Finite-Time Coordinated Path Following Control of Leader-Following Multi-Agent Systems

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Research Article

Keywords: Finite‐time coordinated control, multi‐agent systems.

Posted Date: July 7th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-560294/v1

Title

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Abstract

The applications of the continuous feedback method to achieve both path following and a formation moving along desired orbits at a finite time is presented. It is assumed that the topology among the virtual leader and the followers is directed. An additional condition of so called *barrier function* to yield all the agents moving within a limited area is designed. A novel continuous finite-time path following control law is first designed based on the barrier function and backstepping. Then a novel continuous finite-time formation algorithm is designed by regarding the path following errors as disturbances. The settling time properties of the resulting system are studied in detail. Simulations are presented to validate the proposed strategies.

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Declarations

Funding

This work was supported by the National Natural Science Foundation of China (61973074).

Conflicts of interest/Competing interests

The authors declare that there is no conflict of interests regarding the publication of this article.

Availability of data and material

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Code availability

The simulation code used during the current study are available from the corresponding author on reasonable request.

Authors' contributions

Weibing Chen and Yang-Yang Chen contributed equally to this work and should be considered co-first authors.

Ethics approval

Not applicable.

Consent to participate

Not applicable.

Consent for publication

Not applicable.

Noname manuscript No.
(will be inserted by the editor)

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Received: date / Accepted: date

Abstract The applications of the continuous feedback method to achieve both path following and a formation moving along desired orbits at a finite time is presented. It is assumed that the topology among the virtual leader and the followers is directed. An additional condition of so called *barrier function* to yield all the agents moving within a limited area is designed. A novel continuous finite-time path following control law is first designed based on the barrier function and backstepping. Then a novel continuous finite-time formation algorithm is designed by regarding the path following errors as disturbances. The settling time properties of the resulting system are studied in detail. Simulations are presented to validate the proposed strategies.

Keywords Finite-time coordinated control · multi-agent systems

1 Introduction

Currently the theory of formation control problem has emerged as a hot spot and attracted great attention of researchers. To achieve better performance in seeking measurements of biological variables across a range of spatial and temporal scales in the applications of oceanic and planetary explorations [Bertozzi *et al.*, 2005, Fiorelli *et al.*, 2006], unmanned systems are required to simultaneous follow a set of given orbits with a desired formation, which is a special formation control problem named as the *coordinated path following* control problem.

In the area of the coordinated path following control most results focus on the asymptotic stability of the resulting multi-agent systems. In [Cao *et al.*, 2009] a discrete-time consensus-based algorithm is developed to force each follower tracking a leader with the desired dynamics, which is also called as the *consensus tracking* control problem. The continuous-time consensus tracking control laws are given in the cases of the time-invariant formation in [Cao & Ren, 2012], the time-varying formation in [Yu *et al.*, 2018] and the containment motion in [Zhang & Chen, 2020]. In [Ghabcheloo, 2007] a coordinated path following control law is designed by parameterizing the desired trajectories and at the same time synchronizing the orbital parameters. Such idea is used in the case of uncertain dynamics in [Peng *et al.*, 2013]. Noting that the geometry of the orbit, a novel geometry extension method is proposed and then intergraded into the consensus of the generalized arc-lengths (that is the smooth functions of arc-length) to achieve the coordinated path following task in [Zhang & Leonard, 2007, Chen & Tian, 2015]. The geometry extension method is also used to solve the asymptotic coordinated path following problem with the time-varying flows in [Chen *et al.*, 2020, Chen *et al.*, 2020]. However, the coordinated path following control problem within a finite settling time is still unsolved yet.

This work was supported by the National Natural Science Foundation of China (61973074). Weibing Chen and Yang-Yang Chen contributed equally to this work and should be considered co-first authors. Corresponding author Y.-Y. Chen. Tel. +8613951618987.

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Recently, the finite-time control laws in the multi-agent systems concentrates on the consensus or consensus tracking problems. In [Xiao *et al.*, 2009] a finite-time consensus tracking law is designed for the structure consisting with one leader and bidirectional connected followers based on the sliding-mode method. The sliding-mode method can be used in the cases of directed topologies in [Cao *et al.*, 2010, Wang & Xiao, 2010], uncertainties in [Khoo *et al.*, 2009] and under-actuated systems in [Li *et al.*, 2018]. By integrating into the saturation, the sliding-mode-based finite-time consensus tracking system can analyzed by using the degree of homogeneity. The details can be found in [Gu *et al.*, 2012, Dou *et al.*, 2019]. It is noted that the above control laws are non-smooth and thus sometimes they can not be directly used in the actual continuous systems [Qian & Lin, 2001]. There is a trend to design the continuous finite-time controller for the coordinated control problem. In [Li *et al.*, 2011] a continuous finite-time consensus law is designed for second-order multiagent systems under one leader and bidirectional connected followers. A similar idea is used in [Du *et al.*, 2013] by dynamic output feedback. In [Huang *et al.*, 2015] an adaptive finite-time consensus algorithm is designed for uncertain nonlinear mechanical systems. Simultaneously the continuous finite-time consensus method can be developed to deal with high-order nonholonomic mobile robots with bidirectional topologies in [Du *et al.*, 2017] and surface vehicles under the assumption that all the follower can access to the leader in [Wang & Li, 2020]. Note that the objectives of coordinated path following control problem include path following and formation, which is different from the consensus problem. It is essential to give a finite-time method to the coordinated path following problem.

This paper gives a continuous solution to the finite-time control problem of coordinated path following under directed topologies. In order to solve the trajectory restriction problem, a new definition of barrier function is given, which is integrated into backstepping to design a novel continuous finite-time path following control input projected on the normal vector on the orbit. Another continuous finite-time formation control input projected on the tangential vector on the orbit is designed by regarding the path following errors as disturbances. It is noted that the proposed method in this paper is different from our previous adaptive method in [Chen *et al.*, 2020] in the conditions: 1) Directed networked second-order agents are under consideration and replace the first-order systems with bidirectional topologies; 2) A continuous finite-time design method is used to replace the adaptive methods.

The paper has the following outline. Section 2 provides some preliminaries and formulates the finite-time coordinated path following control problem. In Section 3 we first give a continuous finite-time control law to the path following subsystem and then the formation subsystem. Simulation results are presented in Section 4. Conclusions are given in Section 5.

