PAPER

Computing Spatio-Temporal Multiple View Geometry from Mutual Projections of Multiple Cameras

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SUMMARY The spatio-temporal multiple view geometry can represent the geometry of multiple images in the case where non-rigid arbitrary motions are viewed from multiple translational cameras. However, it requires many corresponding points and is sensitive to the image noise. In this paper, we investigate mutual projections of cameras in four-dimensional space and show that it enables us to reduce the number of corresponding points required for computing the spatio-temporal multiple view geometry. Surprisingly, take three views for instance, we no longer need any corresponding point to calculate the spatio-temporal multiple view geometry, if all the cameras are projected to the other cameras mutually for two time intervals. We also show that the stability of the computation of spatio-temporal multiple view geometry is drastically improved by considering the mutual projections of cameras.

key words: *multiple view geometry, spatio-temporal, mutual projections, multifocal tensor, multiple cameras, camera calibration*

1. Introduction

Over the past two decades there has been a significant development in the understanding and modeling of the geometry of multiple views in computer vision, which is very important for describing the relationship among images taken from multiple cameras and for recovering 3D geometry from images. Based on the classical analysis on multiple views [1], multiple view geometries under more general point-camera configurations have been studied in recent years [2]–[7].

Wolf et al. [6] and Wan et al. [7] showed that by considering the projections in higher-dimensional space such as a 4D space-time, the multiple view geometry for dynamic configurations can be derived, in which a single multifocal tensor describes camera motions as well as camera positions and orientations. These multiple view geometries are very useful for recovering non-rigid object motions from moving cameras and for generating arbitrary views of dynamic scenes. However, one disadvantage of such generalized multiple view geometry in higher-dimensional space is that it requires more corresponding points than the classical multiple view geometry. For example, 26 corresponding points are required for computing trifocal tensors in [6], and 13 corresponding points are required for computing trifocal tensors in [7]. They are also more sensitive to the image noise than the classical one. For the real use of these generalized multiple view geometries, it is necessary to cope with these problems.

Recently, it has been shown that if some cameras are projected to the other cameras, the multiple view geometry can be computed more stably from fewer corresponding points [8]. This is called mutual projections of cameras. However, the existing mutual projection method is only applicable to the classical multiple view geometry, in which all the cameras are assumed to be static and epipoles are single points in images. Whereas in [7], the spatio-temporal multiple view geometry describes the relationship among images viewed from multiple translational cameras. In this case, a set of epipoles derived from a translational camera is no longer a single point but a line. We call it “epipole line”. Note that the epipole line is different from an epipolar line.

In this paper, we investigate mutual projections of cameras in four-dimensional space, and show that these epipole lines enable us to reduce the number of corresponding points required for computing the spatio-temporal multiple view geometry. Surprisingly, take three views for instance, we no longer need any corresponding points for computing the spatio-temporal multiple view geometry, if all the cameras are projected to the other cameras mutually for two time intervals. We also show that the stability of the computation of multiple view geometry in space-time is drastically improved by using the epipole lines under mutual projections.

2. Spatio-Temporal Multiple View Geometry

Let us consider five translational cameras with constant speed and with no rotation, which observe a moving point $\mathbf{X}(t)$ as shown in Fig. 1. Suppose the point $\mathbf{X}(t)$ is projected to these five cameras as $\mathbf{x}(t) = [x^1, x^2, x^3]^\top$, $\mathbf{x}'(t) = [x'^1, x'^2, x'^3]^\top$, $\mathbf{x}''(t) = [x''^1, x''^2, x''^3]^\top$, $\mathbf{x}'''(t) = [x'''^1, x'''^2, x'''^3]^\top$ and $\mathbf{x}''''(t) = [x''''^1, x''''^2, x''''^3]^\top$ at time t. Then, Wan et al. showed that the following trilinear, quadrilinear and quintilinear relationships hold under extended camera projections from 4D space to 2D images [7]:

$$x^i(t)x'^j(t)x''^k(t)\epsilon_{kra}\mathcal{T}_{ij}^r = 0_a \tag{1}$$

$$x^i(t)x'^j(t)x''^k(t)x'''^l(t)\epsilon_{jpa}\epsilon_{kqb}\epsilon_{lrc}\mathcal{Q}_i^{pqr} = 0_{abc} \tag{2}$$

$$x^i(t)x'^j(t)x''^k(t)x'''^l(t)x''''^m(t) \epsilon_{ipa}\epsilon_{jqb}\epsilon_{krc}\epsilon_{lsd}\epsilon_{mte}\mathcal{R}^{pqrst} = 0_{abcde} \tag{3}$$

where ϵ_{ipa} denotes a tensor, which represents a sign based on permutation from $\{i, p, a\}$ to $\{1, 2, 3\}$. It takes 1 if it is even permutation and takes -1, if it is odd permutation. It takes 0

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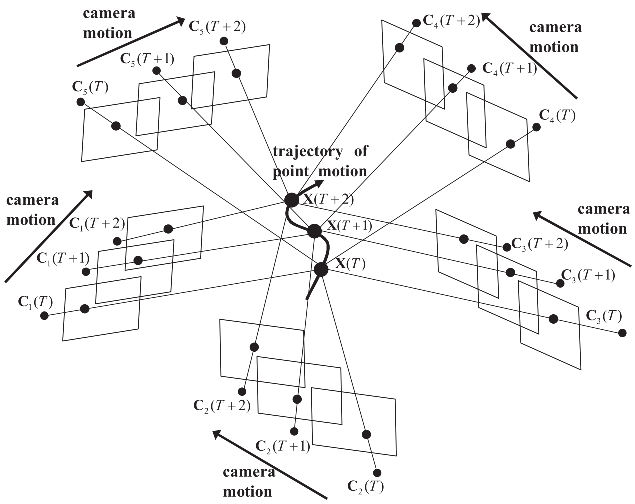


Fig. 1 A moving point in 3D space and its projections on the image planes of five translational cameras.

in the other cases. $\mathcal{T}_{ij}^r$, $\mathcal{Q}_i^{lmn}$ and $\mathcal{R}^{lmnfg}$ denote trifocal tensor, quadrifocal tensor and quintifocal tensor respectively. These tensors represent relative camera positions as well as camera motions, and are invariant, even if the cameras are moving with different velocities and directions. The developed theory can be applied to the cases of non-rigid arbitrary motions viewed from multiple independently translational cameras.

Since corresponding points with time marks induce linear constraints, for computing multifocal tensors, for instance, trifocal tensor $\mathcal{T}_{ij}^r$, we reformulate (1) as follows:

$$\mathbf{M}(t)\mathbf{t} = \mathbf{0} \tag{4}$$

where $\mathbf{t} = [\mathcal{T}_{11}^1, \cdots, \mathcal{T}_{33}^3]^\top$, and $\mathbf{M}(t)$ is a 3×27 matrix whose elements are calculated from the corresponding points $\mathbf{x}(t)$, $\mathbf{x}'(t)$ and $\mathbf{x}''(t)$. Then, $\mathbf{t}$ can be computed by solving the following linear equations.

$$[\mathbf{M}(t_1)^\top, \cdots, \mathbf{M}(t_N)^\top]^\top \mathbf{t} = \mathbf{0} \tag{5}$$

where $N \geq 13$. It has been shown that minimum of 13, 10 and 9 corresponding points are required to compute $\mathcal{T}_{ij}^r$, $\mathcal{Q}_i^{lmn}$ and $\mathcal{R}^{lmnfg}$ linearly [7].

3. Computing Spatio-Temporal Multiple View Geometry from Mutual Projections

Although there are three types of multilinear relationships under the projection from 4D space to 2D space as shown in (1), (2) and (3), we only consider trilinear relationship in this paper, since basic properties of mutual projections of moving cameras can be analyzed in the trilinear relationship.

In (1), $\mathcal{T}_{ij}^r$ is the trifocal tensor for the extended cameras and has the following form:

$$\mathcal{T}_{ij}^r = \epsilon_{ilm}\epsilon_{jqu} \det \begin{bmatrix} \mathbf{a}^l \\ \mathbf{a}^m \\ \mathbf{b}^q \\ \mathbf{b}^u \\ \mathbf{c}^r \end{bmatrix} \tag{6}$$

where $\mathbf{a}^i$, $\mathbf{b}^i$ and $\mathbf{c}^i$ denote the ith row of the three camera matrices respectively. The trifocal tensor $\mathcal{T}_{ij}^r$ is $3\times3\times3$ and has 27 entries. Except a scale ambiguity, we have only 26 free parameters in $\mathcal{T}_{ij}^r$. In addition, (1) provides us 3 linear equations on $\mathcal{T}_{ij}^r$, but only 2 of them are linearly independent. Thus, at least 13 corresponding points are required to compute $\mathcal{T}_{ij}^r$ from images linearly. We call it 13 point method.

