PAPER

User-Perceived Reliability of M-for-N (M:N) Shared Protection Systems

Hirokazu OZAKI[†a)], Atsushi KARA[††], *and* Zixue CHENG[†††], *Members*

SUMMARY In this paper we investigate the reliability of general type shared protection systems i.e. M for N (M:N) that can typically be applied to various telecommunication network devices. We focus on the reliability that is perceived by an end user of one of N units. We assume that any failed unit is instantly replaced by one of the M units (if available). We describe the effectiveness of such a protection system in a quantitative manner. The mathematical analysis gives the closed-form solution of the availability, the recursive computing algorithm of the MTTFF (Mean Time to First Failure) and the MTTF (Mean Time to Failure) perceived by an arbitrary end user. We also show that, under a certain condition, the probability distribution of TTFF (Time to First Failure) can be approximated by a simple exponential distribution. The analysis provides useful information for the analysis and the design of not only the telecommunication network devices but also other general shared protection systems that are subject to service level agreements (SLA) involving user-perceived reliability measures.

key words: *user-perceived reliability, shared protection systems, availability, MTTFF, MTTF, TTFF, probability distribution*

1. Introduction

In today's broadband access networks, a failure of an interface card in a high density node device such as a digital subscriber line access multiplexer for ADSL/VDSL [1], [2], an optical line terminal for passive optical networks [3], [4] brings severe disruption of traffic to a large number of end users. It may cause an irrevocable loss to an enterprise if the network is utilized for business transactions. The same situation holds in other high density network devices such as servers, switches, carrier-grade routers and NAPT (Network Address & Port Translator). Providing a recovery mechanism from such a failure is a crucial issue in order to realize resilient and reliable telecommunication networks. Redundancy is quite an effective measure because restoration by rerouting based on standard IP-layer routing protocols often requires very long time to converge and such a rerouting scheme can not be used in the networks which have point-to-point or point-to-multipoint connections such as access networks. Shared protection e.g. one for N or two for N etc. is a cost-effective (and often space- and energy-saving) scheme for network node devices which have multiple identical units. Figure 1 shows an example of a passive optical network (PON) which has an access node device with a hot-standby protection interface card. In this case Optical Line Terminal (OLT) corresponds to the access node device in which N working interface cards share one protection interface card. When a working interface card fails, the protection interface card takes over the role by switching and the data path is maintained. The protection interface card is usually housed in a fixed card slot of the device in order to simplify the structure of the switch.

An N-unit hot standby system with M spares and $M+N$ repairers i.e. an $(M + N)$ parallel redundant system was analyzed extensively in the long research history of reliability and various reliability quantities were obtained [13]–[15]. Most of the analyses were based on the system administrator's viewpoint. Namely, it was assumed that $(M + N)$ units as a whole made up a system and the failure of a single unit was regarded as the failure of the system. However, there is another type of redundant systems composed of $(M + N)$ units. M spare units are shared by mutually independent users of N units. We call this type of system as M-for-N shared protection system. The M-for-N system requires analysis based on each end user's view point. However, there have been very few research papers that deal with user-perceived reliability measures. J. Fried and P. Kubat studied the customer perceived failure rate of a telecommunication system with a hot-standby shared protection mechanism under the condition that maintenance/repair visits in a constant frequency and they derived a strict solution which needs the approximation for practical use [7]. The unavailability and MTTF (Mean Time to Failure) of

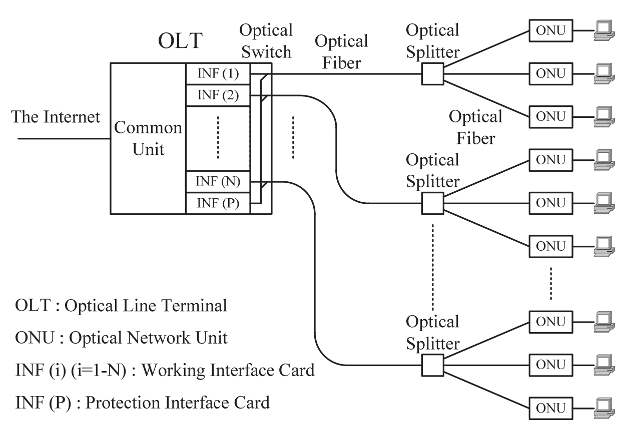

Fig. 1 Passive optical network.

