LETTER

Parameterized Multisurface Fitting for Multi-Frame Superresolution

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SUMMARY We propose a parameterized multisurface fitting method for multi-frame super-resolution (SR) processing. A parameter assumed for the unknown high-resolution (HR) pixel is used for multisurface fitting. Each surface fitted at each low-resolution (LR) pixel is an expression of the parameter. Final SR result is obtained by fusing the sampling values from these surfaces in the maximum a posteriori fashion. Experimental results demonstrate the superiority of the proposed method.

key words: *super-resolution, Taylor series, intensity estimation, parameter multi-surface fitting*

1. Introduction

SR has been studied for decades [1]. Among various SR methods, the interpolation-based approach is the most intuitive one. A pixelwise average algorithm [2] is implemented in the maximum-likelihood sense with Gaussian additive noise. A median average algorithm [3] is adopted which is robust to errors in motion and blur estimation. In [4], an interpolation-based approach using Delaunay triangulation models each triangle patch as a bivariate polynomial. Recently, a method based on interpolation by using multi-surface fitting is presented in [5], which takes local spatial structures into account. However, all the methods mentioned above only use the LR pixels in the neighborhood to estimate the HR pixel.

The only information that has not been used is that of the HR pixel. If we use the information of the HR pixel in the process of multisurface fitting, we have more information to form the surface and make the result more accurate. So it would be beneficial to use the unknown HR pixels as a parameter in paradigm of interpolation-based SR. In spired by this, we propose an image SR method named parameter multi-surface fitting. Specifically, we fit one surface at each LR pixel using the value of HR pixel as a parameter and the values of LR pixels as constants in the neighborhood. Therefore, more information can be utilized to reconstruct the HR pixels. In addition, by using parameter multi-surface fitting, the final expression shows that our method has the ability of reducing fitting errors.

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2. Methodology

Suppose that each HR pixel p_H has a set of LR pixels in the neighborhood after subpixel registration [6], denoted by $p_{L1} \cdots p_{Li} \cdots p_{LK}$, where K is the number of LR pixels in the neighborhood. As shown in Fig. 1, pixels from different LR images are positioned in an HR grid after subpixel registration. The size of neighborhood is chosen as 1*1 HR pixel. If no pixel exists in the chosen neighborhood, we enlarge its size in a stepwise manner. And then we can determine the number of LR pixels in the neighborhood and use those pixels to form the surfaces. We fit one surface at each LR pixel and obtain K values $f_{Li}(p_H)$ by resampling each surface, i.e.,

$$f_{Li}(p_H) \triangleq S(x_H, y_H, \Gamma_{Li}), \quad 1 \le i \le K \tag{1}$$

where Γ_{Li} is the fitted surface for LR pixel p_{Li}. And the intensity of p_H can be obtained by MAP estimation [5]:

$$\begin{aligned}\hat{f}(p_H) &= \arg\max_{f(p_H)} q\left(f(p_H) \,|\, f_{L1}(p_H), \cdot\cdot, f_{LK}(p_H)\right)\\ &= \arg\max_{f(p_H)} q\left(f_{L1}(p_H), \cdot\cdot, f_{LK}(p_H) \,|\, f(p_H)\right) q\left(f(p_H)\right)\end{aligned} \tag{2}$$

where $q(\cdots)$ is the probability density function.

We use the 2-D Taylor series to fit each surface. Suppose that p_{Li} has M_i LR pixels in the neighborhood and the value of the HR pixel p_H is assumed to be the parameter ph. The actual values of K and M_i are different in different neighborhood and dependent on subpixel registration and positional relationship of LR pixels. With the method in the first paragraph in Sect. 2, we can determine the actual value. Then we have M_i+1 equations:

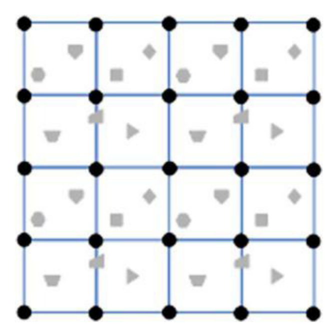

Fig. 1 Illustration of subpixel registration and neighborhood.