2 Preliminaries and problem formulation

The network topology of the coordinated path following system can be described by a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where the nodes $\mathcal{V} = \{\mathcal{V}_0, \cdots, \mathcal{V}_n\}$ are associated to a virtual leader labeled by $\mathcal{V}_0$ and n vehicles labeled by $\{\mathcal{V}_1, \cdots, \mathcal{V}_n\}$, respectively, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of network links. A directed path from node $\mathcal{V}_j$ to $\mathcal{V}_i$ is a sequence of edges $(\mathcal{V}_i, \mathcal{V}_{i_1})$, $(\mathcal{V}_{i_1}, \mathcal{V}_{i_2}), \ldots, (\mathcal{V}_{i_{l-1}}, \mathcal{V}_{i_l})$, $(\mathcal{V}_{i_l}, \mathcal{V}_j)$ in the network topology with distinct nodes $\mathcal{V}_{i_k}, k = 0, \ldots, l$. A digraph is called a directed tree if there exists a node, called the root, that has directed paths to all other nodes in the digraph. Let, for $i, j = 0, \ldots, n$, $a_{ii} = 0$, $a_{ij} = 1$ if $(\mathcal{V}_j, \mathcal{V}_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. In addition define the Laplacian matrix $L \triangleq [l_{ij}]_{i,j=0}^n$ with $l_{ii} = \sum_{j=1}^n a_{ij}$ and $l_{ij} = -a_{ij}$, for any $i \neq j$, $i, j = 0, \ldots, n$. Let $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^{n\times m}$ denotes the sets of $n \times m$ real matrices and $\mathbf{0}$ denotes the zero matrix of appropriate dimension. For the considered coordinated path following system the Laplacian matrix L can be written as

$$L = \begin{bmatrix} 0 & \mathbf{0} \\ l_0 & L_1 \end{bmatrix},$$

where $l_0 = [l_{10}, \cdots, l_{n0}]^T$ and $L_1 \in \mathbb{R}^{n\times n}$. Suppose that Assumption 1 holds. L_1 is a nonsingular *M-matrix* and all eigenvalues of L_1 have positive real parts [Zhang & Tian, 2009].

Assumption 1. The digraph consisting of a virtual leader and n vehicles contains a directed spanning tree with root $\mathcal{V}_0$.

To solve the trajectory restriction problem in the geometry extension method a new definition of the barrier function $\varPsi_i$ is given.

Definition 1. A C^2 function $\varPsi_i : (-\varepsilon_i, \varepsilon_i) \to \mathbb{R}$ is a barrier function with barrier $2\varepsilon_i > 0$ if

(C1) $\lim\limits_{\lambda_i \to -\varepsilon_i^+} \varPsi_i(\lambda_i) = +\infty$ and $\lim\limits_{\lambda_i \to -\varepsilon_i^+} \nabla\varPsi_i(\lambda_i) = -\infty$.

(C2) $\lim\limits_{\lambda_i \to \varepsilon_i^-} \varPsi_i(\lambda_i) = +\infty$ and $\lim\limits_{\lambda_i \to \varepsilon_i^-} \nabla\varPsi_i(\lambda_i) = +\infty$.

(C3) $\nabla\varPsi_i(0) = 0$.

(C4) $|\varPsi_i| \geq c_\varPsi |\lambda_i|$ with a bounded positive constant $c_\varPsi$.

Remark 1. (C4) is an additional condition in contrast with Ψ_i in [Chen & Tian, 2015, Chen *et al.*, 2020].

2.1 Problem formulation

In a fixed inertial reference frame the model of the virtual leader under Assumption 2 is the first-order dynamics such that $\dot{p}_0 = 0$, where $p_0 = [p_{x_0}, p_{y_0}]^T \in \mathbb{R}^2$ is its position. Let $i = 1, \cdots, n$. The dynamic equation for the ith follower satisfying the second-order dynamics is given by

$$\begin{aligned}\dot{p}_i &= v_i,\\ \dot{v}_i &= u_i,\end{aligned} \tag{1}$$

where $p_i = [p_{x_i}, p_{y_i}]^T \in \mathbb{R}^2$ and $v_i = [v_{x_i}, v_{y_i}]^T \in \mathbb{R}^2$ denotes the position and velocity variables, respectively. $u_i = [u_{x_i}, u_{y_i}]^T \in \mathbb{R}^2$ denotes the control input.

Assumption 2. The virtual leader is stationary, that is the position of the virtual leader is fixed on a virtual orbit.

Suppose that the desired orbit associated to each agent is a simple, closed and regular curve with nonzero curvature. In [Chen & Tian, 2015] such curve can be geometry extended to be a set of level curves, in which can be defined by a smooth function (that is *the orbit function*) $\lambda_i : \Omega_i \to (-\varepsilon_i, \varepsilon_i)$ and the desired orbit can be defined by $\lambda_i(p_i) = 0$, where $\Omega_i \subset \mathbb{R}^2$ is an open set and $p \in \mathbb{R}^2$. Let the arc-lengthes are given by

$$s_i(\lambda_i, \phi_i) \triangleq \int_{\phi_i^*}^{\phi_i} \frac{\partial s_i(\lambda_i, \tau)}{\partial \tau} d\tau, \tag{2}$$

where ϕ_i^* is the parameter associated with the starting point of the arc of s_i. The generalized arc-lengthes $\xi_i : \mathbb{R} \to \mathbb{R}$, the functions of the arc-lengths s_i such that $\partial\xi_i/\partial s_i$ is a constant and $c_{\underline{\xi}} \le |\partial\xi_i/\partial s_i| \le c_{\bar{\xi}}$ with two positive constants $c_{\underline{\xi}}$ and $c_{\bar{\xi}}$, are used to describe the formation along the curves.

Finite-time coordinated path following control problem. Let $i \in [1, n]$. Consider the system (1) and the initial position $p_i(0) \in \Omega_i$. Suppose Assumptions 1 and 2 hold. Design a finite-time coordinated path following control u_i such that

$$\lim_{t\to T} \lambda_i(p_i(t)) = 0 \tag{3}$$

with a finite time $T > 0$, $p_i(t) \in \Omega_i$, for all $t \ge 0$, where

$$\Omega_i = \left\{ p_i \in \mathbb{R}^2 \mid |\lambda_i(p_i(t))| < \varepsilon_i \right\}, \tag{4}$$

and

$$\lim_{t\to T} \xi_i(t) = \xi_0(0). \tag{5}$$

Remark 2. This paper devotes to designing a finite-time control law for directed networking second-order agents for the coordinated path following problem. However, [Chen *et al.*, 2020] deals with the adaptive design for first-order agents with unknown time-varying parameters and bidirectional topologies.

