LETTER

On-Line Model Parameter Estimations for Time-Delay Systems

Jung Hun PARK†, **Soohee HAN**††, ***and*** **Bokyu KWON**†††a), ***Members***

SUMMARY This paper concerns a problem of on-line model parameter estimations for multiple time-delay systems. In order to estimate unknown model parameters from measured state variables, we propose two schemes using Lyapunov's direct method, called parallel and series-parallel model estimators. It is shown through a numerical example that the proposed parallel and series-parallel model estimators can be effective when sufficiently rich inputs are applied.

key words: *model parameter estimation, time-delay system, Lyapunov's direct method*

1. Introduction

Over the past several decades, a great deal of attention has been given to the research area of system identification for a model with unknown parameters [1], [2]. In many real plants, some of their model parameters are not available, and thus they should be estimated from known measurable values. Since unknown model parameters to be estimated may be time-varying due to changes in operating conditions, aging of plant equipment, and so on, it is generally known that on-line model parameter estimation techniques are more effective than off-line ones. Specially, an on-line model parameter estimation is essential in many adaptive control schemes. How on-line model parameter estimations and adaptive controls work together has been widely researched for many applications including mechatronics, aerospace, transportation, traffic, chemical process, and network communication [3]–[12]. Since time-delays on states and/or control inputs are often encountered in material transportation delays, data transmission delays, sensor delays, and so on, many real plants can be modeled as time-delay systems [13], [14]. It would be meaningful to develop estimation algorithms for unknown model parameters in such time-delay systems. Even though there are many unknown model parameter estimation algorithms for MIMO ordinary systems [15]–[17] and SISO single time-delay systems [18]–[20], MIMO multiple time-delay systems with unknown parameters have not been considered for a model parameter estimation problem.

In this paper, we propose two on-line model parameter estimation algorithms for multiple time-delay systems under the assumption that the system matrix is stable and all state variables are available. The proposed algorithms are developed with Lyapunov's direct method, which are well-known for non-time-delay systems as the parallel and series-parallel model estimators. Their stability is guaranteed if some linear matrix inequality (LMI) conditions are met. It is shown through a numerical example that two proposed methods can effectively estimate unknown model parameters, and are they compared for several cases. In the presence of noise, the series-parallel model estimator is also shown to be more sensitive than the parallel one.

The rest of this paper is organized as follows. In Sect. 2, the parallel and series-parallel model estimators for multiple time-delay systems with unknown parameters are proposed. Section 3 provides a numerical example to illustrate the performances of two proposed algorithms and compare their performance. Finally, conclusions are given in Sect. 4.

2. Two On-line Model Parameter Estimations

2.1 Parallel Model Estimators

Let us consider the following linear time-invariant system with multiple fixed state delays:

$$\dot{x}(t) = A_p x(t) + \sum_{i=1}^{k} A_{p_i} x(t-h_i) + B_p u(t), \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^r$, k, and h_i are the state, the input, the number of delayed states, and the delay sizes, respectively. Here, k and h_i are assumed to be known. The system matrices $A_p \in \Re^{n\times n}$, $A_{p_i} \in \Re^{n\times n}$, and $B_p \in \Re^{n\times r}$ are unknown and thus should be estimated from known states and inputs. It is also assumed that the system (1) is stable and $u(t) \in \mathcal{L}_\infty$ so that $x(t) \in \mathcal{L}_\infty$.

In order to obtain a parallel model estimator, we first take the following parallel model:

$$\dot{\hat{x}}(t) = \hat{A}_p(t)\hat{x}(t) + \sum_{i=1}^{k} \hat{A}_{p_i}(t)\hat{x}(t-h_i) + \hat{B}_p(t)u(t), \tag{2}$$

where $\hat{x}(t)$ is the estimate of $x(t)$ and $\hat{A}_p(t)$, $\hat{A}_{p_i}(t)$, and $\hat{B}_p(t)$ are the estimates of A_p, A_{p_i}, and B_p at time t, respectively. How to choose $\hat{A}_p(t)$, $\hat{A}_{p_i}(t)$, and $\hat{B}_p(t)$ will be discussed later

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†The author is with the Technical Research Laboratories, POSCO, Phang, Gyeongsangbuk-do, Korea.
††The author is with the Department of Electric Engineering, Konkuk University, Seoul, Korea.
†††The author is with the Department of Control and Instrumentation Engineering, Kangwon National University, Samcheok, Gangwon-do, Korea.
a) E-mail: bkkwon@kangwon.ac.kr

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on.