On the other hand, when we have three moving cameras, we can derive at most 3 pairs of epipoles, $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$, $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$ and $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$, at an instance. Here, $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$, for example, denotes a pair of epipoles which can be regarded as the projections of camera 1 to camera 2 and 3.

At time t, the epipole $\mathbf{e}_{21}(t)$ is a point in view 2, which corresponds to any point in view 1. Also, the epipole $\mathbf{e}_{31}(t)$ at time t is a point in view 3, which corresponds to any point in view 1. This means, by substituting $\mathbf{e}_{21}(t)$ and $\mathbf{e}_{31}(t)$ into $\mathbf{x}'$ and $\mathbf{x}''$ in (1), we have the following trilinear relationship which must hold for any point $\mathbf{m}$ in view 1:

$$m^i e_{21}^j(t) e_{31}^k(t) \epsilon_{krv} \mathcal{T}_{ij}^r = 0_v \qquad {}^\forall\mathbf{m}. \tag{7}$$

Since (7) must hold for any $\mathbf{m}$, the remaining part, $e_{21}^j(t)e_{31}^k(t)\epsilon_{krv}\mathcal{T}_{ij}^r$, must be zero tensor. The similar discussions hold for the pairs of epipoles $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$ and $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$. Thus, we have the following relationships between epipoles and trifocal tensors:

$$e_{21}^j(t) e_{31}^k(t) \epsilon_{krv} \mathcal{T}_{ij}^r = 0_{iv} \tag{8}$$

$$e_{12}^i(t) e_{32}^k(t) \epsilon_{krv} \mathcal{T}_{ij}^r = 0_{jv} \tag{9}$$

$$e_{13}^i(t) e_{23}^j(t) \mathcal{T}_{ij}^r = 0^r. \tag{10}$$

These equations can be combined with (1) for computing $\mathcal{T}_{ij}^r$ from given epipoles and corresponding points. The important point here is that the trifocal tensor $\mathcal{T}_{ij}^r$ is constant, and thus we can use epipoles $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$ at N_1 different time for computing a single trifocal tensor $\mathcal{T}_{ij}^r$. From (8), we have the following equations on $\mathbf{t}$.

$$[\mathbf{M}_1(t_1)^\top, \cdots, \mathbf{M}_1(t_{N_1})^\top]^\top \mathbf{t} = \mathbf{0} \tag{11}$$

where $\mathbf{M}_1(t)$ denotes 9×27 matrix calculated from $\mathbf{e}_{21}$ and $\mathbf{e}_{31}$. Similarly, if we have epipoles $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$ at N_2 time instants and epipoles $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$ at N_3 time instants, (9) and (10) provide us the following constraints on $\mathbf{t}$:

$$[\mathbf{M}_2(t_1)^\top, \cdots, \mathbf{M}_2(t_{N_2})^\top]^\top \mathbf{t} = \mathbf{0} \tag{12}$$

$$[\mathbf{M}_3(t_1)^\top, \cdots, \mathbf{M}_3(t_{N_3})^\top]^\top \mathbf{t} = \mathbf{0} \tag{13}$$

where $\mathbf{M}_2(t)$ denotes 9×27 matrix depending on $\mathbf{e}_{12}$ and $\mathbf{e}_{32}$, and $\mathbf{M}_3(t)$ denotes 3×27 matrix lying on $\mathbf{e}_{13}$ and $\mathbf{e}_{23}$. Then, (11), (12) and (13) can be combined with (5) for computing $\mathcal{T}_{ij}^r$ as follows:

$$[\mathbf{M}(t_1)^\top, \cdots, \mathbf{M}(t_N)^\top, \mathbf{M}_1(t_1)^\top, \cdots, \mathbf{M}_1(t_{N_1})^\top, \mathbf{M}_2(t_1)^\top, \cdots, \mathbf{M}_2(t_{N_2})^\top, \mathbf{M}_3(t_1)^\top, \cdots, \mathbf{M}_3(t_{N_3})^\top]^\top \mathbf{t} = \mathbf{0}. \quad (14)$$

Here, we make use of Least Squares Method to solve the entries of trifocal tensor in $\mathbf{t}$. Rewrite (14) as $\mathbf{E}\mathbf{t} = \mathbf{0}$. The solution on $\mathbf{t}$ is the eigenvector which corresponds to the smallest eigenvalue of $\mathbf{E}^\top\mathbf{E}$. By using such combination, we can compute $\mathcal{T}_{ij}^r$ linearly from fewer corresponding points.

Although (8) provides us 9 linear equations on trifocal tensor, only 6 of them are linearly independent. However, if we use $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$ at N different time, the number of independent equations derived from (8) is not always $6N$. The same thing happens to other two pairs of epipoles. Furthermore, if we combine some pairs of epipoles and use N of them respectively to compute trifocal tensor, the results are very different.

In the following sections, we consider the number of independent equations and the minimum number of corresponding points required for computing spatio-temporal trifocal tensors under mutual projection of cameras by using one, two and all three epipole pairs respectively when these cameras have translational motions.

4. Using One Epipole Pair

4.1 Using Epipole Pair $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$ or $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$

We first consider why (8) has only 6 independent equations. In general, a tensor $m^i\epsilon_{ijk}$ represents three lines which go through a point $\mathbf{m}$. Thus $e_{31}^k(t)\epsilon_{krv}$ in (8) represents three epipolar lines in view 3, which go through an epipole $\mathbf{e}_{31}(t)$. So, (8) describes relationships between epipole $\mathbf{e}_{21}(t)$ in view 2 and epipolar lines $\mathbf{l}''(t)$ which go through $\mathbf{e}_{31}(t)$ in view 3 as follows:

$$e_{21}^j(t)l_k''(t)\mathcal{T}_{ij}^k = 0_i \quad (15)$$

Since (15) must hold for any point $\mathbf{m}$ in view 1, (15) is considered as a point-point-line incidence on any point $\mathbf{m}$ in view 1, epipole $\mathbf{e}_{21}(t)$ in view 2 and any epipolar line $\mathbf{l}''(t)$ which goes through $\mathbf{e}_{31}(t)$ in view 3 as follows:

$$m^i e_{21}^j(t)l_k''(t)\mathcal{T}_{ij}^k = 0 \quad (16)$$

For finding the number of independent equations in (8) in sequential images, we need to count the number of independent incidence relations described by (16) when we use N pairs of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$, $(t = t_1, \cdots, t_N)$.

We note that any point $\mathbf{m}$ in view 1 can be described by a linear combination of three basis points $\mathbf{m}_1$, $\mathbf{m}_2$ and $\mathbf{m}_3$. And any epipolar line $\mathbf{l}''(t)$ which goes through $\mathbf{e}_{31}(t)$ in view 3 can be described by a linear combination of two basis lines. Since $\mathbf{e}_{31}(t)$ of any t are collinear†, one of these two basis lines can be a line which goes through all the epipoles $\mathbf{e}_{31}(t)$ $(t = t_1, \cdots, t_N)$ as shown in Fig. 2. We call the line "epipole line" and denote it by $\mathbf{l}_0''$. Suppose $\mathbf{l}_1''$, $\mathbf{l}_2''$ and $\mathbf{l}_3''$ go through $\mathbf{e}_{31}(t_1)$, $\mathbf{e}_{31}(t_2)$ and $\mathbf{e}_{31}(t_3)$ respectively as shown in Fig. 2. Then, if we have one pair of epipoles $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at t_1, any incidence relation represented by (15) at time t_1

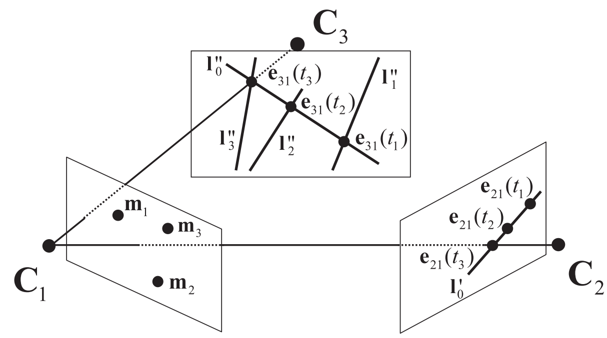


Fig. 2 The basis points, basis lines and epipole lines for representing incidence relations in three views. $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$ show three basis points in view 1. $\mathbf{l}_0'$ shows epipole line which goes through $\mathbf{e}_{21}$ in view 2. $\mathbf{l}_0''$ shows epipole line going through $\mathbf{e}_{31}$ in view 3. $\{\mathbf{l}_1'', \mathbf{l}_2'', \mathbf{l}_3''\}$ show three basis lines in view 3.

can be described by a linear combination of the following 6 basis incidence relations:

$$\begin{aligned} m_1^i e_{21}^j(t_1)l_{0k}''\mathcal{T}_{ij}^k &= 0 & m_1^i e_{21}^j(t_1)l_{1k}''\mathcal{T}_{ij}^k &= 0 \\ m_2^i e_{21}^j(t_1)l_{0k}''\mathcal{T}_{ij}^k &= 0 & m_2^i e_{21}^j(t_1)l_{1k}''\mathcal{T}_{ij}^k &= 0 \\ m_3^i e_{21}^j(t_1)l_{0k}''\mathcal{T}_{ij}^k &= 0 & m_3^i e_{21}^j(t_1)l_{1k}''\mathcal{T}_{ij}^k &= 0 \end{aligned} \quad (17)$$

Therefore, we have only 6 linearly independent equations in (8).