3 Main results

3.1 Open-loop system

The path following dynamics by differentiating λ_i is

$$\dot{\lambda}_i = \|\nabla\lambda_i\| \, v_{N_i}, \tag{6}$$

where $v_{N_i} = N_i^T v_i$ denotes the velocity projected on the normal vector $N_i = \frac{\nabla\lambda_i}{\|\nabla\lambda_i\|}$ on the level orbit of the current position of the vehicle. Differentiating both sides of v_{N_i} yields

$$\dot{v}_{N_i} = u_{N_i} + \Delta_{N_i}, \tag{7}$$

where $u_{N_i} = N_i^T u_i$ denotes the control input projected on the normal vector N_i, $\Delta_{N_i} = v_i^T \dot{N}_i$ and $\dot{N}_i = \frac{\nabla^2\lambda_i v_i}{\|\nabla\lambda_i\|} - \frac{N_i N_i^T \nabla^2\lambda_i v_i}{\|\nabla\lambda_i\|}$.

Differentiating (2) one has $\dot{s}_i = v_{T_i} + \frac{\partial s_i}{\partial\lambda_i}\|\nabla\lambda_i\| \, v_{N_i}$, where $v_{T_i} = T_i^T v_i$ denotes the velocity projected on the tangent vector $T_i = [R_1, R_2]^T N_i$ to the level orbit of the current position of the vehicle with $R_1 = [0, 1]^T$ and $R_2 = [-1, 0]^T$. Then the dynamics of ξ_i is given by

$$\dot{\xi}_i = \frac{\partial\xi_i}{\partial s_i} v_{T_i} + \Delta_{\xi_i}, \tag{8}$$

where $\Delta_{\xi_i} = \frac{\partial\xi_i}{\partial s_i}\frac{\partial s_i}{\partial\lambda_i}\|\nabla\lambda_i\| \, v_{N_i}$. Differentiating both sides of v_{T_i} yields

$$\dot{v}_{T_i} = u_{T_i} + \Delta_{T_i}, \tag{9}$$

where $u_{T_i} = T_i^T u_i$ denotes the control input projected on the tangent vector N_i and $\Delta_{T_i} = v_i^T R \dot{N}_i$.

Let $\varsigma_i = \sum_{j=0}^{n} a_{ij}(\xi_i - \xi_j)$ denote the formation errors. The dynamics of ς_i is described by the equation

$$\dot{\varsigma}_i = \sum_{j=0}^{n} a_{ij}\left(\frac{\partial\xi_i}{\partial s_i} v_{T_i} + \Delta_{\xi_i} - \frac{\partial\xi_j}{\partial s_j} v_{T_j} - \Delta_{\xi_j}\right). \tag{10}$$

As a result, the equations of the formation tracking control system are given by (6), (7), (9) and (10).

3.2 Controller design

Let us first consider the path following subsystem consisting of (6) and (7) and the virtual control $\hat{v}_{N_i}$ be

$$\hat{v}_{N_i} = -k_1 \left(\nabla \Psi_i\right)^{1/\alpha}, \tag{11}$$

where the control gain k_1 will be selected later. Consider the candidate Lyapunov function

$$V_P = \sum_{i=1}^{n} \Psi_i + \gamma_1 \sum_{i=1}^{n} \int_{\hat{v}_{N_i}}^{v_{N_i}} \left(\tau^\alpha - \hat{v}_{N_i}^\alpha\right)^{2-1/\alpha} d\tau. \tag{12}$$

where $\gamma_1 = \frac{1}{(2-1/\alpha)k_1^{1+\alpha}}$. The first term in (12) contributes to achieving the path following objective, i.e., the equations (3) and (4). The second term contributes to guaranteeing the convergence of the differences $\bar{v}_{N_i} = v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha$. Differentiating both sides of (12) along the trajectories of (6), (7) and (11) yields

$$\begin{aligned}
\dot{V}_P = &- \sum_{i=1}^{n} k_1 \left\|\nabla \lambda_i\right\| \nabla \Psi_i^{1+1/\alpha} + \sum_{i=1}^{n} \nabla \Psi_i \left\|\nabla \lambda_i\right\| \tilde{v}_{N_i} \\
&+ \gamma_1 \sum_{i=1}^{n} \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{2-1/\alpha} \left(u_{N_i} + \Delta_{N_i}\right) + \sum_{i=1}^{n} f_{\lambda_i} \dot{\lambda}_i \\
\le &- \sum_{i=1}^{n} k_1 \left\|\nabla \lambda_i\right\| \nabla \Psi_i^{1+1/\alpha} + \sum_{i=1}^{n} \left\|\nabla \lambda_i\right\| \left|\nabla \Psi_i \tilde{v}_{N_i}\right| \\
&+ \gamma_1 \sum_{i=1}^{n} \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{2-1/\alpha} \left(u_{N_i} + \Delta_{N_i}\right) + f_P,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
f_{\lambda_i} &= -\frac{1}{k_1^{1+\alpha}} \frac{\partial \hat{v}_{N_i}^\alpha}{\partial \lambda_i} \int_{\hat{v}_{N_i}}^{v_{N_i}} \left(\tau^\alpha - \hat{v}_{N_i}^\alpha\right)^{1-1/\alpha} d\tau \\
&= k_1^{-1} \nabla^2 \Psi_i \int_{\hat{v}_{N_i}}^{v_{N_i}} \left(\tau^\alpha - \hat{v}_{N_i}^\alpha\right)^{1-1/\alpha} d\tau, \\
f_P &= \sum_{i=1}^{n} k_1^{-1} \left\|\nabla \lambda_i\right\| \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{1-1/\alpha} \\
&\quad \times \left|v_{N_i} \nabla^2 \Psi_i \left(v_{N_i} - \hat{v}_{N_i}\right)\right|, \\
\tilde{v}_{N_i} &= v_{N_i} - \hat{v}_{N_i}
\end{aligned}$$

