

LETTER

Estimation of Optimal Parameter in ε -Filter Based on Signal-Noise Decorrelation

Mitsuharu MATSUMOTO^{†a)} and Shuji HASHIMOTO^{††}, *Members*

SUMMARY ε -filter is a nonlinear filter for reducing noise and is applicable not only to speech signals but also to image signals. The filter design is simple and it can effectively reduce noise with an adequate filter parameter. This paper presents a method for estimating the optimal filter parameter of ε -filter based on signal-noise decorrelation and shows that it yields the optimal filter parameter concerning a wide range of noise levels. The proposed method is applicable where the noise to be removed is uncorrelated with signal, and it does not require any other knowledge such as noise variance and training data.

key words: parameter optimization, nonlinear filter, ε -filter

1. Introduction

Noise reduction plays an important role in signal processing. It is crucial not only for acoustical signal processing but also for image processing. In acoustical signal processing, the approaches to noise reduction can be categorized into two types: multi-channel signal processing and single-channel signal processing. There are many studies about multi-channel signal processing for noise reduction in acoustical signal processing such as microphone array [1], independent component analysis [2], and sparseness approaches [3]. However, single-channel approaches have several advantages compared to multi-channel approaches, e.g. system downsizing, system applicability and system simplification.

The spectral subtraction (SS) is a well-known approach for reducing the noise from a single-channel signal [4]. It can reduce the noise effectively despite the simple procedure. However, it can handle only the stationary noise. It also needs to estimate the noise in advance. Although Kalman filter technique has been used for noise reduction [5], [6], the calculation cost is relatively large due to its statistical calculation. Some authors have reported a model-based approach for noise reduction [7]. In this approach, we can extract the objective sound by learning the sound model in advance. However, it is not applicable to the signals with the unknown noise as well as SS.

To solve the problems, we look to a nonlinear filter labeled ε -filter, which has various applications in signal pro-

cessing [8]–[10]. It can reduce the noise with preserving the speech signal in acoustical signal processing, while it can reduce the noise with preserving the edge in image signal processing. ε -filter is simple and has some desirable features for noise reduction. It does not need to have the signal and noise models in advance. It is easy to be designed and the calculation cost is small because it requires only switching and linear operation. In image processing, although many studies have been reported to reduce the noise with preserving the edge [11]–[16], it is considered that ε -filter is a promising approach because of its simple design.

However, empirical operations were required to set the adequate parameter of ε -filter to reduce the noise effectively. Moreover, we only have a single-channel noisy signal, that is, the original signal and noise are unknown. Hence, even if we set the parameter of ε -filter manually, it is difficult to evaluate whether the parameter is optimal or not. We cannot know the difference between the original signal and the filter output from the observed signal. So far, there are no studies on the appropriateness of the parameter setting of ε -filter.

Recently, decorrelation criterion is proposed to select the optimal stopping time for nonlinear diffusion filtering [17]. The algorithm is simple and can be applied where the noise to be removed is uncorrelated with signal, and it does not require any other knowledge such as noise variance and training data. In this paper, we employ the criterion to obtain the optimal parameter of ε -filter. By using the proposed method, we can obtain the optimal parameter of ε -filter based on decorrelation criterion without any other knowledge such as noise variance and training data as well as the study [17].

2. ε -Filter

We firstly explain the algorithm of ε -filter. To clarify the feature of ε -filter, we first describe the one dimensional case. Let us define $x(k)$ as the input signal (For instance, the signal including speech signal with noise) at time k . Let us also define $y(k)$ as output signal of ε -filter at time k as follows:

$$y(k) = x(k) + \sum_{i=-K}^K a(i)F(x(k+i) - x(k)). \quad (1)$$

where $a(i)$ represents the filter coefficient. $a(i)$ is usually constrained as follows:

$$\sum_{i=-K}^K a(i) = 1. \quad (2)$$

Manuscript received October 6, 2008.

Manuscript revised February 10, 2009.