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[†]The author is with NEC, Abiko-shi, 270–1198 Japan.
[††]The author is with Nara National College of Technology, Yamato-Koriyama-shi, 639–1080 Japan.
[†††]The author is with The University of Aizu, Aizu-wakamatsu-shi, 965–8580 Japan.
a) E-mail: h-ozaki@bc.jp.nec.com
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a single member of a group of N functionally identical systems which work in parallel and share a pool of r backup systems was analyzed in [8]. However, the analysis adopted very rough approximation where user-perceived unavailability was computed by simply dividing the entire system unavailability by the number of units. As they derived MTTF based on such inaccurate unavailability, they did not obtain correct solutions. There are several related works that analyzed the path availability in WDM networks [9], [10]. However, the maintenance manner of geographically fixed optical links considered in the analysis of WDM networks is different from that of the devices including routinely replaceable units. In our previous research, we derived simple closed-form formula for the availability and MTTF of one-for-N and two-for-N shared protection systems with an ordinary on-demand repair discipline from the standpoint of the end users [11].

As a first contribution of this paper, we establish a closed form solution (formula) of the availability that is perceived by an arbitrary end user of general M-for-N (M:N) shared protection systems under the fair switching and the fair maintenance disciplines. As a second contribution, we establish the algorithm for computing the expectation of TTFF i.e. MTTFF (Mean Time to First Failure) numerically without approximation. As a third contribution, we show a computation method of the MTTF (Mean Time to Failure). As a fourth contribution, we show that the TTFF probability distribution could be approximated with a level of accuracy by using a simple exponential distribution which has MTTFF as the expectation under a certain condition. We believe our solutions, computation methods, and analysis provide useful information for the design and the maintenance of not only the various network devices but also other general shared protection systems that are subject to service level agreements (SLA) involving user-perceived reliability measures.

This paper is organized as follows: In Sect. 2 we introduce the concept of the user-perceived reliability and establish a model of M-for-N shared protection systems. In Sect. 3 we formulate the problems of the availability, the MTTFF, and the MTTF such a protection system. And we show a closed form solution and computation method for them. In Sect. 4 shows some numerical solutions of the availability, MTTFF and MTTF. In Sect. 5, we show that the probability distribution of Time to First Failure (TTFF) could be approximated by a simple exponential distribution. In Sect. 6, we describe the applications of the results. Finally in Sect. 7 we conclude this paper and describe our future research.

2. Modeling of M-for-N Shared Protection Systems

In this section we first introduce the concept of the user-perceived reliability and then describe some assumptions for modeling and analysis of M-for-N shared protection systems. We proceed to establish its state transition diagrams for computing the reliability indices.

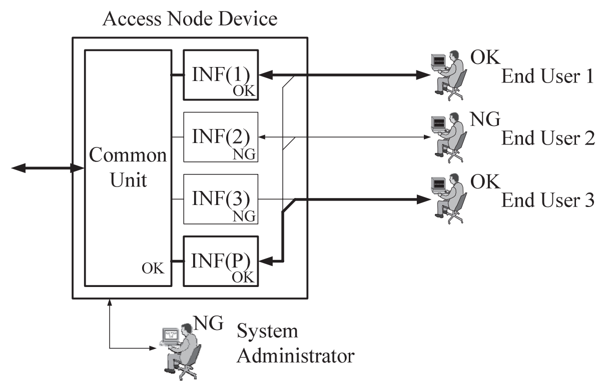

Fig. 2 User-perceived system state.

2.1 User-Perceived Reliability

A system administrator may regard the protection system as failed, when multiple failures occur at the same time and one or more end users become out of service. For example, it is the case when two or more failures exist at the same time in a one-for-N protection system. On the other hand, an end user is interested in the availability and MTTF or MTTFF of his/her own service only. The service for him/her is affected only by the status of his/her working unit and the protection unit which is shared by all working units, if the common unit of the device does not fail. The service is in the normal state, if his/her working unit is normal or it is backed up by the protection unit. The service is stopped, if and only if his/her working unit is out of order and it is not backed up by the protection unit. Figure 2 shows an example of the access node device with one-for-3 protection mechanism.

In the figure, although two interface cards (INF(2) and INF(3)) are broken, only one end user (End User 2) is out of service. The system is operational for other two end users (End User 1 and 3).

It is expected that the reliability perceived by an arbitrary end user improves due to the existence of the protection unit, even though it may be used by one of the other users or it may be broken. Our aim in this paper is to evaluate the improvement of the reliability in a quantitative manner.

2.2 Assumptions for Modeling and Analysis

To simplify the formulation and computing, we take some common and widely accepted assumptions for modeling and analysis as follows.

(1) There are M protection unit(s) and N working unit(s) in the system.
(2) Any failed working unit is instantly replaced by one of the M units (if available). (switch-over)
(3) A failed unit is immediately taken out of the system and is repaired by the repair crew.
(4) A repaired unit is housed in a vacant working slot prior to a vacant protection slot. Namely, a vacant protection slot is filled when there is no vacant working slot.