According to

$$\left|\tilde{v}_{N_i}\right| = \left|\left(v_{N_i}^\alpha\right)^{1/\alpha} - \left(\hat{v}_{N_i}^\alpha\right)^{1/\alpha}\right| \le 2^{1-1/\alpha} \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right|^{1/\alpha}$$

and Lemmas A.1 and A.2 in [Qian & Lin, 2001] we have that

$$\begin{aligned}
\left|\nabla \Psi_i \tilde{v}_{N_i}\right| &\le 2^{1-1/\alpha} \left|\nabla \Psi_i\right| \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right|^{1/\alpha} \\
&\le \left(\nabla \Psi_i\right)^{1+1/\alpha} + c_{\phi_1} \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{1+1/\alpha},
\end{aligned} \tag{14}$$

where $c_{\phi_1} = 2^{1-1/\alpha} \frac{1}{1+\alpha} \phi_\Psi^{-\alpha}$ and $\phi_\Psi = \left(2^{1-1/\alpha} \frac{\alpha}{1+\alpha}\right)^{-1}$. Note that

$$\begin{aligned}
&\left|v_{N_i} \left(v_{N_i} - \hat{v}_{N_i}\right)\right| \le \left(v_{N_i} - \hat{v}_{N_i}\right)^2 + \left|\hat{v}_{N_i}\right| \left|v_{N_i} - \hat{v}_{N_i}\right|, \\
&\quad \left(v_{N_i} - \hat{v}_{N_i}\right)^2 \le 2^{2-2/\alpha} \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{2/\alpha}, \\
&\left|\hat{v}_{N_i}\right| \left|v_{N_i} - \hat{v}_{N_i}\right| \le k_1 2^{1-1/\alpha} \left|\nabla \Psi_i\right|^{1/\alpha} \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right|^{1/\alpha}, \\
&\left|\nabla \Psi_i\right|^{1/\alpha} \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right| \le \frac{1/\alpha}{1+1/\alpha} \phi_2 \left|\nabla \Psi_i\right|^{1+1/\alpha} \\
&\qquad + \frac{1}{1+1/\alpha} \phi_2^{-1/\alpha} \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right|^{1+1/\alpha} \\
&\qquad = \left|\nabla \Psi_i\right|^{1+1/\alpha} + c_{\phi_2} \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right|^{1+1/\alpha},
\end{aligned} \tag{15}$$

where $c_{\phi_2} = \frac{\alpha}{1+\alpha} \phi_2^{-1/\alpha}$ and $\phi_2 = 1 + \alpha$. We conclude that

$$\begin{aligned}
f_P \le &\sum_{i=1}^{n} k_1^{-1} \left\|\lambda_i\right\| \left|\nabla^2 \Psi_i\right| \left(2^{2-2/\alpha} \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{1+1/\alpha}\right. \\
&\left. + k_1 2^{1-1/\alpha} \left(\left|\nabla \Psi_i\right|^{1+1/\alpha} + c_{\phi_2} \left|v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right|^{1+1/\alpha}\right)\right).
\end{aligned} \tag{16}$$

On the set $\Phi_P = \{(\lambda_i, \tilde{v}_{N_i}) \,|V_F \le c_P\}$, for some $c_P > 0$, one has

$$c_{\underline{\lambda}} \le \left\|\nabla \lambda_i\right\| \le c_{\bar{\lambda}}, \;\; \left|\nabla^2 \Psi_i\right| \le c_{\Psi^2} \tag{17}$$

with some $c_{\underline{\lambda}} > 0$, $c_{\bar{\lambda}} > 0$ and $c_{\Psi^2} > 0$. Exploiting the inequalities (14), (16) and (17) we conclude that

$$\begin{aligned}
\dot{V}_P \le &- \sum_{i=1}^{n} \left(k_1 c_{\underline{\lambda}} - c_{\bar{\lambda}} - c_{\bar{\lambda}} 2^{1-1/\alpha} c_{\Psi^2}\right) \left(\nabla \Psi_i\right)^{1+1/\alpha} \\
&+ \sum_{i=1}^{n} c_N \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{1+1/\alpha} \\
&+ \gamma_1 \sum_{i=1}^{n} \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{2-1/\alpha} \left(u_{N_i} + \Delta_{N_i}\right),
\end{aligned} \tag{18}$$

which yields

$$\begin{aligned}
&u_{N_i} = -\Delta_{N_i} - k_2 \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{-1+2/\alpha}, \\
&k_1 > c^{-1} \left(c_{\bar{\lambda}} + c^{-\alpha} 2^{1-1/\alpha} c_{\Psi^2} + \beta_{p_1}\right), \\
&k_2 > (2 - 1/\alpha)\, k_1^{1+\alpha} c_N,
\end{aligned} \tag{19}$$

where $c_N = c_{\bar{\lambda}} \left(c_{\phi_1} + k_1^{-1} 2^{2-2/\alpha} c_{\Psi^2} + 2^{1-1/\alpha} c_{\phi_2} c_{\Psi^2}\right)$ and β_{p_1} is an arbitrary positive constant. Note that

$$\begin{aligned}
&\left(\nabla \Psi_i\right)^{1+1/\alpha} + \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^{1+1/\alpha} \\
&\ge \left(\left(\nabla \Psi_i\right)^2 + \left(v_{N_i}^\alpha - \hat{v}_{N_i}^\alpha\right)^2\right)^{(1+1/\alpha)/2}.
\end{aligned} \tag{20}$$

Suppose that $V_P(t) \neq 0$. Substituting (19) into (18) yields

$$\dot{V}_P \le -\beta_{p_1} \sum_{i=1}^{n} \left((\nabla \Psi_i)^2 + \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^2 \right)^{(1+1/\alpha)/2} = -g_P V_P^{(1+1/\alpha)/2}, \tag{21}$$

where $g_P = \beta_{p_1} V_P^{-(1+1/\alpha)/2} \sum_{i=1}^{n} \left((\nabla \Psi_i)^2 + \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^2 \right)^{(1+1/\alpha)/2}$. By equation (19) the closed-loop equations associated to the path following subsystem for the ith follower are

$$\begin{aligned} \dot{\lambda}_i &= -k_1 \|\nabla \lambda_i\| (\nabla \psi_i)^{1/\alpha} + \|\nabla \lambda_i\| \tilde{v}_{N_i}, \\ \dot{\bar{v}}_{N_i} &= -k_2^{\alpha} \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^{2-\alpha} - \dot{\hat{v}}_{N_i}^{\alpha}. \end{aligned} \tag{22}$$

Remark 3. It is obvious that the closed-loop system (22) for path following is not homogeneous. Therefore, the finite-time stability analysis method given in [Gu *et al.*, 2012,Dou *et al.*, 2019] can not applied in this paper.