[†]The author is with the Education and Research Center for Frontier Science, The University of Electro-Communications, Chofu-shi, 182-8585 Japan.

^{††}The author is with the Faculty of Science and Engineering, Waseda University, Tokyo, 169-8555 Japan.

a) E-mail: mitsuharu.matsumoto@iee.org

DOI: 10.1587/transinf.E92.D.1312

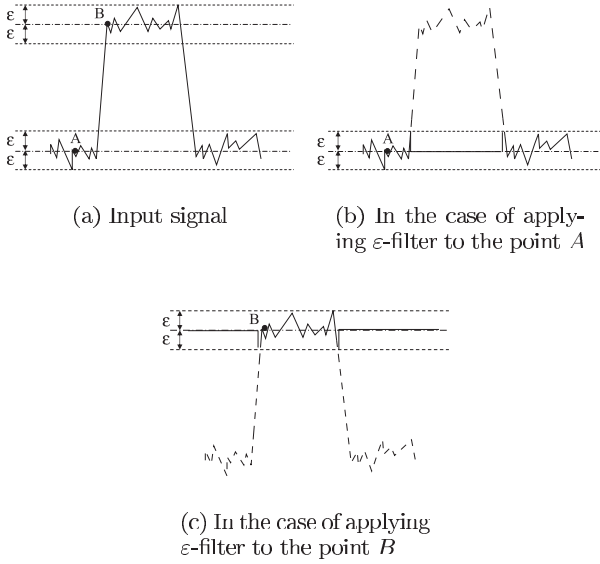


Fig. 1 Basic concept of ε -filter.

The window size of ε -filter is $2K + 1$. $F(x)$ is the nonlinear function described as follows:

$$|F(x)| \leq \varepsilon : -\infty \leq x \leq \infty, \quad (3)$$

where ε is the constant number. This method can reduce small amplitude noise while preserving the speech signal. For example, we can set the nonlinear function $F(x)$ as follows:

$$F(x) = \begin{cases} x & (-\varepsilon \leq x \leq \varepsilon) \\ 0 & (\text{else}). \end{cases} \quad (4)$$

Figure 1 shows the basic concept of ε -filter in case that we utilize Eq. 4 as $F(x)$. Fig. 1 (a) shows the waveform of the input signal. Executing ε -filter at the point A in Fig. 1 (a), we replace all the points where the difference from A is larger than ε by the value of the point A. We then summate the signals in the same window. Fig. 1 (b) shows the basic concept of this procedure. In Fig. 1 (b), the dotted line represents the points where the difference from A is larger than ε . In Fig. 1 (b), the solid line represents the values replaced through this procedure. As a result, if the points are far from A, the points are ignored. On the other hands, if the points are close to A, the points are smoothed. Because of this procedure, ε -filter reduces the noise with preserving the precipitous attack and decay of the speech signal. In the same way, executing ε -filter at the point B in Fig. 1 (c), we replace all the points where the difference from B is larger than ε by the value of the point B. The points are ignored if they are far from B, while the points are smoothed if the points are close to B. Consequently, we can reduce the small amplitude noise near by the processed point while preserving the speech signal.

ε -filter can easily be improved not only for one dimension but also for two dimension. Let us define $x(k, l)$ as the two dimensional input signal at (k, l) . When we apply ε -filter to two dimensional data such as image, ε -filter is designed as follows:

$$y(k, l) = x(k, l) + \sum_{i=-K}^K \sum_{j=-K}^K a(i, j) F(x(k+i, l+j) - x(k, l)), \quad (5)$$

where $a(i, j)$ represents the filter coefficient. $a(i, j)$ is usually constrained as follows:

$$\sum_{i=-K}^K \sum_{j=-K}^K a(i, j) = 1. \quad (6)$$

The feature of two dimensional ε -filter is similar to that of one dimensional ε -filter. We can smooth the small amplitude noise near by the processed point while preserving the edge. It requires fewer calculation compared to conventional methods because it requires only switching and linear operation. It is necessary to estimate the optimal parameter to reduce the noise with preserving the signal effectively.