$$\begin{bmatrix} \Delta x_{1,i} & \Delta y_{1,i} & \frac{\Delta x_{1,i}^2}{2} & \frac{\Delta y_{1,i}^2}{2} & \Delta x_{1,i}\Delta y_{1,i} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta x_{j,i} & \Delta y_{j,i} & \frac{\Delta x_{j,i}^2}{2} & \frac{\Delta y_{j,i}^2}{2} & \Delta x_{j,i}\Delta y_{j,i} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta x_{M_i,i} & \Delta y_{M_i,i} & \frac{\Delta x_{M_i,i}^2}{2} & \frac{\Delta y_{M_i,i}^2}{2} & \Delta x_{M_i,i}\Delta y_{M_i,i} \\ \Delta x_{H,i} & \Delta y_{H,i} & \frac{\Delta x_{H,i}^2}{2} & \frac{\Delta y_{H,i}^2}{2} & \Delta x_{H,i}\Delta y_{H,i} \end{bmatrix} \begin{bmatrix} \frac{\partial f(p_{L_i})}{\partial x} \\ \frac{\partial f(p_{L_i})}{\partial y} \\ \frac{\partial^2 f(p_{L_i})}{\partial x^2} \\ \frac{\partial^2 f(p_{L_i})}{\partial y^2} \\ \frac{\partial^2 f(p_{L_i})}{\partial x \partial y} \end{bmatrix}$$
$$= \left[\Delta f_{1,i}, \cdots, \Delta f_{j,i}, \cdots, \Delta f_{M_i,i}, \Delta f_{H,i}\right]^T \tag{3}$$

where $1 \le j \le M_i$, $\Delta x_{j,i} = x_{Lj} - x_{Li}$, $\Delta y_{j,i} = y_{Lj} - y_{Li}$, $\Delta f_{j,i} = f(p_{Lj}) - f(p_{Li})$ and $\Delta x_{H,i} = x_H - x_{Li}$, $\Delta y_{H,i} = y_H - y_{Li}$, $\Delta f_{H,i} = ph - f(p_{Li})$. It is worthwhile to note that $\Delta f_{H,i} = ph - f(p_{Li})$ is not a constant but an unknown parameter. After using the method of the least square solution, we obtain the Taylor coefficients, of which each element is a polynomial of ph,

$$\vec{t} = [a_{il} + b_{il}ph], \quad 1 \le i \le K, 1 \le l \le 5 \tag{4}$$

where t is the Taylor coefficients, and a_{il}, b_{il} are constants related with intensities and positions of the LR pixels.

Thus, we obtain the expression of each surface and the sampling value on the surface. It is easy to prove that the sampling value is also a polynomial of ph.

$$f_{Li}(p_H) = d_i + e_i ph, \quad 1 \le i \le K \tag{5}$$

where d_i and e_i can be regarded as constant dependent on intensities and positions of the LR pixels.

In this letter, once we obtain Γ_{Li}, we can obtain ξ for each surface by the following equation:

$$f\left(p_{Lj}\right) = f_{Li}\left(p_{Lj}\right) + \xi_{ij}, \quad 1 \le i \le K, 1 \le j \le M_i \tag{6}$$
$$f(p_H) = f_{Li}(p_H) + \xi_{iH}, \quad 1 \le i \le K \tag{7}$$

where p_{Lj} and p_H are the LR and HR pixels that are used to fit Γ_{Li}, and ξ_{ij} and ξ_{iH} are the estimation errors at p_{Lj} and p_H on Γ_{Li}, respectively. The estimation errors are still the polynomials of ph.

$$\xi_{ij} = \alpha_{ij} + \beta_{ij}ph, \quad 1 \le i \le K, 1 \le j \le M_i \tag{8}$$
$$\xi_{iH} = \alpha_{iH} + \beta_{iH}ph, \quad 1 \le i \le K \tag{9}$$