For simplification in (17), we define a new notation for describing all the 6 equations as follows:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_1)\} \begin{Bmatrix} \mathbf{l}_0'' \\ \mathbf{l}_1'' \end{Bmatrix} \quad (18)$$

The number of equations is the product of the number of rows of each column. So, the number of equations in (18) is $3 \times 1 \times 2 = 6$.

Thus, if we have a pair of epipoles $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at two different time, t_1 and t_2, then we can derive the following equations:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_1)\} \begin{Bmatrix} \mathbf{l}_0'' \\ \mathbf{l}_1'' \end{Bmatrix}, \begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_2)\} \begin{Bmatrix} \mathbf{l}_0'' \\ \mathbf{l}_2'' \end{Bmatrix} \quad (19)$$

Then, the number of equations in (19) is $3 \times 1 \times 2 + 3 \times 1 \times 2 = 12$. It means that by using a pair of epipoles $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at two different time, there exist 12 independent equations.

How about the case of 3 different time, t_1, t_2 and t_3? At time t_3, we have:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_3)\} \begin{Bmatrix} \mathbf{l}_0'' \\ \mathbf{l}_3'' \end{Bmatrix} \quad (20)$$

†Suppose the motions of $\mathbf{C}_1$ and $\mathbf{C}_3$ in X, Y, Z axes are $[\Delta X_1, \Delta Y_1, \Delta Z_1]^\top$ and $[\Delta X_3, \Delta Y_3, \Delta Z_3]^\top$ respectively. Since the translational motion is constant in each camera, ΔX_1, ΔY_1, ΔZ_1, ΔX_3, ΔY_3, ΔZ_3 are invariable. Then, view 3 observes the motion of $\mathbf{C}_1$ as $[\Delta X_1 - \Delta X_3, \Delta Y_1 - \Delta Y_3, \Delta Z_1 - \Delta Z_3]^\top$, which is still a translational motion on a line in the 3D space. Since a line in 3D is projected to a line in the 2D image, the projection of the trajectory of $\mathbf{C}_1$ to $\mathbf{C}_3$ is a line.

Since $\mathbf{e}_{21}(t_1)$, $\mathbf{e}_{21}(t_2)$ and $\mathbf{e}_{21}(t_3)$ are collinear, $\mathbf{e}_{21}(t_3)$ can be written by the linear combination of $\mathbf{e}_{21}(t_1)$ and $\mathbf{e}_{21}(t_2)$ as:

$$\mathbf{e}_{21}(t_3) = c_1\mathbf{e}_{21}(t_1) + c_2\mathbf{e}_{21}(t_2) \tag{21}$$

Then (20) can be described as follows:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{c_1\mathbf{e}_{21}(t_1) + c_2\mathbf{e}_{21}(t_2)\} \begin{Bmatrix} \mathbf{l}''_0 \\ \mathbf{l}''_3 \end{Bmatrix} \tag{22}$$

Now, since

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{c_1\mathbf{e}_{21}(t_1) + c_2\mathbf{e}_{21}(t_2)\} \{\mathbf{l}''_0\} \tag{23}$$

can be described by a linear combination of $\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_1)\} \{\mathbf{l}''_0\}$ and $\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_2)\} \{\mathbf{l}''_0\}$, it is linearly dependent with (19). Therefore, only

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{c_1\mathbf{e}_{21}(t_1) + c_2\mathbf{e}_{21}(t_2)\} \{\mathbf{l}''_3\} \tag{24}$$

is linearly independent, and then we have only 3 independent equations in (20). Thus, we find that from a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at 3 different time, 15 independent equations can be derived.

At time t_4, it seems that we have another independent equation as follows:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_4)\} \{\mathbf{l}''_4\} \tag{25}$$

However, this is not the case. Since any line on the plane can be described by a set of three basis lines, $\mathbf{l}''_3$ can be described by $\mathbf{l}''_0$, $\mathbf{l}''_1$ and $\mathbf{l}''_2$ as follows:

$$\mathbf{l}''_3 = d_1\mathbf{l}''_0 + d_2\mathbf{l}''_1 + d_3\mathbf{l}''_2$$

Then, at t_3, (24) can be represented as:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{c_1\mathbf{e}_{21}(t_1) + c_2\mathbf{e}_{21}(t_2)\} \{d_1\mathbf{l}''_0 + d_2\mathbf{l}''_1 + d_3\mathbf{l}''_2\} \tag{26}$$

Simplifying (26), we have:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \left\{\mathbf{e}_{21}(t_2)\mathbf{l}''_1 + \frac{c_1 d_3}{c_2 d_2}\mathbf{e}_{21}(t_1)\mathbf{l}''_2\right\} \tag{27}$$

Similarly, at t_4, (25) can also be described as:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{c_3\mathbf{e}_{21}(t_1) + c_4\mathbf{e}_{21}(t_2)\} \{d_4\mathbf{l}''_0 + d_5\mathbf{l}''_1 + d_6\mathbf{l}''_2\} \tag{28}$$

and their simplified forms are:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \left\{\mathbf{e}_{21}(t_2)\mathbf{l}''_1 + \frac{c_3 d_6}{c_4 d_5}\mathbf{e}_{21}(t_1)\mathbf{l}''_2\right\} \tag{29}$$

We find that (27) and (29) are same, except the coefficient $\frac{c_1 d_3}{c_2 d_2}$ and $\frac{c_3 d_6}{c_4 d_5}$, but in fact they are equal and relate to the initial position of camera 1, camera motions, camera matrices, t_1 and t_2, all of which are constants. As a result, there is no independent equation at t_4. The same thing happens at time t_5, $t_6, \cdots$. Thus when we use 3 or more pairs of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$, the number of independent equations we can derive is 15.

So far, we find that 6, 12 and 15 independent equations are available from one pair of epipoles at 1, 2 and 3 time. Since N sets of corresponding points provide us $2N$ linearly independent equations from (1), the following inequality must hold for computing 26 free parameters of the trifocal tensor $\mathcal{T}_{ij}^k$, by using a pair of epipoles $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at time t_1:

$$2N + 6 \geq 26$$

Thus we require 10 corresponding points except epipoles for computing the trifocal tensor. Similarly, if we have a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at time t_1 and t_2, the number of corresponding points required for computing $\mathcal{T}_{ij}^k$ is $(26 - 12)/2 = 7$, and if we have a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at time t_1, t_2 and t_3, we require 6 corresponding points.

The case of epipole pair $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$ is exactly the same as that of $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$. We summarize the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras in Table 1.

4.2 Using Epipole Pair $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$

The case of epipole pair $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$ is much simpler than that of other two epipole pairs and the analysis can refer to the previous section. The number of independent equations by using N pairs of $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$, and the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras are summarized in Table 2.

Table 1 The number of independent equations derived by using $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ for N_1 time ($t = t_1, \cdots, t_{N_1}$) or $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ for N_2 time ($t = t_1, \cdots, t_{N_2}$), and the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras. Note, $3^\star$ denotes 3 or greater than 3.

N_1 or N_2	# of Eq.	# of points
1	6	10
2	12	7
$3^\star$	15	6

Table 2 The number of independent equations derived by using $\{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ for N_3 time ($t = t_1, \cdots, t_{N_3}$), and the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras. Note, $3^\star$ denotes 3 or greater than 3.

N_3	# of Eq.	# of points
1	3	12
2	6	10
$3^\star$	9	9

5. Using Two Epipole Pairs

We next consider the number of corresponding points required for computing $\mathcal{T}_{ij}^{r}$ by using two pairs of epipoles observed during N time intervals.