Note that $0 < (1+1/\alpha)/2 < 1$. To apply Theorem 4.2 in [Bhat & Bernstein, 2000] we will show that g_P has lower bound. From (C4) we have that

$$\begin{aligned} \Psi_i &= \int_{\lambda_{i0}}^{\lambda_i} \nabla \Psi_i(\tau) d\tau \le c_{\Psi}^{-1} |\nabla \Psi_i|^2 + |\nabla \Psi_i| \varepsilon_i, \\ &\int_{\hat{v}_{N_i}}^{v_{N_i}} \left(\tau^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^{2-1/\alpha} d\tau \le 2^{1-1/\alpha} \left| v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right|^2 \end{aligned} \tag{23}$$

with $\psi_{i0} = \psi_i(\lambda_{i0})$ and $\lambda_{i0} = \lambda_i(0)$, which yields

$$V_P \le \beta_{p_3} \sum_{i=1}^{n} \left[(\nabla \Psi_i)^2 + \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^2 \right] + c_{p_3}, \tag{24}$$

where $\beta_{p_3} = \max \left\{ c_{\Psi}^{-1}, \frac{2^{1-1/\alpha}}{(2-1/\alpha)k_1^{1+\alpha}} \right\}$, $c_{p_3} = \sum_{i=1}^{n} |\nabla \Psi_i| \varepsilon_i$ and β_{p_4} is positive and bounded. As a result,

$$g_P \ge \frac{\beta_{p_1} \sum_{i=1}^{n} \left((\nabla \psi_i)^2 + \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^2 \right)^{(1+1/\alpha)/2}}{\left(\beta_{p_3} \sum_{i=1}^{n} \left((\nabla \psi_i)^2 + \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^2 \right) + c_{p_3} \right)^{(1+1/\alpha)/2}} \ge \beta_{p_4}. \tag{25}$$

According the Theorem 4.2 in [Bhat & Bernstein, 2000] we establish the following theorem.

Theorem 1 *Suppose that the initial positions of vehicles are such that $p_i(0) \in \Omega_i$. Assume moreover that Assumption 2 holds. Then the path following objectives (3) and (4) can be achieved by the finite-time control u_{N_i} given in (19) for $i = 1, \cdots, n$.*

Proof: From (21) we conclude that the function V_P is bounded all the time, which implies the objective (4) is satisfied according to (C1) and (C2). From (21) and (25) we conclude that $\lambda_i = 0$ and $\bar{v}_{N_i}$ are the finite-time-stable equilibria of the closed-loop path following equations (22). ■

In the following we will consider the formation subsystem consisting of (9) and (10). Let the virtual control $\hat{v}_{T_i}$ be

$$\hat{v}_{T_i} = -\left(\frac{\partial \xi_i}{\partial s_i} \right)^{-1} k_3 {\varsigma_i}^{1/\alpha}, \tag{26}$$

where k_3 is a positive control gain and will be selected later. Consider the candidate Lyapunov function

$$V_F = \frac{1}{2} \sum_{i=1}^{n} \varsigma_i^2 + \gamma_2 \sum_{i=1}^{n} \int_{\hat{v}_{T_i}}^{v_{T_i}} \left(\tau^{\alpha} - \hat{v}_{T_i}^{\alpha} \right)^{2-1/\alpha} d\tau, \tag{27}$$

where $\gamma_2 = \frac{1}{(2-1/\alpha)k_3^{1+\alpha}}$. In (27) the first term contributes to achieving the formation objective, i.e., the equation (5). The second term contributes to guaranteeing the convergence of the differences $\bar{v}_{T_i} = v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}$. Differentiating both sides of (27) along the trajectories of (9), (10) and (26) yields

$$\begin{aligned} \dot{V}_F = &-k_3 \sum_{i=1}^{n} \varsigma_i \sum_{j=0}^{n} a_{ij} \left({\varsigma_i}^{1/\alpha} - {\varsigma_j}^{1/\alpha} \right) + f_F \\ &+ \gamma_2 \sum_{i=1}^{n} \left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha} \right)^{2-1/\alpha} (u_{T_i} + \Delta_{T_i}) + g_{F_1} + g_{F_2}, \end{aligned} \tag{28}$$

where $\tilde{v}_{T_j} = v_{T_i} - \hat{v}_{T_i}$

$$f_F = \sum_{i=1}^{n} \varsigma_i \sum_{j=0}^{n} a_{ij} \left(\frac{\partial \xi_i}{\partial s_i} \tilde{v}_{T_i} - \frac{\partial \xi_j}{\partial s_j} \tilde{v}_{T_j} \right) \tag{29}$$

$$g_{F_1} = \sum_{i=1}^{n} \varsigma_i \sum_{j=0}^{n} a_{ij} \left(\Delta_{\xi_i} - \Delta_{\xi_j} \right), \tag{30}$$

$$\begin{aligned} g_{F_2} = &-\frac{1}{k_3} \sum_{i=1}^{n} \left(\frac{\partial \xi_i}{\partial s_i} \right)^{-\alpha} \sum_{i=1}^{n} \int_{\hat{v}_{T_i}}^{v_{T_i}} \left(\tau^{\alpha} - \hat{v}_{T_i}^{\alpha} \right)^{1-1/\alpha} d\tau \\ &\times \sum_{j=0}^{n} a_{ij} \left(\frac{\partial \xi_i}{\partial s_i} v_{T_i} + \Delta_{\xi_i} - \frac{\partial \xi_j}{\partial s_j} v_{T_j} - \Delta_{\xi_j} \right). \end{aligned} \tag{31}$$