If the state estimation error and model parameter estimation errors are defined as

$$e(t) \triangleq x(t) - \hat{x}(t), \quad \tilde{A}_p(t) \triangleq \hat{A}_p(t) - A_p,$$
$$\tilde{B}_p(t) \triangleq \hat{B}_p(t) - B_p, \quad \tilde{A}_{p_i}(t) \triangleq \hat{A}_{p_i}(t) - A_{p_i},$$

then the error dynamics is given as

$$\begin{aligned}\dot{e}(t) &= A_p e(t) + \sum_{i=1}^{k} A_{p_i} e(t-h_i)\\ &\quad - \tilde{A}_p(t)\hat{x}(t) - \sum_{i=1}^{k} \tilde{A}_{p_i}(t)\hat{x}(t-h_i) - \tilde{B}_p(t)u(t), \end{aligned} \tag{3}$$

for the parallel model (2).

In order to derive the adaptive laws of $\hat{A}_p(t)$, $\hat{A}_{p_i}(t)$, and $\hat{B}_p(t)$, we start off by considering the following Lyapunov function candidate:

$$\begin{aligned} &V\left(e(t), \tilde{A}_p(t), \tilde{A}_{p_i}(t), \tilde{B}_p(t)\right) = e^T(t)Pe(t)\\ &+\mathrm{tr}\left(\frac{\tilde{A}_p^T(t)P\tilde{A}_p(t)}{\gamma_1}\right) + \mathrm{tr}\left(\frac{\tilde{B}_p^T(t)P\tilde{B}_p(t)}{\gamma_2}\right)\\ &+\sum_{i=1}^{k}\mathrm{tr}\left(\frac{\tilde{A}_{p_i}^T(t)P\tilde{A}_{p_i}(t)}{\delta_i}\right) + \sum_{i=1}^{k}\int_{t-h_i}^{t} e^T(\tau)Qe(\tau)d\tau, \end{aligned} \tag{4}$$

where $\mathrm{tr}(A)$ denotes the trace of a matrix A, γ_1, γ_2, and δ_i are positive constants, and $P = P^T > 0$ and $Q = Q^T > 0$ will be chosen to satisfy some condition for the Lyapunov stability later in this paper. The time derivative $\dot{V}$ of V in (4) can be represented as

$$\begin{aligned} \dot{V} &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t)\\ &+ \mathrm{tr}\left(\frac{\dot{\tilde{A}}_p^T(t)P\tilde{A}_p(t)}{\gamma_1} + \frac{\tilde{A}_p^T(t)P\dot{\tilde{A}}_p(t)}{\gamma_1}\right)\\ &+ \mathrm{tr}\left(\frac{\dot{\tilde{B}}_p^T(t)P\tilde{B}_p(t)}{\gamma_2} + \frac{\tilde{B}_p^T(t)P\dot{\tilde{B}}_p(t)}{\gamma_2}\right)\\ &+ \sum_{i=1}^{k}\mathrm{tr}\left(\frac{\dot{\tilde{A}}_{p_i}^T(t)P\tilde{A}_{p_i}(t)}{\delta_i} + \frac{\tilde{A}_{p_i}^T(t)P\dot{\tilde{A}}_{p_i}(t)}{\delta_i}\right)\\ &+ \sum_{i=1}^{k}\left\{e^T(t)Qe(t) - e^T(t-h_i)Qe(t-h_i)\right\}. \end{aligned} \tag{5}$$

In order to guarantee the stability of the error dynamic (3), that is, to make $\dot{V}$ in (5) along the trajectory of (3) negative, the obvious choices for $\dot{\tilde{A}}_p(t)$, $\dot{\tilde{A}}_{p_i}(t)$, and $\dot{\tilde{B}}_p(t)$ are intuitively given as follows:

$$\dot{\tilde{A}}_p(t) = \dot{\hat{A}}_p(t) = \gamma_1 e(t)\hat{x}^T(t), \tag{6}$$
$$\dot{\tilde{B}}_p(t) = \dot{\hat{B}}_p(t) = \gamma_2 e(t)u^T(t), \tag{7}$$
$$\dot{\tilde{A}}_{p_i}(t) = \dot{\hat{A}}_{p_i}(t) = \delta_i e(t)\hat{x}^T(t-h_i), \tag{8}$$

and then $\dot{V}$ can be represented as

$$\dot{V} = \Psi^T\Lambda\Psi < 0, \tag{9}$$

where Ψ and Λ are defined by

$$\Psi^T \triangleq \begin{bmatrix} e^T(t) & e^T(t-h_1) & \cdots & e^T(t-h_k) \end{bmatrix},$$
$$\Lambda \triangleq \begin{bmatrix} A_p^T P + PA_p + kQ & PA_{p_1} & \cdots & PA_{p_k}\\ A_{p_1}^T P & -Q & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ A_{p_k}^T P & 0 & \cdots & -Q \end{bmatrix}.$$