In ε -filter, ε is an essential parameter to reduce the noise appropriately. If ε is set to an excessively large value, the ε -filter becomes similar to linear filter and smoothes not only the noise but also the signal. On the other hand, if ε is set to an excessively small value, it does nothing to reduce the noise anymore. Due to these reasons, ε values should be set adequately.

3. Estimation of Optimal Parameters Based on Signal-Noise Decorrelation

Let us consider a vector $\mathbf{x}$ including the signal vector $\mathbf{s}$ with the noise $\mathbf{n}$. x_i is the element of $\mathbf{x}$ and corresponds to $x(i)$ in the acoustical signal or is the sorting of $x(k, l)$ in the image. s_i and n_i are the elements of $\mathbf{s}$ and $\mathbf{n}$, respectively. N is the number of the elements. For instance, when we consider L [pixel] $\times$ M [pixel] image ($N = L \times M$), we can sort the element of the image such that i is constrained as

$$i = k + (l - 1) \times L. \quad (7)$$

$\mathbf{x} = (x_1, x_2, \dots, x_N)^T$ can be represented as

$$\mathbf{x} = \mathbf{s} + \mathbf{n}. \quad (8)$$

$\bar{x}$ is defined as the mean of a vector $\mathbf{x}$ and is defined as

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i. \quad (9)$$

We define the variance of the vector $\mathbf{x}$ as

$$\text{var}(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2. \quad (10)$$

The covariance of two vectors $\mathbf{x}$ and $\mathbf{y}$ is given by

$$\text{cov}(\mathbf{x}, \mathbf{y}) = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y}). \quad (11)$$

where $\bar{y}$ is the average of $\mathbf{y}$. y_i is the i th element of $\mathbf{y}$. The normalized form of the covariance is called the coefficient

of correlation and is defined as

$$\text{corr}(\mathbf{x}, \mathbf{y}) = \frac{\text{cov}(\mathbf{x}, \mathbf{y})}{\sqrt{\text{var}(\mathbf{x})} \sqrt{\text{var}(\mathbf{y})}}. \quad (12)$$

The correlation between $\mathbf{s}$ and $\mathbf{n}$ often becomes small in both cases of acoustical and image signals. Although the original $\mathbf{s}$ and $\mathbf{n}$ cannot be obtained, the filter output $\mathbf{y}$ is similar to $\mathbf{s}$ when the parameters of ε -filter is set optimally. The estimated noise $\tilde{\mathbf{n}}$ is described as follows:

$$\tilde{\mathbf{n}} = \mathbf{x} - \mathbf{y}. \quad (13)$$

Hence, the optimal ε_{opt} can be obtained as

$$\varepsilon_{opt} = \arg \min_{\varepsilon} \text{corr}(\mathbf{y}, \mathbf{x} - \mathbf{y}). \quad (14)$$

Let us test and evaluate this criterion experimentally.

4. Experiment

To evaluate the effectiveness of the proposed method, we conducted the evaluation experiments using various types of image data. As an example, we show the results using “Lena” as shown in Fig. 2. We added various levels of random noise with uniform distribution to the original image. The maximal intensity of noise changes from 10 to 40. When the maximal intensity of noise is J , the noise range is $[-J, J]$. Throughout the experiments, the filter coefficient a_i is set to $1/(2K + 1)^2$ to make it uniform weight. To test the robustness of the the proposed method concerning the window size, the window size was changed from 3×3 to 9×9 . We show the results when the window size was set to 7×7 as examples. Similar results could be obtained throughout all the experiments regardless of the window size. The mean absolute errors between the original image and the input image with noise were 2.37, 4.72, 7.10 and 9.45, respectively. Figure 3 shows the relation between the correlation and ε values. Figure 4 shows the relation between the mean absolute error and ε values. As the value of image data is constrained $[0, 255]$, the ε values were changed from 10 to 250 with 10 interval in the experiments. As shown in Figs. 3 and 4, the mean absolute error was minimal when the correlation was minimal throughout all the experiments. The