Note that

$$f_F \le \sum_{i=1}^{n} |\varsigma_i| \left(\gamma_1 c_{\bar{\xi}} \left| \tilde{v}_{T_i} \right| + \gamma_2 c_{\bar{\xi}} \sum_{j=0}^{n} \left| \tilde{v}_{T_j} \right| \right), \tag{32}$$

where $\gamma_3 = \max_{\forall i}\left\{\sum_{j=0}^{n} a_{ij}\right\}$ and $\gamma_4 = \max_{\forall i,j}\{a_{ij}\}$. Lemmas A.1 and A.2 in [Qian & Lin, 2001] yield

$$\begin{aligned}
&|\tilde{v}_{T_i}| = \left|\left(v_{T_i}^{\alpha}\right)^{1/\alpha} - \left(\hat{v}_{T_i}^{\alpha}\right)^{1/\alpha}\right| \le 2^{1-1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha},\\
&|\varsigma_i|\,|\tilde{v}_{T_i}| \le |\varsigma_i|^{1+1/\alpha} + c_{F_1}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1+1/\alpha},\\
&|\varsigma_i|\,\left|\tilde{v}_{T_j}\right| \le |\varsigma_i|^{1+1/\alpha} + c_{F_1}\left|v_{T_j}^{\alpha} - \hat{v}_{T_j}^{\alpha}\right|^{1+1/\alpha},
\end{aligned}\tag{33}$$

where $c_{F_1} = 2^{1-1/\alpha}n^{1-1/\alpha}\frac{1}{1+\alpha}\phi_{F_1}^{-\alpha}$ and $\phi_{F_1} = (2^{1-1/\alpha} n^{1-1/\alpha}\frac{\alpha}{1+\alpha})^{-1}$. Substituting equations (33) into (32) yields

$$f_F \le c_{\varsigma_1}\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha} + c_{vT1}\sum_{j=0}^{n}\left|v_{T_j}^{\alpha} - \hat{v}_{T_j}^{\alpha}\right|^{1+1/\alpha}, \tag{34}$$

where $c_{\varsigma_1} = \gamma_3 c_{\bar{\xi}} + \gamma_4 c_{\bar{\xi}}$ and $c_{vT1} = \gamma_3 c_{\bar{\xi}} c_{F_1} + \gamma_4 c_{\bar{\xi}} n c_{F_1} c_{F_1}$. Note that

$$g_{F_2} \le g_{F_{21}} + g_{F_{22}}, \tag{35}$$

where

$$\begin{aligned}
g_{F_{21}} &= \sum_{i=1}^{n} k_3^{-1} c_{\underline{\xi}}^{-\alpha}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1-1/\alpha}\left|v_{T_i} - \hat{v}_{T_i}\right|\\
&\quad\times\sum_{j=1}^{n} a_{ij}\left|\frac{\partial\xi_i}{\partial s_i}v_{T_i} - \frac{\partial\xi_j}{\partial s_j}v_{T_j}\right|,\\
g_{F_{22}} &= \sum_{i=1}^{n} k_3^{-1} c_{\underline{\xi}}^{-\alpha}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1-1/\alpha}\left|v_{T_i} - \hat{v}_{T_i}\right|\\
&\quad\times\sum_{j=1}^{n} a_{ij}\left|\Delta_{\xi_i} - \Delta_{\xi_j}\right|.
\end{aligned}\tag{36}$$

Due to the fact that

$$\begin{aligned}
\left|\hat{v}_{T_i}\right|\left|v_{T_i} - \hat{v}_{T_i}\right| &\le \left(v_{T_i} - \hat{v}_{T_i}\right)^2 + \left|\hat{v}_{T_i}\right|\left|v_{T_i} - \hat{v}_{T_i}\right|\\
&\le k_3 c_{\underline{\xi}}^{-1} 2^{1-1/\alpha}|\varsigma_i|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha},\\
\left|\hat{v}_{T_j}\right|\left|v_{T_i} - \hat{v}_{T_i}\right| &\le k_3 c_{\underline{\xi}}^{-1} 2^{1-1/\alpha}|\varsigma_j|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha},\\
\left|v_{T_i}\right|\left|v_{T_i} - \hat{v}_{T_i}\right| &\le \left(v_{T_i} - \hat{v}_{T_i}\right)^2 + \left|\hat{v}_{T_i}\right|\left|v_{T_i} - \hat{v}_{T_i}\right|\\
&\le 2^{2-2/\alpha}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{2/\alpha}\\
&\quad + k_3 c_{\underline{\xi}}^{-1} 2^{1-1/\alpha}|\varsigma_i|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha},\\
\left|v_{T_j}\right|\left|v_{T_i} - \hat{v}_{T_i}\right| &\le 2^{2-2/\alpha}\left|v_{T_j}^{\alpha} - \hat{v}_{T_j}^{\alpha}\right|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha}\\
&\quad + k_3 c_{\underline{\xi}}^{-1} 2^{1-1/\alpha}|\varsigma_j|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha},
\end{aligned}\tag{37}$$

from (36), one has

$$g_{F_{21}} \le \sum_{i=1}^{n} k_3^{-1} c_{\underline{\xi}}^{-\alpha}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1-1/\alpha} g_{F_{23}}, \tag{38}$$

where

$$\begin{aligned}
g_{F_{23}} &= \gamma_1 c_{\bar{\xi}}\Big(2^{2-2/\alpha}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{2/\alpha}\\
&\quad + k_3 c_{\underline{\xi}}^{-1} 2^{1-1/\alpha}|\varsigma_i|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha}\Big)\\
&\quad + \gamma_2 c_{\bar{\xi}}\sum_{j=0}^{n}\Big(2^{2-2/\alpha}\left|v_{T_j}^{\alpha} - \hat{v}_{T_j}^{\alpha}\right|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha}\\
&\quad + k_3 c_{\underline{\xi}}^{-1} 2^{1-1/\alpha}|\varsigma_j|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1/\alpha}\Big).
\end{aligned}\tag{39}$$