It is noted that $\hat{A}_p(t)$, $\hat{A}_{p_i}(t)$, and $\hat{B}_p(t)$ can be computed by using the measured state $x(t)$, the measured input $u(t)$, and the estimated state $\hat{x}(t)$ from the parallel model (2).

2.2 Series-Parallel Model Estimators

In order to obtain a series-parallel model estimator, we choose arbitrary stable matrices, $A_m \in \Re^{n\times n}$ and $A_{m_i} \in \Re^{n\times n}$, and then adding and subtracting the terms $A_m x(t)$ and $\sum_{i=1}^{k} A_{m_i}x(t-h_i)$ in the left-hand side of (1) yield

$$\begin{aligned}\dot{x}(t) &= A_m x(t) + (A_p - A_m)x(t) + \sum_{i=1}^{k} A_{m_i}x(t-h_i)\\ &\quad + \sum_{i=1}^{k}(A_{p_i} - A_{m_i})x(t-h_i) + B_p u(t), \end{aligned} \tag{10}$$

and we have the following series-parallel model:

$$\begin{aligned}\dot{\hat{x}}(t) &= A_m\hat{x}(t) + (\hat{A}_p(t) - A_m)x(t) + \sum_{i=1}^{k} A_{m_i}\hat{x}(t-h_i)\\ &\quad + \sum_{i=1}^{k}(\hat{A}_{p_i}(t) - A_{m_i})x(t-h_i) + \hat{B}_p(t)u(t). \end{aligned} \tag{11}$$

The error dynamic for the series-parallel model (11) is given as

$$\begin{aligned}\dot{e}(t) &= A_m e(t) - \sum_{i=1}^{k} A_{m_i} e(t-h_i)\\ &\quad - \tilde{A}_p(t)x(t) - \sum_{i=1}^{k}\tilde{A}_{p_i}(t)x(t-h_i) - \tilde{B}_p(t)u(t). \end{aligned} \tag{12}$$

As in the case of parallel model estimators, make $\dot{V}$ in (5) along the trajectory of (12) negative for guaranteeing the stability of the series-parallel model estimator, $\dot{\tilde{A}}_p(t)$, $\dot{\tilde{A}}_{p_i}(t)$, and $\dot{\tilde{B}}_p(t)$ are easily determined as follows:

$$\dot{\tilde{A}}_p(t) = \dot{\hat{A}}_p(t) = \gamma_1 e(t)x^T(t), \tag{13}$$
$$\dot{\tilde{B}}_p(t) = \dot{\hat{B}}_p(t) = \gamma_2 e(t)u^T(t), \tag{14}$$
$$\dot{\tilde{A}}_{p_i}(t) = \dot{\hat{A}}_{p_i}(t) = \delta_i e(t)x^T(t-h_i), \tag{15}$$

and then $\dot{V}$ is given as

$$\dot{V} = \Psi^T\Gamma\Psi < 0, \tag{16}$$

where Γ is defined by

$$\Gamma \triangleq \begin{bmatrix} A_m^T P + PA_m + kQ & -PA_{m_1} & \cdots & -PA_{m_n} \\ -A_{m_1}^T P & -Q & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -A_{m_n}^T P & 0 & \cdots & -Q \end{bmatrix}.$$

2.3 Properties and Stabilities for Two Estimators

It is noted that Λ in (9) depends on the unknown parameters A_p and A_{p_i}, whereas Γ in (16) depends on the known design parameters A_m and A_{m_i}. Furthermore, the considerable difference between the parallel and series-parallel model estimators comes from their performance in the presence of noise. If the measurement $x(t)$ is corrupted by some noise v, the estimated results for the series-parallel model estimator will depend on v^2 and v, whereas for the parallel one only on v. The quality of estimation for the parallel model estimator may be less sensitive for a noise.