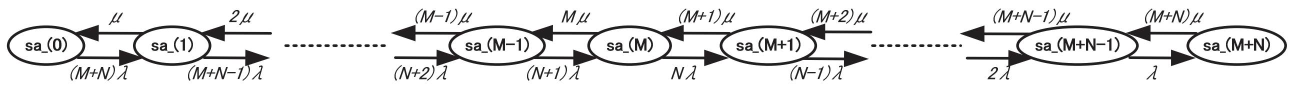

Fig. 3 State transition diagram of M-for-N shared protection systems (System administrator's perspective).

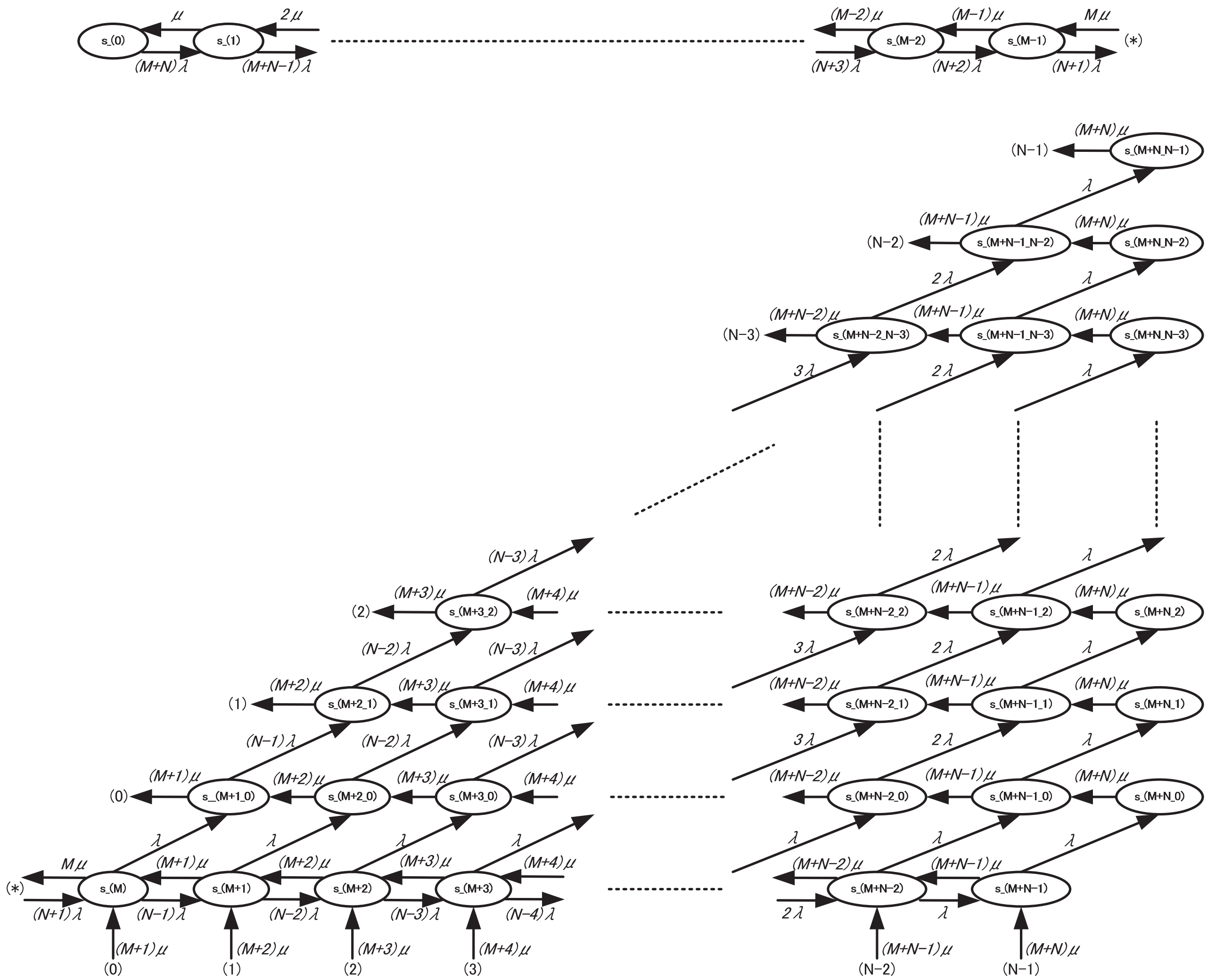

Fig. 4 State transition diagram of M-for-N shared protection systems for availability computation (An arbitrary end user's perspective).