From Lemmas A.1 and A.2 in [Qian & Lin, 2001] we have

$$\begin{aligned}
&|\varsigma_i|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right| \le |\varsigma_i|^{1+1/\alpha} + c_{F_2}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1+1/\alpha},\\
&|\varsigma_j|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right| \le |\varsigma_j|^{1+1/\alpha} + c_{F_2}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1+1/\alpha},\\
&\left|v_{T_j}^{\alpha} - \hat{v}_{T_j}^{\alpha}\right|^{1/\alpha}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right| \le \left|v_{T_j}^{\alpha} - \hat{v}_{T_j}^{\alpha}\right|^{1+1/\alpha}\\
&\qquad + c_{F_2}\left|v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right|^{1+1/\alpha}
\end{aligned}\tag{40}$$

with $c_{F_2} = n^{1-1/\alpha}\frac{\alpha}{1+\alpha}\phi_{F_2}^{-\alpha}$ and $\phi_{F_2} = \left(n^{1-1/\alpha}\frac{1}{1+\alpha}\right)^{-1}$, which yields

$$\begin{aligned}
&\sum_{i=1}^{n} c_{F_1}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1-1/\alpha} g_{F_{23}}\\
&\le c_{vT2}\sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha} + c_{\varsigma_2}\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha},
\end{aligned}\tag{41}$$

where

$$\begin{aligned}
c_{vT2} &= k_3^{-1} c_{\underline{\xi}}^{-\alpha}\gamma_3 c_{\bar{\xi}} 2^{2-2/\alpha} + c_{\underline{\xi}}^{-\alpha-1}\gamma_1 c_{\bar{\xi}} 2^{1-1/\alpha} c_{F_2}\\
&\quad + k_3^{-1} c_{\underline{\xi}}^{-\alpha}\gamma_4 c_{\bar{\xi}} 2^{2-2/\alpha} c_{F_2} n + c_{\underline{\xi}}^{-\alpha-1}\gamma_4 c_{\bar{\xi}} 2^{1-1/\alpha} c_{F_2} n
\end{aligned}$$

and

$$c_{\varsigma_2} = c_{\underline{\xi}}^{-\alpha-1}\gamma_3 c_{\bar{\xi}} 2^{1-1/\alpha} + k_3^{-1} c_{\underline{\xi}}^{-\alpha}\gamma_4 c_{\bar{\xi}} 2^{2-2/\alpha} n.$$

From (35), (38) and (41) we conclude that

$$\begin{aligned}
g_{F_2} &\le c_{vT2}\sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha} + c_{\varsigma_2}\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha}\\
&\quad + g_{F_{22}}.
\end{aligned}$$

Substituting the above equation and (34) into (28) yields

$$\begin{aligned}
&\dot{V}_F \le -k_3\sum_{i=1}^{n}\varsigma_i\sum_{j=0}^{n} a_{ij}\left({\varsigma_i}^{1/\alpha} - {\varsigma_j}^{1/\alpha}\right) + (c_{\varsigma_1} + c_{\varsigma_2})\\
&\times\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha} + \gamma_2\sum_{i=1}^{n}\left(v_{N_i}^{\alpha} - \hat{v}_{N_i}^{\alpha}\right)^{2-1/\alpha}\left(u_{T_i} + \Delta_{T_i}\right)\\
&+ (c_{vT1} + c_{vT2})\sum_{j=0}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha} + g_{F_1} + g_{F_{22}},
\end{aligned}\tag{42}$$

in which makes the choices such that

$$u_{T_i} = -\Delta_{T_i} - k_4\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{2/\alpha-1}, \tag{43}$$

where the control gain k_4 is set later. As a result, the closed-loop formation subsystem for the ith follower is

$$\begin{aligned}
\dot{\varsigma}_i &= -k_3\sum_{j=0}^{n} a_{ij}\left({\varsigma_i}^{1/\alpha} - {\varsigma_j}^{1/\alpha}\right) + \sum_{j=0}^{n} a_{ij}\left(\Delta_{\xi_i} - \Delta_{\xi_j}\right)\\
&\quad + \sum_{j=0}^{n} a_{ij}\left(\frac{\partial \xi_i}{\partial s_i}\tilde{v}_{T_i} - \frac{\partial \xi_j}{\partial s_j}\tilde{v}_{T_j}\right),\\
\dot{v}_{T_i} &= -k_4^{\alpha}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{2-\alpha} - \dot{\hat{v}}_{T_i}^{\alpha}.
\end{aligned} \tag{44}$$

Substituting $-k_3\varsigma^T L_1 \varsigma^{1/\alpha} \leq -k_3\rho_{L_1} n^{-1}\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha}$ and (43) into (42) yields

$$\begin{aligned}
\dot{V}_F &\leq -\left(k_3\rho_{L_1}n^{-1} - c_{\varsigma_1} - c_{\varsigma_2}\right)\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha} - (-c_{vT1}\\
&\quad + k_4\gamma_2 - c_{vT2})\sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha} + g_{F_1} + g_{F_{22}},
\end{aligned}$$

which makes the choices of the control gains as follows:

$$\begin{aligned}
k_3 &\geq \rho_{L_1}^{-1} n\left(c_{\varsigma_1} + c_{\varsigma_2} + \beta_{F_1}\right),\\
k_4 &\geq (2 - 1/\alpha)\, k_3^{1+\alpha}\left(c_{vT1} + c_{vT2} + \beta_{F_1}\right),
\end{aligned} \tag{45}$$

where β_{F_1} is an arbitrary positive constant. As a result,

$$\begin{aligned}
\dot{V}_F &\leq -\beta_{F_1}\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha} - \beta_{F_1}\sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha}\\
&\quad + g_{F_1} + g_{F_{22}}.
\end{aligned} \tag{46}$$

Let $l_F = \max\left\{1, \frac{2^{1-1/\alpha}}{(2-1/\alpha)k_3^{1+\alpha}}\right\}$. Then

$$\begin{aligned}
V_F &\leq \sum_{i=1}^{n}\left[\varsigma_i^2 + \frac{2^{1-1/\alpha}}{(2-1/\alpha)\,k_3^{1+\alpha}}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^2\right]\\
&\leq l_F\sum_{i=1}^{n}\left[\varsigma_i^2 + \left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^2\right],
\end{aligned} \tag{47}$$

which yields $V_F^{(1+1/\alpha)/2} \leq l_F^{(1+1/\alpha)/2}\left(\sum_{i=1}^{n}\varsigma^{1+1/\alpha} + \sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha}\right)$. Suppose that $V_F(t) \neq 0$. Equation (46) can be rewritten as

$$\dot{V}_F \leq -\beta_{F_2}V_F^{(1+1/\alpha)/2} + g_{F_3}, \tag{48}$$

where

$$\begin{aligned}
\beta_{F_2} &= \frac{\beta_{F_1}\sum_{i=1}^{n}|\varsigma_i|^{1+1/\alpha} + \beta_{F_1}\sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha}}{l_F^{(1+1/\alpha)/2}\left(\sum_{i=1}^{n}\varsigma^{1+1/\alpha} + \sum_{i=1}^{n}\left(v_{T_i}^{\alpha} - \hat{v}_{T_i}^{\alpha}\right)^{1+1/\alpha}\right)},\\
g_{F_3} &= \frac{g_{F_1} + g_{F_{22}}}{V_{2F}^{(1+1/\alpha)/2}}.
\end{aligned}$$

Due to $0 < (1 + 1/\alpha)/2 < 1$, β_{F_2} has lower bound. Also g_{F_3} disappears as $\lim_{t\to T}\lambda_i(t) = 0$ and $\lim_{t\to T}\bar{v}_{N_i}(t) = 0$ as proven in Theorem 1. We give the following result directly.