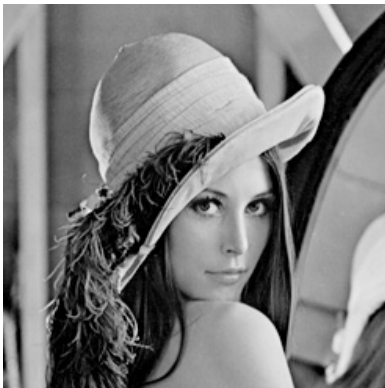


Fig. 2 Image of “Lena”.

improved mean absolute errors between the original image and the output image were 1.81, 2.88, 3.73 and 4.33, respectively. In other words, we could obtain the optimal ε_{opt} based on signal-noise decorrelation. Figures 5 and 6 show the image with two types of random noise and the output image of ε -filter with obtained ε_{opt} . As shown in Figs. 5 and 6, we could obtain the smoothed images with preserving edge regardless of the noise levels. We also could obtain similar results concerning other images.

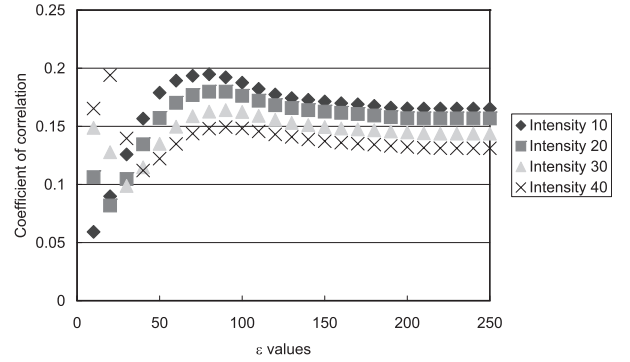


Fig. 3 Relation between coefficient of correlation and ε values depending on noise intensity.

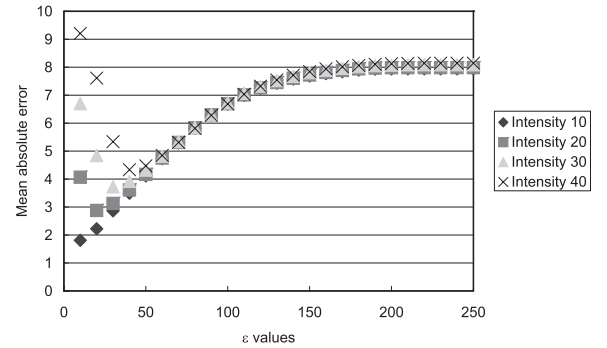


Fig. 4 Relation between mean absolute error and ε values depending on noise intensity.



(a) Image with random noise whose maximal intensity is 20



(b) Output image

Fig. 5 Experimental result. (Noise intensity is 20.)



(a) Image with random noise whose maximal intensity is 40

(b) Output image

Fig. 6 Experimental result. (Noise intensity is 40.)

5. Conclusions

In this paper we proposed a method for optimal parameter settings based on signal-noise decorrelation. The algorithm is simple and the optimal parameters could be obtained throughout all the experiments without any other information except signal-noise decorrelation. We reported the experimental results concerning the image signal in the paper. We also would like to investigate the effectiveness of the proposed method in acoustical signal processing.

Acknowledgement

This research was supported by the research grant of Support Center for Advanced Telecommunications Technology Research (SCAT), by the research grant of Foundation for the Fusion of Science and Technology, by the research grant of Tateisi Science and Technology Foundation, by the Ministry of Education, Culture, Sports, Science and Technology, Grant-in-Aid for Young Scientists (B), 20700168, 2008, by the CREST project "Foundation of technology supporting the creation of digital media contents" of JST, and by the Global-COE Program, "Global Robot Academia".