(5) Under (4), a repaired unit is housed in the slot of the device which has been vacant for a longer time at the point. It causes a release of a protection unit. (switch-back)
(6) There is no failure in the common unit and the switch itself in the device.
(7) A shared protection system behaves as a continuous time Markov chain (CTMC) [12].
(8) The time between failures and the time to repair for one unit are both subject to exponential distribution.
(9) Mean numbers of failures and repairs in a unit time are expressed by λ and μ respectively. Assume $\lambda \ll \mu$. (The assumption $\lambda \ll \mu$ is not necessary for MTTFF and MTTF computation.)
(10) All units are identical in their functionality and performance.
(11) Individual units are not discriminated.
(12) The switching discipline is FCFS i.e. first come (fail) first served (back up) [12].
(13) There are ample repair crews. (at least $M + N$)
(14) The time for switching control and switching operations (switch-over or switch-back) is ignored.

2.3 State Transition Diagrams for the Availability Computation of M-for-N Shared Protection Systems

We show a state transition diagram of M-for-N shared protection systems viewed from the system administrator in Fig. 3. The notation is as follows.

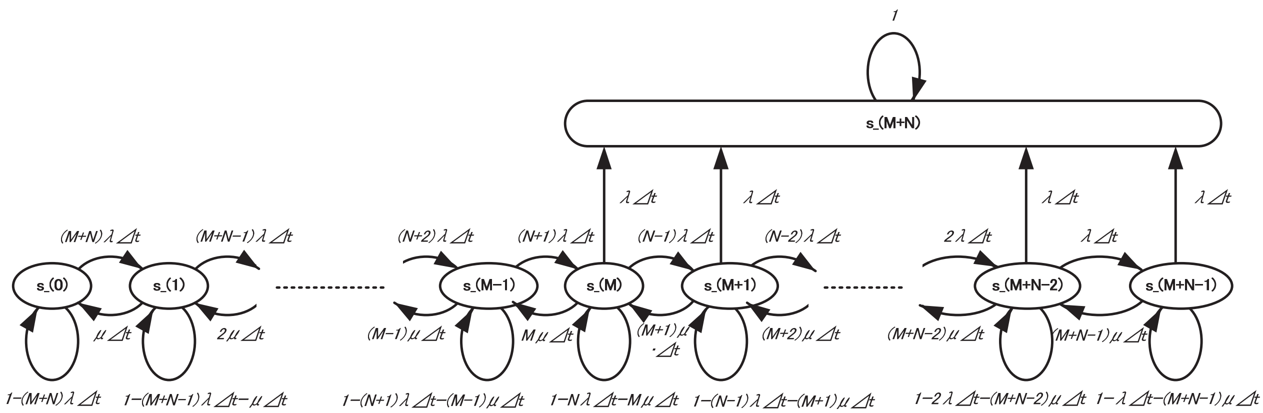

Fig. 5 State transition diagram of M-for-N shared protection systems for MTTFF and MTTF computation (An arbitrary end user's perspective).

$sa\_(i)$: the state which has i failure(s) in the system

We show a state transition diagram of the M-for-N shared protection systems in Fig. 4 that is viewed from an arbitrary end user. We call this end user "him/her".

This diagram is used for the computation of the availability. There are $(M+N+N(N+1)/2)$ different states viewed from him/her. The difference between Fig. 3 and Fig. 4 is the standpoint of the observer i.e. the system administrator or an end user. The notation of the diagram is as follows.

$s\_(i)$: the state which has i failure(s) in the system and his/her service is normal

$s\_(i\_j)$: the state which has i failure(s) in the system and his/her service is out. There is j user(s) behind him/her in a virtual queue which waits for service restoration

2.4 State Transition Diagrams for the MTTFF and the MTTF Computation of M-for-N Shared Protection Systems

We show a state transition diagram of M-for-N shared protection systems in Fig. 5 that is used for the computation of the user-perceived MTTFF and MTTF. There are $(M+N+1)$ different states that are seen from the end user of an arbitrary working unit. The notation of the diagram is as follows.

$s\_(i)$ ($i = 0$ to $M+N-1$): the state which has i failure(s) in the system and his/her service is normal

$s\_(M+N)$: the state in which his/her service is out

Δt means an infinitesimally small time interval.