5.1 Using Epipole Pair $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$ and $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$

5.1.1 $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{1} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$

Suppose we have a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at time t_1 and a pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ at time t_2 respectively. Then we have the following constraints on $\mathcal{T}_{ij}^{r}$:

$$e_{21}^{j}(t_1)e_{31}^{k}(t_1)\epsilon_{krv}\mathcal{T}_{ij}^{r} = 0_{iv} \tag{30}$$

$$e_{12}^{i}(t_2)e_{32}^{k}(t_2)\epsilon_{krv}\mathcal{T}_{ij}^{r} = 0_{jv} \tag{31}$$

We have known that (30) can be written as:

$$m^{i}e_{21}^{j}(t_1)l_k''(t_1)\mathcal{T}_{ij}^{k} = 0 \tag{32}$$

where $\mathbf{m}$ denotes any point in view 1, and $\mathbf{l}''(t_1)$ denotes any epipolar line which goes through $\mathbf{e}_{31}(t_1)$ in view 3. Similarly, (31) can be described as follows:

$$e_{12}^{i}(t_2)m'^{j}l_k''(t_2)\mathcal{T}_{ij}^{k} = 0 \tag{33}$$

where $\mathbf{m}'$ denotes any point in view 2, and $\mathbf{l}''(t_2)$ denotes any epipolar line which goes through $\mathbf{e}_{32}(t_2)$ in view 3. Now, let us consider $\mathbf{l}_1''$ and $\mathbf{l}_2''$ which go through $\mathbf{e}_{31}(t_1)$, and $\mathbf{l}_2''$ and $\mathbf{l}_3''$ which go through $\mathbf{e}_{32}(t_2)$ as shown in Fig. 3. The three lines do not coincide. Then (32) can be written as:

$$\left\{\begin{matrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{matrix}\right\}\{\mathbf{e}_{21}(t_1)\}\left\{\begin{matrix}\mathbf{l}_1''\\ \mathbf{l}_2''\end{matrix}\right\} \tag{34}$$

which has 6 independent equations. Similarly, (33) can also be written as:

$$\{\mathbf{e}_{12}(t_2)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\\ \mathbf{m}_3'\end{matrix}\right\}\left\{\begin{matrix}\mathbf{l}_3''\\ \mathbf{l}_2''\end{matrix}\right\} \tag{35}$$

However, (35) provides us less than 6 independent equations

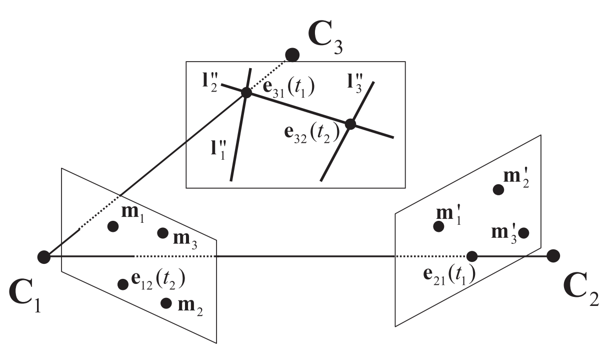


Fig. 3 The basis points and lines for representing incidence relations in three views.

if we combine it with (34). Let us explain it in detail.

(35) can be described as two parts:

$$\{\mathbf{e}_{12}(t_2)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\\ \mathbf{m}_3'\end{matrix}\right\}\{\mathbf{l}_3''\} \tag{36}$$

$$\{\mathbf{e}_{12}(t_2)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\\ \mathbf{m}_3'\end{matrix}\right\}\{\mathbf{l}_2''\} \tag{37}$$

Since (36) is independent relative to (34), (36) provides us 3 independent equations. However, (37) does not.

If we consider one of the basis points $\mathbf{m}_3'$ as $\mathbf{e}_{21}(t_1)$, (37) becomes:

$$\{\mathbf{e}_{12}(t_2)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\\ \mathbf{e}_{21}(t_1)\end{matrix}\right\}\{\mathbf{l}_2''\} \tag{38}$$

where

$$\{\mathbf{e}_{12}(t_2)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\end{matrix}\right\}\{\mathbf{l}_2''\} \tag{39}$$

are independent with (34). However, the third equation

$$\{\mathbf{e}_{12}(t_2)\}\{\mathbf{e}_{21}(t_1)\}\{\mathbf{l}_2''\} \tag{40}$$

is dependent with (34), since $\mathbf{e}_{12}(t_2)$ can be represented by the three basis points $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$ in view 1. Thus, (35) provides us $3 + 2 = 5$ independent equations. Therefore, 1 pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ and 1 pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ provide us $6 + 5 = 11$ independent equations.

5.1.2 $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$

If we have a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at time t_1, and a pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ at time t_2 and t_3, then we have the following constraints on $\mathcal{T}_{ij}^{r}$:

$$e_{21}^{j}(t_1)e_{31}^{k}(t_1)\epsilon_{krv}\mathcal{T}_{ij}^{r} = 0_{iv} \tag{41}$$

$$e_{12}^{i}(t_2)e_{32}^{k}(t_2)\epsilon_{krv}\mathcal{T}_{ij}^{r} = 0_{jv} \tag{42}$$

$$e_{12}^{i}(t_3)e_{32}^{k}(t_3)\epsilon_{krv}\mathcal{T}_{ij}^{r} = 0_{jv} \tag{43}$$

Then, we can obtain their simplified forms as follows:

$$\left\{\begin{matrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{matrix}\right\}\{\mathbf{e}_{21}(t_1)\}\left\{\begin{matrix}\mathbf{l}_2''\\ \mathbf{l}_3''\end{matrix}\right\} \tag{44}$$

$$\{\mathbf{e}_{12}(t_2)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\\ \mathbf{m}_3'\end{matrix}\right\}\left\{\begin{matrix}\mathbf{l}_1''\\ \mathbf{l}_2''\end{matrix}\right\} \tag{45}$$

$$\{\mathbf{e}_{12}(t_3)\}\left\{\begin{matrix}\mathbf{m}_1'\\ \mathbf{m}_2'\\ \mathbf{m}_3'\end{matrix}\right\}\left\{\begin{matrix}\mathbf{l}_1''\\ \mathbf{l}_3''\end{matrix}\right\} \tag{46}$$

where $\mathbf{l}_2''$ and $\mathbf{l}_3''$ go through $\mathbf{e}_{31}(t_1)$, $\mathbf{l}_1''$ and $\mathbf{l}_2''$ go through $\mathbf{e}_{32}(t_2)$, and, $\mathbf{l}_1''$ and $\mathbf{l}_3''$ go through $\mathbf{e}_{32}(t_3)$ as shown in Fig. 4.

The former discussions also hold here, and we can see

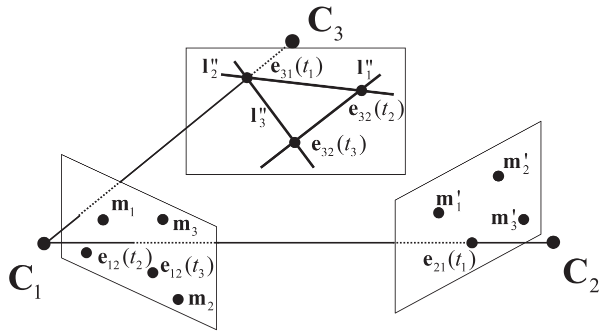


Fig. 4 The basis points, basis lines and epipole line for representing incidence relations in three views.

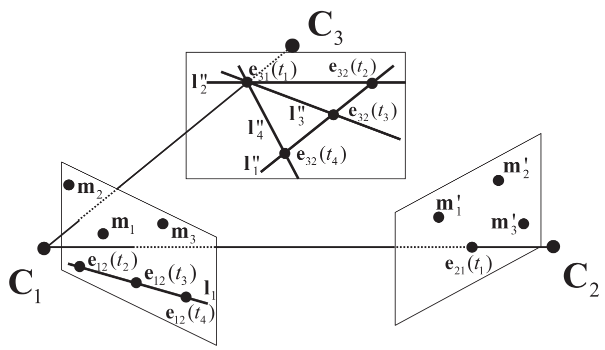


Fig. 5 The basis points, basis lines and epipole lines for representing incidence relations in three views.

that (44) provides us 6 independent equations, (45) and (46) provides us 5 independent equations respectively. Thus, a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at 1 time and a pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ at 2 different time provide us $6 + 5 + 5 = 16$ independent equations.

5.1.3 $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3}^\star \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$

Here, $n^\star$ denotes n or greater than n. We next consider the case where a pair of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ is available at a single time, and a pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ is available at three time intervals of more.

We first consider the case of $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$, in which (44), (45), (46), and the following equations are available:

$$\{\mathbf{e}_{12}(t_4)\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{m}'_3 \end{Bmatrix} \begin{Bmatrix} \mathbf{l}''_1 \\ \mathbf{l}''_4 \end{Bmatrix} \tag{47}$$

where $\mathbf{l}''_4$ goes through $\mathbf{e}_{31}(t_1)$ and $\mathbf{e}_{32}(t_4)$ as shown in Fig. 5.

As we have seen, $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ provide us 16 independent equations. Then how many independent equations does (47) include?