From now on, let us consider the convergence of two proposed algorithms. For some positive or negative definite matrix Ω, the following property satisfies

$$\lambda_{\min}(\Omega)\|x\|^2 \le x^T \Omega x \le \lambda_{\max}(\Omega)\|x\|^2, \tag{17}$$

where $\lambda_{\min}(\Omega)$ and $\lambda_{\max}(\Omega)$ stand for the minimum and maximum eigenvalues of a matrix Ω, respectively. Using this inequality (17) and taking integration on both sides of (9) with (6)–(8) or (16) with (13)–(15) yield

$$\int_0^\infty \Psi^T \Psi dt \le \left(\frac{V_\infty - V_0}{\lambda_{\max}(\Lambda)} \text{ or } \frac{V_\infty - V_0}{\lambda_{\max}(\Gamma)} \right) < \infty,$$

where V_∞ and V_0 are defined by

$$\lim_{t\to\infty} V\left(e(t), \tilde{A}_p(t), \tilde{A}_{p_i}(t), \tilde{B}_p(t)\right) = V_\infty < \infty,$$
$$V(t=0) = V_0,$$

therefore, we have

$$\int_0^\infty \Psi^T \Psi dt = \int_0^\infty \left(\|e(t)\|^2 + \sum_{i=1}^k \|e(t-h_i)\|^2 \right) dt < \infty,$$

which implies that $e(t) \in \mathcal{L}_2$. Because $e(t) = x(t) - \hat{x}(t)$ and $x(t),\ \hat{x}(t) \in \mathcal{L}_\infty$, we have that $e(t) \in \mathcal{L}_\infty$. From (3) or (12) and $x(t),\ u(t) \in \mathcal{L}_\infty$ or $\hat{x}(t),\ u(t) \in \mathcal{L}_\infty$, it also follows that $\dot{e}(t) \in \mathcal{L}_\infty$ because of the Lyapunov stability properties. With $e(t),\ \dot{e}(t) \in \mathcal{L}_\infty$ and $e(t) \in \mathcal{L}_2$, it can be inferred by Barbălat's Lemma that $\lim_{t\to\infty} e(t) = 0$, which, in turn, implies that $\dot{\hat{A}}_p(t),\ \dot{\hat{A}}_{p_i}(t),\ \dot{\hat{B}}_p(t) \to 0$ as $t \to \infty$.

It is worth noting that the convergence properties of $\hat{A}_p(t)$, $\hat{A}_{p_i}(t)$, and $\hat{B}_p(t)$ to their true values A_p, A_{p_i}, and B_p, respectively, entirely depend on the properties of the input $u(t)$. If $u(t)$ belongs to the class of sufficiently rich inputs, *i.e.*, $u(t)$ has enough frequencies to excite all the modes of the system, then it is persistently exciting (PE) and guarantees that $\hat{A}_p(t)$, $\hat{A}_{p_i}(t)$, and $\hat{B}_p(t)$ exponentially fast converge to A_p, A_{p_k}, and B_p, respectively.

3. Numerical Example

In this section, a numerical example is presented to illustrate the performance of the proposed parallel and series-parallel model estimators. Consider the following time-delay system:

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} -0.5 & -1 \\ 0.5 & 0 \end{bmatrix} x(t-2) + \begin{bmatrix} -0.5 \\ 1.5 \end{bmatrix} u(t). \tag{18}$$

For the series-parallel model estimator, stable design matrices A_m and A_{m_1} are given by

$$A_m = \begin{bmatrix} -1 & 0 \\ 0 & -1.5 \end{bmatrix}, \quad A_{m_1} = \begin{bmatrix} -0.5 & 0 \\ 0 & 0 \end{bmatrix}.$$

The positive definite matrices P and Q guaranteeing the stability of two proposed algorithms are obtained by solving the LMIs [21], $\Lambda < 0$ and $\Gamma < 0$,