Theorem 2 *Suppose that the initial positions of vehicles are such that $p_i(0) \in \Omega_i$. Assume moreover that Assumptions 1 and 2 hold. Then the formation objective (5) can be achieved by the finite-time control u_{T_i} given in (43) for $i = 1, \cdots, n$.*

Proof: The proof follows the same argument as the proof of Theorem 5.3 in [Bhat & Bernstein, 2000], hence it is omitted. ■

Theorems 1 and 2 yield the following result.

Theorem 3 *Suppose that the initial positions of vehicles are such that $p_i(0) \in \Omega_i$. Assume moreover that Assumptions 1 and 2 hold. For $i = 1, \cdots, n$, the finite-time coordinated path following control problem is solved by the coordinated path following control*

$$u_i = \begin{bmatrix} N_i^T \\ T_i^T \end{bmatrix}^{-1}\begin{bmatrix} u_{N_i} \\ u_{T_i} \end{bmatrix}, \tag{49}$$

where u_{N_i} and u_{T_i} are as given in the equations (19) and (43), respectively.

Remark 4. In this paper, we first give a new barrier function with an additional condition (C4) on page 3. Then a novel continuous finite-time path following control law is designed based on the barrier function and backstepping. Simultaneously, a novel continuous finite-time formation algorithm is designed by regarding the path following errors as disturbances. In [Chen *et al.*, 2020], a so-called *congelation of variables* method is used to design the adaptive updated law for unknown time-varying parameters, and at the same time the coordinated path following law is designed based on a unified Lynapunov function.

Remark 5. The settling time properties of the resulting system are studied according to the finite-time stability theory in this paper, which is different from the asymptotic properties of the resulting adaptive system by using the Lynapunov stability theory in [Chen *et al.*, 2020].

4 Simulation

The desired formation is a triangle pattern with a digraph as shown in Figure 1. The selected trajectories for the agents are concentric ellipses with different semi-major axis and semi-minor axis, that is $C_{l0}:\ p_{x_l}^2/(e_l a)^2 + p_{y_l}^2/(e_l b)^2 = 1$, where $e_l = 1 + 0.5l$,

$a = 3$, $b = 2$, $l = 0, 1, 2, 3, 4$. The parameters are selected as $k_1 = k_3 = 2.7$, $k_2 = k_4 = 34$ and $\alpha = 9/7$. The initial generalized arc-length of the virtual leader is $\xi_0(0) = 0$. The motion of the agents is illustrated in Figure 2, where o, $\square$, $\bigstar$ and $+$ denote the agents' positions at $t = 0$, $t = 1$, $t = 2$ and $t = 7$, respectively. From this figure one can see that the four agents converge to the given orbits and form the desired formation. The time histories of the path following errors λ_i and of the formation errors $\xi_i - \xi_0$ are plotted in Figures 3 and 4, respectively. From above figures we show that path following and formation tracking are achieved.

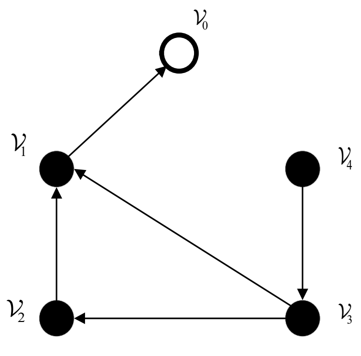

Fig. 1 The leader-following topology.

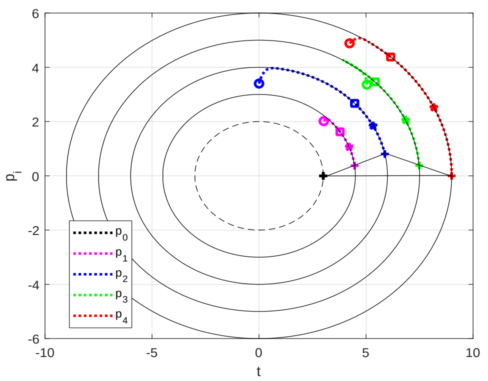

Fig. 2 The motion of the agents.

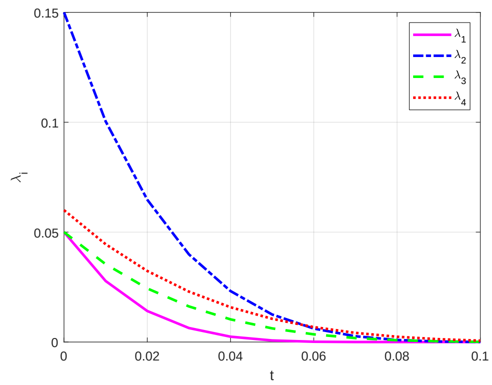

Fig. 3 Path following errors.

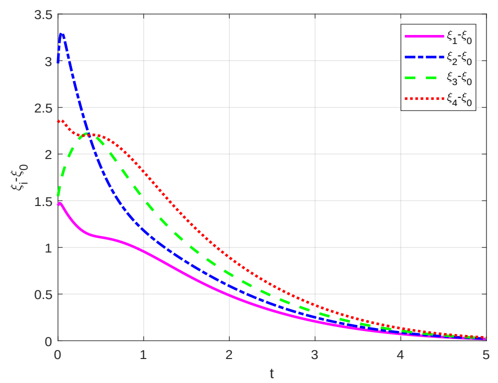

Fig. 4 Formation errors.

5 Conclusion

A continuous feedback method to solve the finite-time coordinated path following control problem has been presented, where the topology among the virtual leader and follows is directed. Since the restriction of the movable ranges of the agents, a novel barrier function is given. The finite-time path following control law and the finite-time formation control law are designed, respectively. Conditions on the control gains to guarantee that the path following errors and the formation errors finite-time converge to zeros are presented.

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