(47) can be written into two parts:

$$\{\mathbf{e}_{12}(t_4)\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{m}'_3 \end{Bmatrix} \{\mathbf{l}''_1\} \tag{48}$$

$$\{\mathbf{e}_{12}(t_4)\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{m}'_3 \end{Bmatrix} \{\mathbf{l}''_4\} \tag{49}$$

Since $\mathbf{e}_{12}(t_2)$, $\mathbf{e}_{12}(t_3)$ and $\mathbf{e}_{12}(t_4)$ are collinear, $\mathbf{e}_{12}(t_4)$ can be described by $\mathbf{e}_{12}(t_2)$ and $\mathbf{e}_{12}(t_3)$. Then, (48) can be rewritten as follows:

$$\{c_1\mathbf{e}_{12}(t_2) + c_2\mathbf{e}_{12}(t_3)\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{m}'_3 \end{Bmatrix} \{\mathbf{l}''_1\} \tag{50}$$

which can be represented by the combination of some equations from (45) and (46). So (48) does not provide us any independent equation. Then how about (49)? If we consider one of the basis points $\mathbf{m}'_3$ as $\mathbf{e}_{21}(t_1)$, (49) can be rewritten as:

$$\{\mathbf{e}_{12}(t_4)\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{e}_{21}(t_1) \end{Bmatrix} \{\mathbf{l}''_4\} \tag{51}$$

The first two equations are independent to others, but the third equation is not. Since $\mathbf{e}_{12}(t_4)$ and $\mathbf{l}''_4$ can be described by $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$ and $\{\mathbf{l}''_2, \mathbf{l}''_3\}$ respectively, the third equation can be represented as follows:

$$\{a_1\mathbf{m}_1 + a_2\mathbf{m}_2 + a_3\mathbf{m}_3\}\{\mathbf{e}_{21}(t_1)\}\{d_1\mathbf{l}''_2 + d_2\mathbf{l}''_3\} \tag{52}$$

which is the combination of 6 equations in (44). Therefore, (49) has only 2 independent equations. Thus, using one more pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$, we can derive 2 more independent equations than $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$, that is, $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ can bring us $16 + 2 = 18$ independent equations.

On the other hand, (49) can also be written as:

$$\{c_1\mathbf{e}_{12}(t_2) + c_2\mathbf{e}_{12}(t_3)\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{m}'_3 \end{Bmatrix} \{d_1\mathbf{l}''_2 + d_2\mathbf{l}''_3\} \tag{53}$$

Simplifying it, we have:

$$\left\{ \frac{c_2 d_1}{c_1 d_2} \mathbf{e}_{12}(t_3)\mathbf{l}''_2 + \mathbf{e}_{12}(t_2)\mathbf{l}''_3 \right\} \begin{Bmatrix} \mathbf{m}'_1 \\ \mathbf{m}'_2 \\ \mathbf{m}'_3 \end{Bmatrix} \tag{54}$$

At time t_n ($n > 4$), we derive the same equations as (54), and cannot derive new independent equations. Thus, $\mathbf{1} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3}^\star \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ still provide us 18 independent equations.

5.1.4 $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$

We next consider the case, where we have $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ at two time t_1 and t_2, and $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ at the other two time t_3 and t_4. In this case, we have the following equations:

$$\begin{Bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{Bmatrix} \{\mathbf{e}_{21}(t_1)\} \begin{Bmatrix} \mathbf{l}''_1 \\ \mathbf{l}''_2 \end{Bmatrix} \tag{55}$$

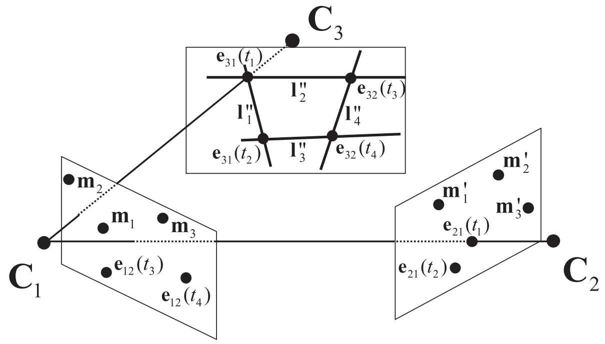


Fig. 6 The basis points, basis lines and epipole lines for representing incidence relations in three views.

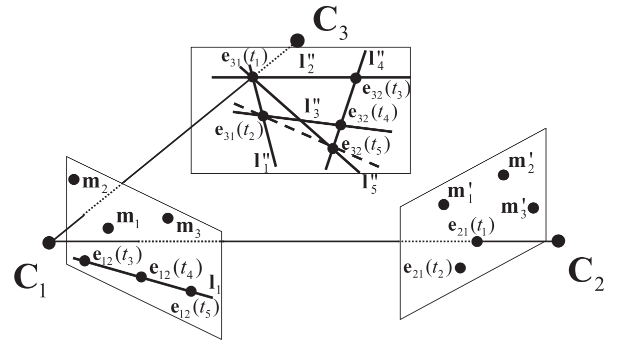


Fig. 7 The basis points, basis lines and epipole lines for representing incidence relations in three views.

$$\left\{\begin{matrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{matrix}\right\}\{\mathbf{e}_{21}(t_2)\}\left\{\begin{matrix}\mathbf{l}''_1\\ \mathbf{l}''_3\end{matrix}\right\} \tag{56}$$

$$\{\mathbf{e}_{12}(t_3)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\left\{\begin{matrix}\mathbf{l}''_4\\ \mathbf{l}''_2\end{matrix}\right\} \tag{57}$$

$$\{\mathbf{e}_{12}(t_4)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\left\{\begin{matrix}\mathbf{l}''_4\\ \mathbf{l}''_3\end{matrix}\right\} \tag{58}$$

where $\mathbf{l}''_1$ and $\mathbf{l}''_2$ go through $\mathbf{e}_{31}(t_1)$, $\mathbf{l}''_1$ and $\mathbf{l}''_3$ go through $\mathbf{e}_{31}(t_2)$, $\mathbf{l}''_4$ and $\mathbf{l}''_2$ go through $\mathbf{e}_{32}(t_3)$, and $\mathbf{l}''_4$ and $\mathbf{l}''_3$ go through $\mathbf{e}_{32}(t_4)$ as shown in Fig. 6. (55) and (56) provide us full 6 independent equations respectively, but (57) and (58) do not. (57) can be separated into:

$$\{\mathbf{e}_{12}(t_3)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\{\mathbf{l}''_4\} \tag{59}$$

$$\{\mathbf{e}_{12}(t_3)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\{\mathbf{l}''_2\} \tag{60}$$

Since (59) is independent with (55) and (56), (59) brings us 3 independent equations. Moreover, it can be written as:

$$\{(c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{e}_{21}(t_2)\end{matrix}\right\}\{\mathbf{l}''_4\} \tag{61}$$

Then (60) can also be rewritten into:

$$\{\mathbf{e}_{12}(t_3)\} \quad \{\mathbf{m}'_1\} \quad \{\mathbf{l}''_2\} \tag{62}$$

$$\{c_1\mathbf{m}_1+c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}\{\mathbf{e}_{21}(t_1)\}\{\mathbf{l}''_2\} \tag{63}$$

$$\{c_1\mathbf{m}_1+c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}\{\mathbf{e}_{21}(t_2)\}\{d_1\mathbf{l}''_1+d_2\mathbf{l}''_3 + d_3\mathbf{l}''_4\} \tag{64}$$

(62) is independent to (55), (56) and (59), but (63) can be described by (55). In addition, (64) can be represented by (56) and (61). Therefore, (60) contributes only 1 independent equation. Thus, (57) provides us 3+1 = 4 independent equations. For the same reason, (58) also brings us 4 independent equations. Then, $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ provides us 6 + 6 + 4 + 4 = 20 independent equations.

5.1.5 $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3}^\star \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$

We have derived 20 independent equations from the case $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$. What will happen when we use one more pair of $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$? We can add the following equations to (55)~(58) in this case:

$$\{\mathbf{e}_{12}(t_5)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\left\{\begin{matrix}\mathbf{l}''_4\\ \mathbf{l}''_5\end{matrix}\right\} \tag{65}$$

where $\mathbf{l}''_4$ is a epipole line which goes through $\mathbf{e}_{32}$, and $\mathbf{l}''_5$ denotes a line going through $\mathbf{e}_{32}(t_5)$ as shown in Fig. 7. Then (65) can be rewritten as follows:

$$\{a_1\mathbf{e}_{12}(t_3) + a_2\mathbf{e}_{12}(t_4)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\{\mathbf{l}''_4\} \tag{66}$$

$$\{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{e}_{21}(t_1)\\ \mathbf{e}_{21}(t_2)\end{matrix}\right\}\{\mathbf{l}''_5\} \tag{67}$$

(66) can be described by (57) and (58), so it does not provide any independent equation. In addition, $\mathbf{l}''_5$ goes through $\mathbf{e}_{32}(t_5)$. Then it can be a line going through not only $\mathbf{e}_{31}(t_1)$ and $\mathbf{e}_{32}(t_5)$, but also $\mathbf{e}_{31}(t_2)$ and $\mathbf{e}_{32}(t_5)$ as shown in Fig. 7. Thus, (67) can be written as:

$$\{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\} \quad \{\mathbf{m}'_1\} \quad \{\mathbf{l}''_5\} \tag{68}$$

$$\{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}\{\mathbf{e}_{21}(t_1)\}\{d_1\mathbf{l}''_1+d_2\mathbf{l}''_2\} \tag{69}$$

$$\{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}\{\mathbf{e}_{21}(t_2)\}\{d_3\mathbf{l}''_1 + d_4\mathbf{l}''_3\} \tag{70}$$

(68) is independent to other equations, but (69) and (70) can be represented by (55) and (56) respectively. Then (65) provides us 1 independent equation. Thus, in the case of $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3} \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$, we have 20 + 1 = 21 independent equations.