References

- [1] M. Matsumoto and S. Hashimoto, "A miniaturized adaptive microphone array under directional constraint utilizing aggregated

- microphones," *J. Acoust. Soc. Am.*, vol.119, no.1, pp.352–359, 2006.
- [2] A.J. Bell and T.J. Sejowski, "An information maximization approach to blind separation and blind deconvolution," *Neural Comput.*, vol.7, pp.1129–1159, 1995.
- [3] S. Rickard and O. Yilmaz, "On the approximate w-disjoint orthogonality of speech," *Proc. IEEE Int'l Conf. on Acoustics, Speech, and Signal Process 2002.*, pp.529–532, Orlando, Florida, 2002.
- [4] S.F. Boll, "Suppression of acoustic noise in speech using spectral subtraction," *IEEE Trans. Acoust. Speech Signal Process.*, vol.ASSP-27, no.2, pp.113–120, 1979.
- [5] R.E. Kalman, "A new approach to linear filtering and prediction problems," *Trans. ASME*, vol.82, pp.35–45, 1960.
- [6] M. Fujimoto and Y. Ariki, "Speech recognition under noisy environments using speech signal estimation method based on kalman filter," *IEICE Trans. Inf. & Syst. (Japanese Edition)*, vol.J85-D-II, no.1, pp.1–11, Jan. 2002.
- [7] P. Daniel, W. Ellis, and R. Weiss, "Model-based monaural source separation using a vector-quantized phase-vocoder representation," *Proc. IEEE Int'l Conf. on Acoustics, Speech, and Signal Process. 2006*, pp.V-957–960, 2006.
- [8] H. Harashima, K. Odajima, Y. Shishikui, and H. Miyakawa, "ε-separating nonlinear digital filter and its applications," *IEICE Trans. Fundamentals (Japanese Edition)*, vol.J65-A, no.4, pp.297–304, April 1982.
- [9] K. Arakawa, K. Matsuura, H. Watabe, and Y. Arakawa, "A method of noise reduction for speech signals using component separating ε-filters," *IEICE Trans. Fundamentals (Japanese Edition)*, vol.J85-A, no.10, pp.1059–1069, Oct. 2002.
- [10] K. Arakawa and T. Okada, "ε-separating nonlinear filter bank and its application to face image beautification," *IEICE Trans. Fundamentals (Japanese Edition)*, vol.J88-A, no.11, pp.1216–1225, Nov. 2005.
- [11] T. Boulton, R.A. Melter, F. Skorina, and I. Stojmenovic, "G-neighbors," *SPIE Conf. on Vision Geometry II*, SPIE, vol.2060, pp.96–109, 1993.
- [12] N. Himayat and S.A. Kassam, "Approximate performance analysis of edge preserving filters," *IEEE Trans. Signal Process.*, vol.41, no.9, pp.2764–2777, 1993.
- [13] L. Yin, R. Yang, M. Gabbouj, and Y. Neuvo, "Weighted median filters: A tutorial," *IEEE Trans. Circuits Syst.*, vol.43, no.3, pp.157–192, 1996.
- [14] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images," *Int'l Conf. on Computer Vision*, IEEE, pp.839–846, 1998.
- [15] K. Inoue, W. Du, and K. Urahama, "Image smoothing with edge enhancement by using band-pass bilateral filter," *IEICE Trans. Fundamentals (Japanese Edition)*, vol.J88-A, no.5, pp.687–690, May 2005.
- [16] S. Fleishman, I. Drori, and D. Cohen-Or, "Bilateral mesh denoising," *ACM SIGGRAPH*, ACM, vol.4667, no.5, pp.950–953, 2003.
- [17] P. Mrazek and M. Navara, "Selection of optimal stopping time for nonlinear diffusion filtering," *Int'l Journal of Computer Vision*, vol.52, no.2/3, pp.189–203, 2003.