3. Computing the User-Perceived Availability, MTTFF, and MTTF of M-for-N Shared Protection Systems

3.1 The Availability Computation

We use PA_i to denote the steady state probability of $sa\_(i)$ ($i = 0$ to $M+N$). The diagram in Fig. 3 and the probability conservation law yield the following formula.

$$PA_i = \frac{(M+N)!}{i!\,(M+N-i)!}\left(\frac{\lambda}{\mu}\right)^i PA_0 \quad (i = 0 \text{ to } M+N) \qquad (1)$$

$$PA_0 = \frac{1}{1+\displaystyle\sum_{j=1}^{M+N}\frac{(M+N)!}{j!\,(M+N-j)!}\left(\frac{\lambda}{\mu}\right)^j}$$

We use P_i and P_{i_j} to denote the steady state probability of $s\_(i)$ and $s\_(i\_j)$. The comparison between two diagrams i.e. Fig. 3 and Fig. 4 yields the following relations.

$$P_i = PA_i \qquad (i = 0 \text{ to } M) \qquad (2)$$

$$P_i + \sum_{j=0}^{i-M-1} P_{i\_j} = PA_i \quad (i = M+1 \text{ to } M+N-1) \qquad (3)$$

$$\sum_{j=0}^{N-1} P_{M+N\_j} = PA_{M+N} \qquad (4)$$

The availability perceived by an arbitrary end user $A_{M:N}$ is expressed as follows.

$$A_{M:N} = \sum_{i=0}^{M+N-1} P_i \qquad (5)$$

From (1) to (5), we have the closed form solution of the availability as follows.

$$A_{M:N} = \frac{\displaystyle\sum_{q=1}^{M}\frac{(M+N)!}{q!(M+N-q)!}\left(\frac{\lambda}{\mu}\right)^q+\sum_{r=1}^{N-1}\frac{(N-r)(M+N)!}{N(M+r)!(N-r)!}\left(\frac{\lambda}{\mu}\right)^{M+r}}{1+\displaystyle\sum_{u=1}^{M+N}\frac{(M+N)!}{u!(M+N-u)!}\left(\frac{\lambda}{\mu}\right)^u} \qquad (6)$$

3.2 The MTTFF Computation

We use $P_i(t)$ ($i = 0, 1, 2, \cdots, M+N-1$) to denote the proportion of the time that the system is in state $s\_(i)$ when an arbitrary end user observes the system from time 0 to time t. The initial state is $s\_(0)$ i.e. $P_0(0) = 1$.

We establish a set of simultaneous differential equations of $P_i(t)$ based on the state transition diagram of Fig. 5 and apply the Laplace transform.

$$P_i(s) = \int_0^{\infty} P_i(t)e^{-st}\,dt \tag{7}$$

We use T_i ($i = 0, 1, 2, \cdots, M{+}N{-}1$) to denote the mean accumulated sojourn time that the system is in state $s\_(i)$ until it falls in $s\_(M + N)$ i.e. the dead state. T_i is expressed as the time integral of $P_i(t)$ as t goes to infinity, and it is equal to $P_i(s)$ as s goes to zero as in (8).

$$T_i = \lim_{t\to\infty}\int_0^t P_i(t)\,dt = \lim_{s\to 0}\left[s\cdot\left\{\frac{P_i(s)}{s}\right\}\right] = \lim_{s\to 0} P_i(s) \tag{8}$$
$$(0 \le i \le M + N - 1)$$

It follows from the equations the following relation and recurrence formulas.

$$\lambda \sum_{k=M}^{M+N-1} T_k = 1 \tag{9}$$

$$T_{M+N-2} = \frac{\lambda + (M + N - 1)\mu}{\lambda} T_{M+N-1} \tag{10}$$

$$T_{M+N-q} = \frac{\{\lambda+(M+N-q+1)\mu\}T_{M+N-q+1} + \lambda \sum_{u=M+N-q+2}^{M+N-1} T_u}{(q-1)\lambda} \tag{11}$$
$$(3 \le q \le N)$$

$$T_{M-r} = \frac{1 + (M + 1 - r)\mu T_{M+1-r}}{(N + r)\lambda} \quad (1 \le r \le M) \tag{12}$$

We obtain each T_i by combining the equations from (9) through (12). The MTTFF $M_{M:N}$ is computed by substituting T_i into the following expression.

$$M_{M:N} = T_0 + T_1 + \cdots + T_{M+N-2} + T_{M+N-1} \tag{13}$$

3.3 The MTTF Computation

The initial state of the system for computing MTTFF is assumed to be $s\_(0)$ i.e. the state in which there is no failure in the entire system. However, when the service for an arbitrary end user is restored after the service outage, he/she is not in such a system initial state. The restored user is located in one of the state $s\_(M)$ to $s\_(M + N - 1)$ in Fig. 5. We choose these states as the initial states based on the stationary state probability distribution. In the process of the availability derivation, we obtain the stationary state probability P_M to P_{M+N-1} of $s\_(M)$ to $s\_(M + N - 1)$ as follows.

$$P_{M+i} = \frac{N - i}{N} PA_{M+i} \qquad (i = 0 \text{ to } N - 1) \tag{14}$$

where

PA_i is given by (1).