On the other hand, (65) can also be described by:

$$\{a_1\mathbf{e}_{12}(t_3) + a_2\mathbf{e}_{12}(t_4)\}\left\{\begin{matrix}\mathbf{m}'_1\\ \mathbf{m}'_2\\ \mathbf{m}'_3\end{matrix}\right\}\left\{b_1\mathbf{l}''_2 + b_2\mathbf{l}''_3 + b_3\mathbf{l}''_4\right\} \tag{71}$$

Modifying it, we have:

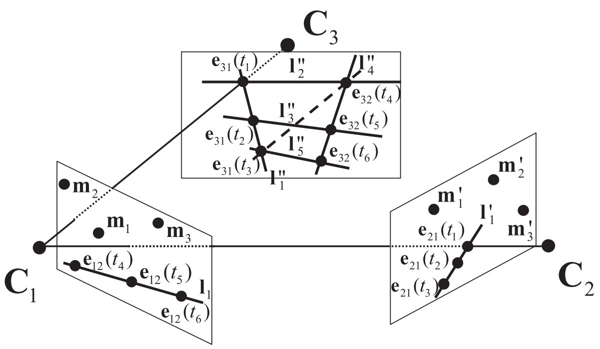


Fig. 8 The basis points, basis lines and epipole lines for representing incidence relations in three views.

Table 3 The number of independent equations derived by using $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ for N_1 time $(t = t_1, \cdots, t_{N_1})$, and $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ for N_2 time $(t = t_1, \cdots, t_{N_2})$, and the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras. Note, $3^\star$ denotes 3 or greater than 3.

$N_1 + N_2$	# of Eq.	# of points
1 + 1	11	8
1 + 2	16	5
$1 + 3^\star$	18	4
2 + 2	20	3
$2 + 3^\star$	21	3
$3^\star + 3^\star$	22	2

$$\left\{\frac{a_2 b_1}{a_1 b_2}\mathbf{e}_{12}(t_4)\mathbf{l}_2'' + \mathbf{e}_{12}(t_3)\mathbf{l}_3''\right\}\left\{\begin{matrix}\mathbf{m}_1' \\ \mathbf{m}_2' \\ \mathbf{m}_3'\end{matrix}\right\} \tag{72}$$

Since $\frac{a_2 b_1}{a_1 b_2}$ is a constant, even if we use more $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$, the number of independent equations could not increase for the same reason mentioned before. Therefore, $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3}^\star \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ provides us 21 independent equations.

5.1.6 $\mathbf{3}^\star \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3}^\star \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$

The discussion on the number of independent equations in this case is very similar to the previous case, $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3}^\star \times \{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$, so we do not give the explanation here, and only the configuration is shown in Fig. 8.

Up to now, we have considered all the cases of using epipole pair $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ and $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$. They are summarized in Table 3.

5.2 Using Epipole Pair $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$ and $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$, or $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$ and $\{\mathbf{e}_{13}, \mathbf{e}_{23}\}$

In such combinations, the number of independent equations and corresponding points required in all the cases are summarized in Table 5. Most of them can be obtained by Tables 1 and 2 directly. Only two cases shown in Table 4 need to be explained:

We first consider the case $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3} \times \{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$.

In this case, we have 5 sets of simplified equations:

Table 4 Two exceptions.

No.	$N_1 \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ or $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ + $N_2 \times \{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$	# of Eq.	# of points
1	$2 + 3^\star$	20	3
2	$3^\star + 3^\star$	22	2

Table 5 The number of independent equations derived by using of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ for N_1 time $(t = t_1, \cdots, t_{N_1})$ or $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ for N_2 time $(t = t_1, \cdots, t_{N_2})$, and $\{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ for N_3 time $(t = t_1, \cdots, t_{N_3})$, and the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras. Note, $3^\star$ denotes 3 or greater than 3.

$(N_1 + N_3)$ or $(N_2 + N_3)$	# of Eq.	# of points
1 + 1	9	9
1 + 2	12	7
$1 + 3^\star$	15	6
2 + 1	15	6
$3^\star + 1$	18	4
2 + 2	18	4
$2 + 3^\star$	20	3
$3^\star + 2$	21	3
$3^\star + 3^\star$	22	2

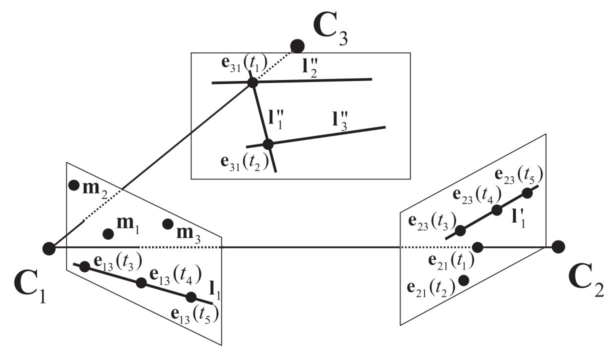


Fig. 9 The basis points, basis lines and epipole lines for representing incidence relations in three views.

$$\left\{\begin{matrix}\mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3\end{matrix}\right\}\{\mathbf{e}_{21}(t_1)\}\left\{\begin{matrix}\mathbf{l}_1'' \\ \mathbf{l}_2''\end{matrix}\right\} \tag{73}$$

$$\left\{\begin{matrix}\mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3\end{matrix}\right\}\{\mathbf{e}_{21}(t_2)\}\left\{\begin{matrix}\mathbf{l}_1'' \\ \mathbf{l}_3''\end{matrix}\right\} \tag{74}$$

$$\{\mathbf{e}_{13}(t_3)\}\{\mathbf{e}_{23}(t_3)\}\left\{\begin{matrix}\mathbf{l}_1'' \\ \mathbf{l}_2'' \\ \mathbf{l}_3''\end{matrix}\right\} \tag{75}$$

$$\{\mathbf{e}_{13}(t_4)\}\{\mathbf{e}_{23}(t_4)\}\left\{\begin{matrix}\mathbf{l}_1'' \\ \mathbf{l}_2'' \\ \mathbf{l}_3''\end{matrix}\right\} \tag{76}$$

$$\{\mathbf{e}_{13}(t_5)\}\{\mathbf{e}_{23}(t_5)\}\left\{\begin{matrix}\mathbf{l}_1'' \\ \mathbf{l}_2'' \\ \mathbf{l}_3''\end{matrix}\right\} \tag{77}$$

where $\mathbf{l}_1''$ and $\mathbf{l}_2''$ go through $\mathbf{e}_{31}(t_1)$, and, $\mathbf{l}_1''$ and $\mathbf{l}_3''$ go through $\mathbf{e}_{31}(t_2)$ as shown in Fig. 9. (73)~(76) represent the case of $\mathbf{2} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$, which provides us 18 independent equations. Then we next consider how many independent equations can be derived from (77).

Since $\mathbf{e}_{13}(t_4)$ and $\mathbf{e}_{23}(t_4)$ can be described by $\{\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3\}$ and $\{\mathbf{e}_{21}(t_1),\ \mathbf{e}_{21}(t_2),\ \mathbf{e}_{23}(t_3)\}$ respectively, the first equation in (76) has the following calculations:

$$\{\mathbf{e}_{13}(t_4)\}\,\{\mathbf{e}_{23}(t_4)\}\,\{\mathbf{l}_1''\} \tag{78}$$

$$= \{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}$$

$$\{d_1\mathbf{e}_{21}(t_1)+d_2\mathbf{e}_{21}(t_2)+d_3\mathbf{e}_{23}(t_3)\}\,\{\mathbf{l}_1''\} \tag{79}$$

$$= \{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}\,\{\mathbf{e}_{23}(t_3)\}\,\{\mathbf{l}_1''\} \tag{80}$$

$$= \{\mathbf{e}_{13}(t_4)\}\,\{\mathbf{e}_{23}(t_3)\}\,\{\mathbf{l}_1''\} \tag{81}$$

For the same reason, the first equation in (77) can also be rewritten as:

$$\{\mathbf{e}_{13}(t_5)\}\,\{\mathbf{e}_{23}(t_3)\}\,\{\mathbf{l}_1''\} \tag{82}$$

Since it can be described by:

$$\{c_4\mathbf{e}_{13}(t_3) + c_5\mathbf{e}_{13}(t_4)\}\,\{\mathbf{e}_{23}(t_3)\}\,\{\mathbf{l}_1''\} \tag{83}$$

and (83) can be represented by (75) and (81), the first equation in (77) is not independent. Whereas the other 2 equations in (77) are independent to all the others. Therefore, (77) brings us only 2 independent equations. Even if one more pair of $\{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ is given, it will not provide us more independent constrains.