Subsequently, we can calculate the fitting error σ_i^2 of the surface Γ_{Li}

$$\sigma_i^2 = \frac{1}{M_i + 1}\left(\sum_{j=1}^{M_i} \xi_{ij}^2 + \xi_{iH}^2\right), \quad 1 \le i \le K \tag{10}$$

It can be proved that σ_i^2 is a quadratic polynomial of ph.

$$\sigma_i^2 = a_i + b_i ph + c_i ph^2, \quad 1 \le i \le K \tag{11}$$

where a_i, b_i and c_i are constants related with intensities and positions of the LR pixels. Once we have (5) and (11), under Gaussian assumption, (2) becomes

$$\hat{f}(p_H) = \arg\min_{f(p_H)} \left[\sum_{i=1}^{K} \sigma_i + \sum_{i=1}^{K} \frac{(f_{Li}(p_H) - f(p_H))^2}{\sigma_i^2} + \lambda (f_0(p_H) - f(p_H))^2\right] \tag{12}$$

where $f_0(p_H)$ is the prior estimation of $f(p_H)$ and λ is an empirical parameter. $f_0(p_H)$ can be obtained in many ways, such as B-spline interpolation. Substituting (5) and (11) to (12), we have

$$\hat{f}(p_H) = \arg\min_{ph} \left[\sum_{i=1}^{K} \left(a_i + b_i ph + c_i ph^2\right)^{\frac{1}{2}} + \sum_{i=1}^{K} \frac{(d_i + e_i ph - ph)^2}{a_i + b_i ph + c_i ph^2} + \lambda (f_0(p_H) - ph)^2\right] \tag{13}$$

where a_i, b_i, c_i, d_i, e_i are the same with those in (5) and (11).

Essentially, the second item of (12) is like the form of weighted sum. The weight is the fitting error, and the surface with smaller noise and error has greater contribution to the final HR pixel value. The first item of (12) limits the fitting error in order to guarantee the surface formed more accurately and less noisy.

3. Experiments

We utilize 25 LR images to reconstruct HR images with Gaussian noise of 15 dB. The LR images are generated by sub-sampling with the factor of 4 in each direction and the positions of sampling are random. To eliminate the effect of prior knowledge on the final performance, we set the value of λ in (13) to 0.

We adopt several image quality assessment (IQA) methods to quantify the results, including visual information fidelity index (VIF) [7], feature-similarity index (FSIM) [8], and peak signal to noise ratio (PSNR). The results are shown in Table 1. Larger VIF, FSIM and PSNR values mean better results of reconstruction. Comparing to other methods, our method achieves a better result. Information of the HR pixels makes the estimated HR pixels more accurate.

Moreover, we provide visual examples in Fig. 2 to compare with other methods intuitively. From Fig. 2, we can observe that other methods fails to reconstruct the details and the reconstructed images has obvious artifacts while our method has a better performance in preserving details and generates fewer artifacts than other methods.

Table 1 Comparisons based on IQA.

Methods/Metrics	VIF	FSIM	PSNR
Elad [2]	0.3757	0.7863	35.3119
Farsiu [3]	0.3621	0.8238	35.6890
Lertrattanapanich[4]	0.4678	0.8088	35.0998
Zhou [5]	0.4629	0.8175	36.2953
Our method	**0.5855**	**0.8726**	**37.8399**

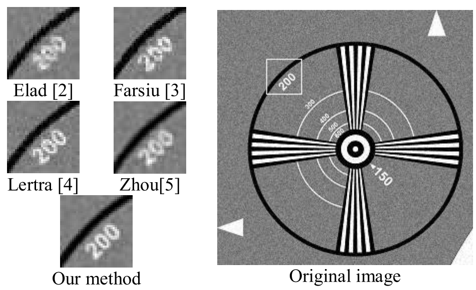

Fig. 2 Visual examples of reconstructed images.

4. Conclusion

We present a SR method by considering the unknown HR pixel as a parameter of a fitted surface. The proposed method can reduce fitting errors in the manner of the MAP approach. Experiments show that our method can achieve better performance with lower reconstruction error.

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