We assume that the probability of the initial state PI_j ($j = M$ to $M + N - 1$) be subject to the distribution of the stationary state probability P_j. It is given by the following normalized probability.

$$PI_j = \frac{P_j}{\sum_{k=M}^{M+N-1} P_k} \qquad (M \le j \le M + N - 1) \tag{15}$$

The relation (9) holds regardless of the initial state for arbitrary M and N.

When the initial state is $s\_(M)$, we have

$$T_{M+N-2} = \frac{\lambda + (M + N - 1)\mu}{\lambda} T_{M+N-1}$$
$$T_{M+N-q} = \frac{\{\lambda+(M+N-q+1)\mu\}T_{M+N-q+1} + \lambda \sum_{u=M+N-q+2}^{M+N-1} T_u}{(q-1)\lambda}$$
$$(3 \le q \le N)$$
$$T_{M-r} = \frac{(M + 1 - r)\mu T_{M+1-r}}{(N + r)\lambda}$$
$$(1 \le r \le M) \tag{16}$$

When the initial state is $s\_(M + p)$ ($N \ge 3$, $1 \le p \le N - 2$), we have

$$T_q = \frac{(M + N - q + 1)\lambda}{q\mu} T_{q-1} \qquad (1 \le q \le M + 1)$$
$$T_{M+r} = \frac{\lambda \sum_{u=M}^{M+r-2} T_u + \lambda T_{M+r-1}}{(M + r)\mu} \qquad (2 \le r \le p)$$
$$T_{M+N-2} = \frac{\lambda + (M + N - 1)\mu}{\lambda} T_{M+N-1}$$
$$T_{M+N-v} = \frac{\{\lambda+(M+N-v+1)\mu\}T_{M+N-v+1} + \lambda \sum_{u=M+N-v+2}^{M+N-1} T_u}{(v-1)\lambda}$$
$$(3 \le v \le N - p) \tag{17}$$

When the initial state is $s\_M + N - 1$, we have

$$T_q = \frac{(M + N - q + 1)\lambda}{q\mu} T_{q-1} \qquad (1 \le q \le M + 1)$$
$$T_{M+r} = \frac{\lambda \sum_{u=M}^{M+r-2} T_k + \lambda T_{M+r-1}}{(M + r)\mu} \qquad (2 \le r \le N - 1) \tag{18}$$

We compute the MTTFF (Mean Time to First Failure) $M_{M:N}$ after the service restoration by the above recurrence formulas, the conservation law (9), and the addition of (13) if the initial state is given.

The MTTF $\overline{M}_{M:N}$ is computed as the weighted mean of the MTTFF which has $s_{-}(M)$ to $s_{-}(M+N-1)$ as the initial state by normalized initial state probabilities (15). Namely,

$$\overline{M}_{M:N} = \sum_{j=M}^{M+N-1} \{PI_j \cdot (M_{M:N}|s_{-}(j))\} \tag{19}$$

where $M_{M:N}|s_{-}(j)$ means $M_{M:N}$ which has $s_{-}(j)$ as the initial state.

4. Numerical Examples of the User-Perceived Availability, MTTFF and MTTF of M-for-N Shared Protection Systems

We show some examples of user-perceived availability, MTTFF and MTTF in M-for-N shared protection systems with concrete numerical values. We assume here that λ is 0.0005/day and μ is 0.1/day. It corresponds to the case that the mean time between failures (MTBF) of an interface card is 5.48 years and that the mean repair turnaround time is 10 days. (We approximate 1 year is equal to 365 days) These values roughly correspond to the maintenance practice of an ordinary interface card in today's access node devices. Without redundancy, the availability is 99.50248% and the MTTFF is 5.48 years. Table 1 shows the calculation result of the availability Fig. 6 depicts the availability curves based on Table 1.

We find that the availability does not indicate deterioration even for a large N. For example, even when $N = 128$, the availability has the value of two-nines, three-nines, four-nines and five-nines for $M = 1, 2, 3$ and 4 respectively. The five nine availability means that service outage is about 5 minutes per year. This level of availability is often required for carrier grade telecommunication services [6]. This level of availability is maintained with $N \le 16$, $N \le 64$, and $N \le 128$ for the case of $M = 2$, $M = 3$, and $M = 4$ respectively.

In Fig. 6, the availability curve of the one for N gives different appearance from other curves i.e. two for N, three for N and four for N. It falls more rapidly as N increases. Although one-for-N system is often used in many actual devices, this result indicates that the use of multiple protection units is an effective method to attain higher availability.