Next, let us discuss the case of $\mathbf{3} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3} \times \{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ in No.2.

All the equations with simplified forms are as follows:

$$\begin{Bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{Bmatrix}\{\mathbf{e}_{21}(t_1)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_2''\end{Bmatrix} \tag{84}$$

$$\begin{Bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{Bmatrix}\{\mathbf{e}_{21}(t_2)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_3''\end{Bmatrix} \tag{85}$$

$$\begin{Bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{Bmatrix}\{\mathbf{e}_{21}(t_3)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_4''\end{Bmatrix} \tag{86}$$

$$\{\mathbf{e}_{13}(t_4)\}\,\{\mathbf{e}_{23}(t_4)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_2''\\ \mathbf{l}_3''\end{Bmatrix} \tag{87}$$

$$\{\mathbf{e}_{13}(t_5)\}\,\{\mathbf{e}_{23}(t_5)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_2''\\ \mathbf{l}_3''\end{Bmatrix} \tag{88}$$

$$\{\mathbf{e}_{13}(t_6)\}\,\{\mathbf{e}_{23}(t_6)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_2''\\ \mathbf{l}_3''\end{Bmatrix} \tag{89}$$

The configuration of them is shown in Fig. 10. If we only focus on (84)~(88), we know that they describe the case of $\mathbf{3} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{2} \times \{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ which brings us 21 independent equations. How about (89)?

Since (88) can be represented by:

$$\{c_1\mathbf{m}_1 + c_2\mathbf{m}_2 + c_3\mathbf{m}_3\}$$

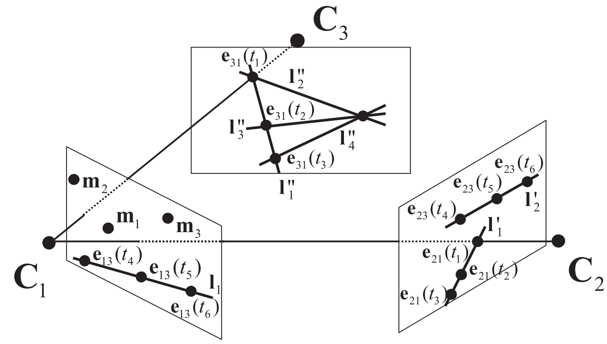


Fig. 10 The basis points, basis lines and epipole lines for representing incidence relations in three views.

$$\{d_1\mathbf{e}_{21}(t_1) + d_2\mathbf{e}_{21}(t_2) + d_3\mathbf{e}_{23}(t_4)\}\begin{Bmatrix}\mathbf{l}_1''\\ \mathbf{l}_2''\\ \mathbf{l}_3''\end{Bmatrix} \tag{90}$$

Expanding and simplifying them, we obtain:

$$\{\mathbf{e}_{13}(t_5)\}\,\{\mathbf{e}_{23}(t_4)\}\,\{\mathbf{l}_1''\} \tag{91}$$

$$\{\mathbf{e}_{13}(t_5)\}\,\{d_2\mathbf{e}_{21}(t_2) + d_3\mathbf{e}_{23}(t_4)\}\,\{\mathbf{l}_2''\} \tag{92}$$

$$\{\mathbf{e}_{13}(t_5)\}\,\{d_1\mathbf{e}_{21}(t_1) + d_3\mathbf{e}_{23}(t_4)\}\,\{\mathbf{l}_3''\} \tag{93}$$

For the same reason, (89) also equals to:

$$\{\mathbf{e}_{13}(t_6)\}\,\{\mathbf{e}_{23}(t_4)\}\,\{\mathbf{l}_1''\} \tag{94}$$

$$\{\mathbf{e}_{13}(t_6)\}\,\{d_2'\mathbf{e}_{21}(t_2) + d_3'\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_2''\} \tag{95}$$

$$\{\mathbf{e}_{13}(t_6)\}\,\{d_1'\mathbf{e}_{21}(t_1) + d_3'\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_3''\} \tag{96}$$

(94) can be described by (87) and (91), so in (89), only 2 independent equation candidates exit.

On the other hand,

$$\begin{Bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{Bmatrix}\{\mathbf{e}_{21}(t_3)\}\,\{\mathbf{l}_4''\} \tag{97}$$

in (86) can be rewritten into:

$$\begin{Bmatrix}\mathbf{e}_{13}(t_5)\\ \mathbf{e}_{13}(t_6)\\ \mathbf{m}_3\end{Bmatrix}\{a_1\mathbf{e}_{21}(t_1) + a_2\mathbf{e}_{21}(t_2)\}\{b_1\mathbf{l}_2'' + b_2\mathbf{l}_3''\} \tag{98}$$

So we have these 2 equations:

$$\frac{a_2 b_1}{a_1 b_2}\,\{\mathbf{e}_{13}(t_5)\}\,\{\mathbf{e}_{21}(t_2)\}\,\{\mathbf{l}_2''\} + \{\mathbf{e}_{13}(t_5)\}\,\{\mathbf{e}_{21}(t_1)\}\,\{\mathbf{l}_3''\} \tag{99}$$

$$\frac{a_2 b_1}{a_1 b_2}\,\{\mathbf{e}_{13}(t_6)\}\,\{\mathbf{e}_{21}(t_2)\}\,\{\mathbf{l}_2''\} + \{\mathbf{e}_{13}(t_6)\}\,\{\mathbf{e}_{21}(t_1)\}\,\{\mathbf{l}_3''\} \tag{100}$$

Combining (96) and (100) we have:

$$-\frac{a_2 b_1 d_1'}{a_1 b_2}\,\{\mathbf{e}_{13}(t_6)\}\,\{\mathbf{e}_{21}(t_2)\}\,\{\mathbf{l}_2''\}+\left\{d_3'\mathbf{e}_{13}(t_6)\right\}\{\mathbf{e}_{23}(t_4)\}\,\{\mathbf{l}_3''\} \tag{101}$$

Then, by substituting (95) into (101), we obtain:

$$-\frac{a_2 b_1 d_1'}{a_1 b_2 d_2'} \{\mathbf{e}_{13}(t_6)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_2''\} + \{\mathbf{e}_{13}(t_6)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_3''\} \tag{102}$$

Similarly, the following equation can also be derived:

$$-\frac{a_2 b_1 d_1}{a_1 b_2 d_2} \{\mathbf{e}_{13}(t_5)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_2''\} + \{\mathbf{e}_{13}(t_5)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_3''\} \tag{103}$$

Since

$$-\frac{a_2 b_1 d_1}{a_1 b_2 d_2} = -\frac{a_2 b_1 d_1'}{a_1 b_2 d_2'}, \tag{104}$$

we denote them as A. For $\mathbf{e}_{13}(t_6) = c_4\mathbf{e}_{13}(t_4) + c_5\mathbf{e}_{13}(t_5)$, (102) can be written as follows:

$$\begin{aligned} &Ac_4\{\mathbf{e}_{13}(t_4)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_2''\} + c_4\{\mathbf{e}_{13}(t_4)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_3''\} \\ &\quad + c_5(A\{\mathbf{e}_{13}(t_5)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_2''\} + \{\mathbf{e}_{13}(t_5)\}\{\mathbf{e}_{23}(t_4)\}\{\mathbf{l}_3''\}) \end{aligned}$$

which is the pure combination of (87) and (103). Therefore, (96) can be described by other equations. Thus, (89) only provides us 1 independent equation. Then, from the case of $\mathbf{3} \times \{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\} + \mathbf{3} \times \{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$, we derive $21 + 1 = 22$ independent equations. Increasing the number of $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ or $\{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ will not bring more independent constrains. So, in the case of No.2, we still have 22 independent equations. And if we change the epipole pair $\{\mathbf{e}_{21}, \mathbf{e}_{31}\}$ to $\{\mathbf{e}_{12}, \mathbf{e}_{32}\}$, the same results will be derived.

