 0.75 (right). In the left, M=2, V_p=10, N_{max}=256, w=0.50. In the right, M=2, V_p=10, N_{max}=256, P_c=0.70.

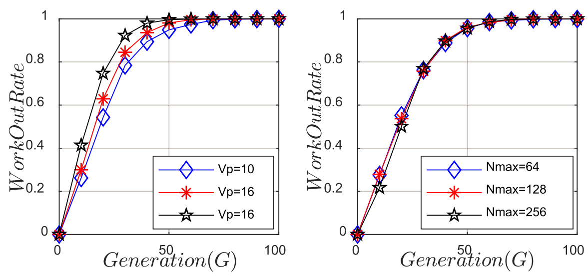

Fig. 4 Relation among *WorkOutRate* and G with V_p=10, 12, 16 (left) and relation among *WorkOutRate* and G with N_{max}=64, 128, 256 (right). In the left, M=2, w=0.50, P_c=0.70, *index*=0.90, N_max =256. In the right, M=2, V_p=10, w=0.50, P_c=0.70, *index*=0.90.

the population is, the faster the solving is, because more individuals bring more genes. We can also see that that factor N_{max} and M have little effect on the convergence speed, which proves the robustness and advantages.

4.2 Performance of System Capacity and BLER

Without loss of generality, we set $\mu = 1$, $N_0 = 1$, simulate the average BLER for the LO-OFDM-IM with BPSK and show in Fig. 5. We select S subcarriers activation patterns out of L total combinations by the lexicographic codebook. According to Eq. (3), only d_o of patterns of [6,2] and [9,4] colud be 2, more than 1. It shows that large diversity gain can bring low BLER. Under the same channel condition, fewer active subcarriers seem to have lower BLER. But the simulation results show that pattern of [9,4] is much better than the [9,2]. This proves the correctness of using the diversity gain as the reference index of patterns selection.

Finally, we compare the capacity and the BLER of LO-OFDM-IM system based on different ativations vector and power allocation modes, as shown in Fig. 6. Of course, the vector **p** is [1/2,1/4,1/8,1/8] when N is 4. It can be seen that the capacity of LO-OFDM-IM is higher than the average ativations vector one, which is named as the convention one. Under the same subcarriers combination conditions, the optimal activation probability vector **p** does have a significant benefit in increasing the system capacity. Because different activation modes have different amounts of mutual information, the LO-OFDM-IM system cannot reach the maximum channel capacity under equal probability. Similarly, the non-uniform distribution of power can also effectively increase the capacity, especially when the SNR ratio is not high. We can see that the optimal **p** brings a lower BLER as a result of higher utilization rate of the better subcarriers. Although the optimal distribution of power will lead to an increase in the block error rate of the system, a better choice can be made by weighing the relationship between the system capacity and the BLER.

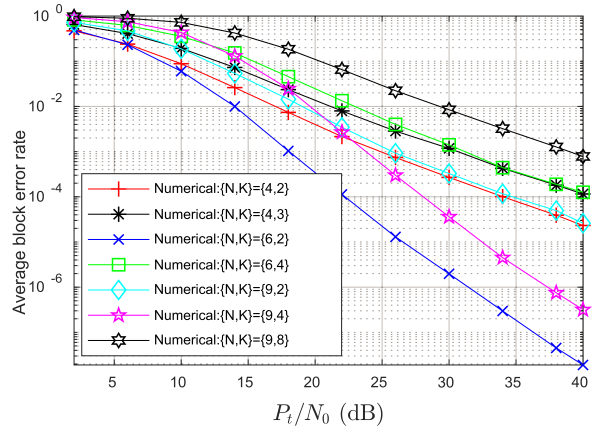

Fig. 5 Average BLER vs. the SNR with different N and K.

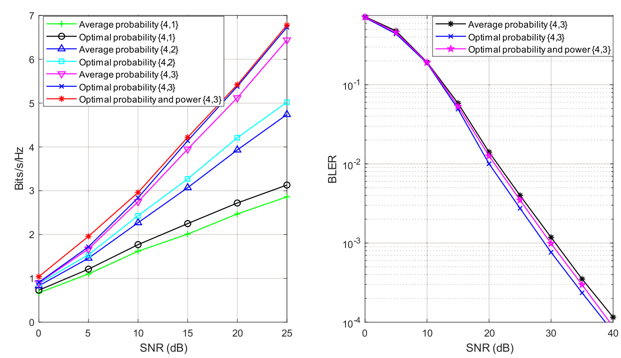

Fig. 6 Relation among Capacity and SNR with different activation probability and power allocation. Relation among BLER and SNR with different activation probability and power allocation.

5. Conclusion

For the problem of subcarriers combination, we give a robust algorithm based on GA. According to the scheme given by the algorithm, we achieve the optimization of $U(N, K, M)$ and BLER. At the same time, it verifies that under the fast power allocation scheme we provide, the system capacity is further improved and the BKER is almost unaffected. In a word, the proposed scheme can effectively improve the system performance.

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