Table 2 shows the calculation results of the MTTF compared with the MTTFF after the system start (The initial state is $s_{-}(0)$). We confirm multiple spares ($M \ge 2$) contribute to the improvement of the user-perceived MTTF more than a single spare. For example, 2-for-128, 3-for-128, and 4-for-128 has the MTTF of 3.46, 16.85, and 106.20 times of that of 1-for-128 shared protection system respectively. We also find the difference between MTTF and MTTFF is quite small in these cases. (The difference is less than 1%.) It is fairly appropriate to use MTTFF instead of the MTTF in these cases.

Table 1 Availability of M-for-N of M-for-N shared protection systems.

N	Availability (%) λ=0.0005/day μ=0.1/day			
	one for N	two for N	three for N	four for N
1	99.9975248137	99.9999876856	99.9999999387	99.9999999997
2	99.9962933778	99.9999754325	99.9999998473	99.9999999991
3	99.9950660262	99.9999591560	99.9999996955	99.9999999979
4	99.9938427437	99.9999388863	99.9999994687	99.9999999958
8	99.9889900025	99.9998184724	99.9999975251	99.9999999705
16	99.9794752806	99.9993949190	99.9999858078	99.9999997190
32	99.9611828910	99.9978670902	99.9999085848	99.9999967593
64	99.9273503451	99.9924555685	99.9993910373	99.9999596673
128	99.8692988487	99.9746641507	99.9961326070	99.9995138493

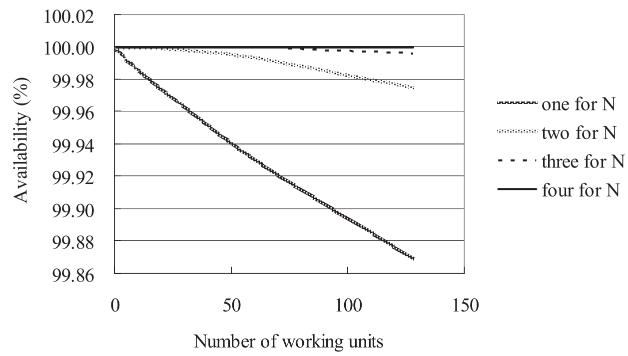

Fig. 6 Availability of M-for-N shared protection systems ($\lambda = 0.0005$/day, $\mu = 0.1$/day).

Table 2 MTTF and MTTFF of M-for-N shared protection systems.

N	MTTF [upper] and MTTFF [lower] (years) (λ=0.0005/day μ=0.1/day)			
	one for N	two for N	three for N	four for N
1	553.42	74160.73	11179731.51	1797700827.40
	556.16	74347.95	11198458.90	1799955501.10
2	369.87	37203.76	4488641.26	601628149.03
	371.69	37298.38	4496195.93	602385339.13
3	278.09	22396.50	2252724.22	258870795.27
	279.46	22453.89	2256533.71	259197738.23
4	223.02	14980.64	1292089.80	129952449.00
	224.12	15019.31	1294285.21	130117146.59
8	125.13	5060.22	278204.15	18672395.47
	125.74	5073.65	278685.96	18696393.49
16	67.56	1528.20	48806.51	1969440.90
	67.89	1532.47	48894.35	1972044.35
32	36.19	439.31	7668.68	172620.07
	36.36	440.65	7683.59	172861.67
64	19.83	127.51	1179.19	14170.93
	19.92	127.96	1181.85	14193.22
128	11.56	39.99	194.83	1227.68
	11.61	40.16	195.39	1230.12

Note: 365 days approximates 1 year.

5. Probability Distribution of TTFF

In this section we analyze the following probability distribution of TTFF (Time to First Failure).

$$P[TTFF \le t] = 1 - \sum_{i=0}^{M+N-1} P_i(t) \tag{20}$$

The probability distribution of TTFF can be obtained by solving a set of differential equations based on Fig. 5 directly in a numerical manner. However, it is complicated and takes quite a long calculation time.

It is not difficult to show that the probability

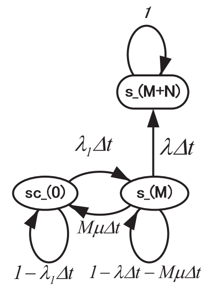

Fig. 7 Simplified state transition diagram.

distribution of TTFF can be approximated by using a simple exponential function. The following is an outline of the proof of this fact.