6. Using All Three Epipole Pairs

The results on the number of independent equations and corresponding points required by using all three epipole pairs are summarized in Table 6, in which the number of independent equations can be derived by:

$$N_1 + N_2 + N_3 = (N_1 + N_2) + (N_2 + N_3) - N_2$$

where, $(N_1 + N_2)$ is from Table 3, $(N_2 + N_3)$ is from Table 5, and, N_2 comes from Table 1.

The most interesting thing is when we have 2 or more samples of each epipole pair, we do not need any corresponding point to derive trifocal tensors anymore.

7. Experiments

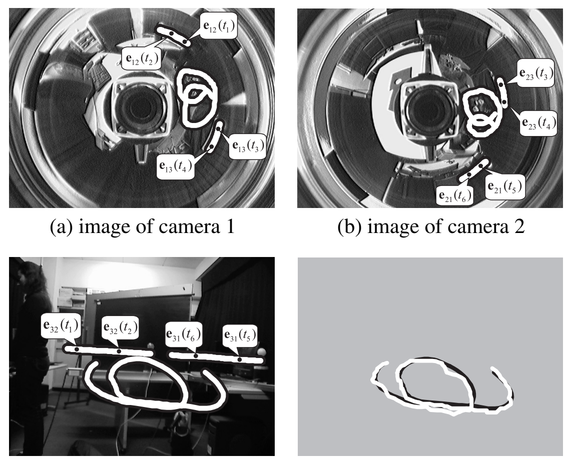
We next show the results of some experiments.

7.1 Real Image Experiment

We first show the results from real image experiments, in which the spatio-temporal trifocal tensor is computed from three epipole pairs at two different time viewed from arbitrary translational cameras. No other corresponding points are used. The extracted trifocal tensor is used for generating the third view from the first view and the second view of moving cameras.

Table 6 The number of independent equations derived by using $\{\mathbf{e}_{21}(t), \mathbf{e}_{31}(t)\}$ for N_1 time ($t = t_1, \cdots, t_{N_1}$), $\{\mathbf{e}_{12}(t), \mathbf{e}_{32}(t)\}$ for N_2 time ($t = t_1, \cdots, t_{N_2}$) and $\{\mathbf{e}_{13}(t), \mathbf{e}_{23}(t)\}$ for N_3 time ($t = t_1, \cdots, t_{N_3}$), and the number of corresponding points required for computing trifocal tensors in each case of mutual projections of cameras. Note, $n^\star$ denotes n or greater than n.

$N_1 + N_2 + N_3$	# of Eq.	# of points
$1+1+1$	14	6
$1+1+2$	17	5
$1+2+1$	19	4
$1+1+3^\star$	20	3
$1+3^\star+1$	21	3
$1+2+2$	22	2
$2+2+1$	23	2
$1+2+3^\star$	24	1
$1+3^\star+2$	24	1
$2+3^\star+1$	24	1
$1+3^\star+3^\star$	25	1
$3^\star+3^\star+1$	25	1
$2^\star+2^\star+2^\star$	26	0



(a) image of camera 1 (b) image of camera 2

(c) image of camera 3 (d) point transfer result

Fig. 11 Real image experiment 1. Images (a), (b) and (c) show epipole lines and image motions of a single point viewed from camera 1, 2 and 3. The black points on epipole lines in each image are used for computing the trifocal tensor. These three cameras are translating with different speeds and in different directions. The white curve in (d) shows image motions in camera 3 computed by the extended trifocal tensor, and the black curve shows the real image motions.

In this experiment, we used two omnidirectional cameras and one normal camera. These three cameras are translating with different constant speeds and in different directions. In Fig. 11, (a), (b) and (c) show image motions of a single moving point and 6 epipole lines in translational camera 1, 2 and 3 respectively. The trifocal tensor is computed from 3 epipole pairs, each of which is sampled at two different time, $\{\mathbf{e}_{12}(t_1), \mathbf{e}_{32}(t_1)\}$, $\{\mathbf{e}_{12}(t_2), \mathbf{e}_{32}(t_2)\}$, $\{\mathbf{e}_{13}(t_3), \mathbf{e}_{23}(t_3)\}$, $\{\mathbf{e}_{13}(t_4), \mathbf{e}_{23}(t_4)\}$, $\{\mathbf{e}_{21}(t_5), \mathbf{e}_{31}(t_5)\}$, $\{\mathbf{e}_{21}(t_6), \mathbf{e}_{31}(t_6)\}$, which are shown by black points in (a), (b) and (c). The extracted trifocal tensor is used for generating image motions in camera 3 from image motions in camera 1 and 2. The white curve in Fig. 11 (d) shows image motions in camera 3 generated from the extended trifocal tensor, and the black curve shows the real image motions viewed from camera 3. As shown in

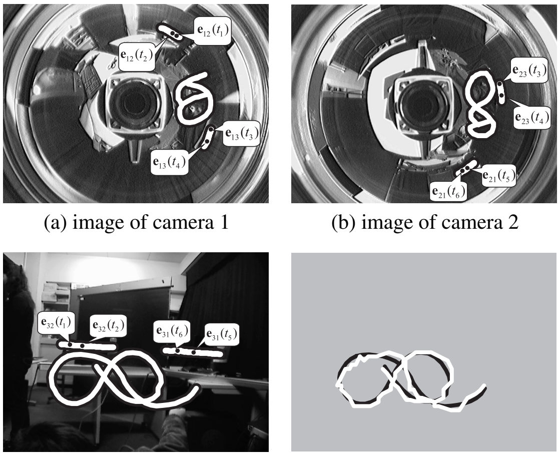


Fig. 12 Real image experiment 2.

Fig. 11 (d), the generated image motions are almost identical with the original image motions even if these 3 cameras have unknown translational motions.

The other experiment is also given in Fig. 12. As we can see, the spatio-temporal trifocal tensor can be derived from only 2 samples of the projection of each camera with arbitrary translational motion, and thus it is very practical for generating images of arbitrary motions viewed from translational cameras.

7.2 Stability Evaluation

We next show the stability of extracted spatio-temporal trifocal tensors with the 13 point method and the proposed mutual projection method.

Figure 13 (a) shows a 3D configuration of three moving cameras and one moving point. The black points show the position of the three cameras, $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$, before translational motions, and the white points show those after the translational motions. The translational motions of these three cameras are different and unknown. The black curve shows a locus of a freely moving point. For evaluating the extracted trifocal tensors, we computed reprojection error using the trifocal tensors. The reprojection error is defined as: $\frac{1}{M}\sum_{i=1}^{M} d(\mathbf{m}_i, \hat{\mathbf{m}}_i)^2$, where M denotes the number of the points, and $d(\mathbf{m}_i, \hat{\mathbf{m}}_i)$ denotes the distance between the true point $\mathbf{m}_i$ and a point $\hat{\mathbf{m}}_i$ computed by the trifocal tensor.

In the mutual projection method, we used three pairs of epipoles in two different time and some corresponding points for computing the trifocal tensors. We increased the number of corresponding points used for computing trifocal tensors in three views from 0 to 25 and evaluated the reprojection errors with Least Squares Method addressed in Sect. 3. In the same way, we also evaluated the 13 point method with same corresponding points from 13 to 25. The Gaussian noise with the standard deviation of 1 pixel is added to each image. Figure 13 (b) shows the relationship between the number of corresponding points and the reprojection errors. The black points show the result from mutual

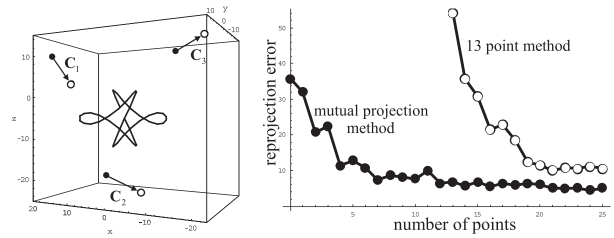


Fig. 13 Stability evaluation.

projection method, and the white points show that from 13 point method. As we can see, with less or even no corresponding points, the mutual projection method can derive more stable trifocal tensors than the 13 point method.

8. Conclusions

In this paper, we analyzed the computation of spatio-temporal multiple view geometry from mutual projections of multiple cameras. Taking three moving cameras for instance, we discussed the number of independent equations and corresponding points to compute the trifocal tensor by using one, two and all three epipole pairs. As a result, with one epipole pair at 3 different time we need 6 corresponding points, with two epipole pairs we at least require 2 corresponding points, and when we use three epipole pairs at 2 different time respectively, we do not need any corresponding point to figure out the trifocal tensor. That means arbitrary image motions tracked by moving cameras can be recovered even if they are coplanar or collinear, as long as we have the projections of cameras. The method was implemented and tested by using real image sequences. The stability of trifocal tensors extracted by using mutual projections of cameras was compared with that of the 13 point method.

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