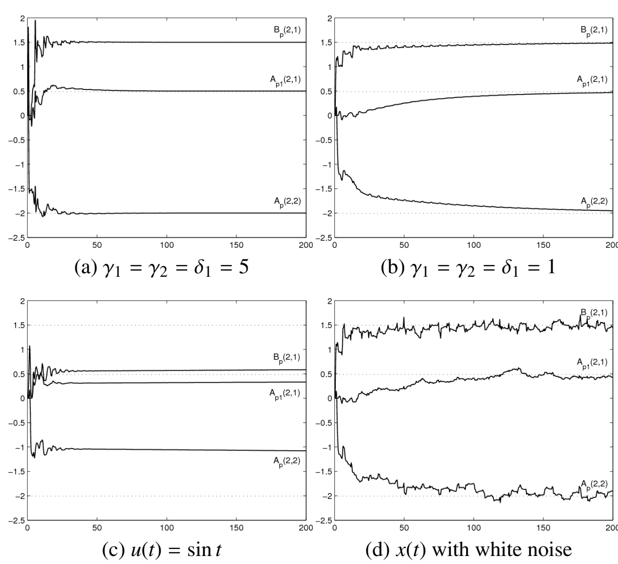
$$P = \begin{bmatrix} 61.3673 & 5.1815 \\ 5.1815 & 51.0960 \end{bmatrix}, \ Q = \begin{bmatrix} 53.3413 & 15.3883 \\ 15.3883 & 109.8872 \end{bmatrix},$$

for the parallel model estimator and

$$P = \begin{bmatrix} 60.7507 & 0 \\ 0 & 49.3106 \end{bmatrix}, \ Q = \begin{bmatrix} 63.5121 & 0 \\ 0 & 72.3222 \end{bmatrix},$$

for the series-parallel one.

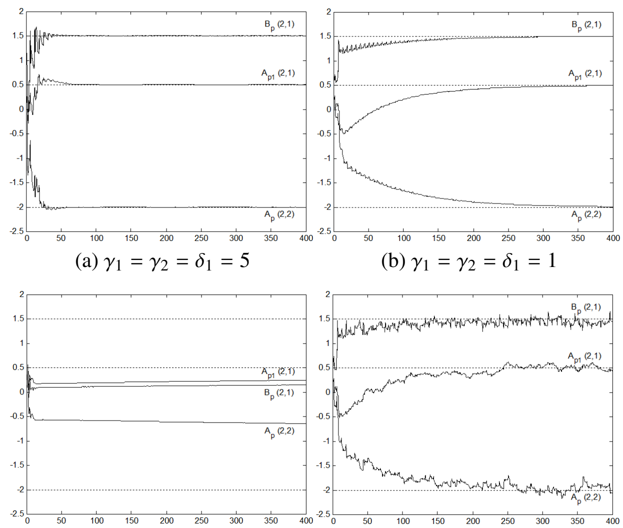
The performance of two proposed algorithms is shown in Fig. 1 and 2 (Note that the different time-scales are adopted on two figures). Plots (a) and (b) in Fig. 1 and 2 give the results when the input $u(t) = \sin t + \sin 2t + \sin 3t$, and the adaptive gain $\gamma_1 = \gamma_2 = \delta_1 = 5$ for (a) and $\gamma_1 = \gamma_2 = \delta_1 = 1$ for (b). It is clear that a larger adaptive gain leads to a faster convergence to true parameters. In each plot (c) of Fig. 1 and



(a) $\gamma_1 = \gamma_2 = \delta_1 = 5$ (b) $\gamma_1 = \gamma_2 = \delta_1 = 1$

(c) $u(t) = \sin t$ (d) $x(t)$ with white noise

Fig. 1 Results of parallel model estimators.



(a) $\gamma_1 = \gamma_2 = \delta_1 = 5$ (b) $\gamma_1 = \gamma_2 = \delta_1 = 1$

(c) $u(t) = \sin t$ (d) $x(t)$ with white noise

Fig. 2 Results of series-parallel model estimators.

2, the parameter convergence to true values is not achieved due to the use of a non-PE input. Each plot (d) of Fig. 1 and 2 shows that two proposed schemes can estimate the true parameters although the measurement is corrupted by $0.05n(t)$, where $n(t)$ is a normally distributed white noise. Also, two plots tell us that that the estimates of the parallel model estimator converge to true parameters much faster than that of series-parallel model estimator.

4. Conclusion

This paper discussed the design of two on-line model parameter estimation algorithms for multiple time-delay systems under the assumption that their system matrix is stable and the state variables are measurable. We proposed the parallel and series-parallel model estimators with Lyapunov's direct method. It was shown that their stability is guaranteed under some LMI condition. It was also verified through a numerical example that two proposed schemes can estimate the true parameters for multiple state-delayed systems with unknown system matrices correctly, even in the presence of measurement noise.

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