The state transition diagrams in Fig. 5 can be reduced to a simplified one shown in Fig. 7 if λ is smaller enough than μ (e.g. if the terms of the form $\left(\frac{\lambda}{\mu}\right)^k$ ($k = 2, 3, 4, \ldots$) can be ignored). $s\_(M+1)$ to $s\_(M+N-1)$ in Fig. 5 could be truncated because these states have very small state probabilities if $\lambda \ll \mu$ and they have the same transition probability $\lambda\,\Delta t$ to $s\_(M+N)$. $s\_(0)$ to $s\_(M-1)$ could be merged into one normal state i.e. $sc\_(0)$ in Fig. 7. In Fig. 7, λ_1 is the rate with which the transition $sc\_(0)$ to $s\_(M)$ occurs. It is a constant determined by the value of M, N, λ and μ.

By solving the differential equations based on Fig. 7 and applying the condition $\lambda \ll \mu$, we obtain the following formula.

$$P_{M:N}[TTFF \le t] \cong 1 - \exp\left(-\frac{\lambda_1 \lambda}{\lambda + \lambda_1 + \mu} \cdot t\right) \tag{21}$$

As the expectation of time which is computed from (21) must correspond to the MTTFF which is computed in a numerical manner described in Sect. 3, (21) is also expressed as follows.

$$P_{M:N}[TTFF \le t] \cong 1 - \exp\left(-\frac{1}{MTTFF} \cdot t\right) \tag{22}$$

In case of 2-for-8 shared protection systems with $\lambda = 0.0005$/day and $\mu = 0.1$/ day, the expression of the exponential approximation is as follows. (See Table 2)

$$P[TTFF \le t] = 1 - \exp(-t/5073.65) \tag{23}$$

In this case, it is confirmed that the correspondence to the numerical solutions of the differential equations has five-decimal-place accuracy at least in $TTFF \ge 500$ years.

6. Applications

In this section, we describe the applications of the aforementioned computation result. The results can be applied to not only the reliability analysis of existing shared protection systems but also the concrete design of new devices. In many cases, the controllable system design parameters are M, N, and μ since λ usually depends on the characteristics of the hardware parts that are not easy to control. The repair rate depends on the maintenance policy that can be controlled to some extent based on economical conditions. Therefore, when λ and μ are given, the designer determines the appropriate values of M and N with taking into account the balance among the reliability, the cost and the other conditions such as size, power consumptions of the device.

Table 3 Typical availability objectives.

Service Class	Availability (%)	Down time (hrs/yr)
Australian DSL Service	99.930	6.1300
Equipment suppliers	99.999	0.0876
Customer expectations		
Consumer class	99.900	8.7600
Business class	99.995	0.4380

Table 3 shows examples of typical availability objectives [16]. The devices are required to have five-nine availability. As a case study, we take the design of a digital subscriber line access multiplexer (DSLAM). For simplicity, we assume that only line cards that consist of an M-for-N shared protection system can fail. We also assume that λ and μ are given as 0.0005/day and 0.1/day respectively. For five-nine availability, M must be equal to or more than 2 (See Table 1). In addition to the availability, if the MTTF is required to be more than 1500 years, N must be equal to or less than 16. (See Table 2) So $M = 2$ and $N = 16$ is determined if there is no other conditions.

The results of our research can be also applied not only to telecommunication network devices but also to various ICT service systems in general that are subject to service level agreements (SLA) involving user-perceived reliability measures.

7. Conclusion

In this paper we analyzed the user-perceived reliability of M-for-N (M:N) shared protection systems. We first introduce the concept of such a kind of reliability. Mathematical analysis based on the state transition diagrams gives a closed form solution of the availability and a computation method of the MTTFF and the MTTF. The solution and computation methods we showed do not include any approximation and give an exact numerical solution for each reliability index on arbitrary system parameters i.e. M, N, λ and μ. We also showed an approximation of the probability distribution of TTFF. We finally described the applications of the analysis. In our future research, we will investigate prioritized controls of shared protection systems.

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Hirokazu Ozaki received the B.S degree in Electrical Engineering and the M.S degree in Electronics from Kyoto University in 1983 and 1985 respectively. He joined NEC Corporation and engaged in research and development of fiber optical transmission systems. He is currently a manager of Broadband Networks Operations Unit.

Atsushi Kara is a professor at Nara National College of Technology, Yamato-koriyama, Japan. He is interested in various aspects of internet engineering.

Zixue Cheng received the M.E. and Ph.D degrees from Tohoku University in 1990 and 1993, respectively. He was an assistant professor from 1993 to 1999, an associate professor from 1999 to 2002, and has been a professor, since Feb. 2002, at the School of Computer Science and Engineering, University of Aizu. Currently he is interested in distributed algorithms, distance education, ubiquitous computing, and functional safety. Dr. Cheng is a member of IEEE, ACM, and IPSJ.