$Regular\ Paper$

Area Recovery under Depth Constraint for Technology Mapping for LUT-based FPGAs *1

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This paper presents Cut Resubstitution; a heuristic algorithm for post-processing of technology mapping for LUT-based FPGAs to minimize area under depth constraint. The concept of Cut Resubstitution is iterating local transformation of an LUT network with considering actual area reduction without using Boolean matching. Cut Resubstitution iterates the following process. At first, Cut Resubstitution substitutes several LUTs in current network in such a way that another LUT is to be redundant. Then Cut Resubstitution eliminates the redundant LUT from network. Experimental results show that a simple depth-minimum mapper followed by Cut Resubstitution generates network whose area is 7%, 7%, 10% smaller than that generated by DAOmap for maximum number of inputs of LUT 4, 5, 6 on average. Our method is similar or slightly faster than DAOmap.

1. Introduction

Recently, designs using LUT (LookUp-Table) based FPGAs (Field Programmable Gate Array) are becoming popular. LUT-based FPGAs require common photomasks for multiple designs, while ASICs (Application Specific Integrated Circuits) require specific photomasks for individual design. Once an LUT-based FPGA is reconfigured for a design, it is available to use immediately. For these reasons, LUT-based FPGAs are often used for prototypes or manufactures required to be developed faster. Main drawbacks of LUT-based FPGAs are performance and power consumption. Thus, technology mapping for LUT-based FPGAs is required to generate high-quality network in short run-time.

Technology mapping for LUT-based FPGAs is a process to convert a given

Boolean network into a functionally equivalent network comprised of K-input LUTs *2. Technology mapping to generate an area-minimal LUT network whose depth is minimum is well studied 1)-6),9),11),13). Area of an LUT network means the number of LUTs in the LUT network. Depth means the length of the longest path *3. Because a problem for area-minimization has been shown to be NPhard ¹⁰⁾, predicting accurately which LUTs are the best for minimizing area in practical time is difficult. Thus, heuristics are likely to be necessary to generate LUT network whose depth and area are minimum. Recent methods take a twostep approach. At first, they generate a depth-minimum LUT network *4 with considering area cost of each K-feasible cone \star^{5} in Boolean network. Area cost is a heuristic metric to indicate how good a K-feasible cone is for area. Then, a post-processing recovers area of LUT network with keeping the depth. Some methods ^{2),9),13)} recover area by iterating global transformation of LUT network. At first, they modify area cost for each K-feasible cone with using the structure of current LUT network. Then, they generate new depth-minimum LUT network with using the area cost. Because the area cost does not have direct relation with actual area, they can fail to find good LUT network. Other methods 6),8),11) recover area by iterating local transformation of LUT network. They iteratively extract a large cone in LUT network, and remap it with fewer LUTs with using Boolean matching. They are likely to be not suitable for large designs because methods using Boolean matching tend to consume significantly long run-time *6.

This paper presents a heuristic algorithm for post-processing of technology mapping; Cut Resubstitution. The concept of Cut Resubstitution is iterating local transformation of LUT network without using Boolean matching. Cut Resubstitution generates a local optimum solution because it does not consider

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 $[\]star 2$ In this paper, K-input LUT is denoted by LUT, and the network comprised of K-input LUTs is denoted by LUT network.

^{★3} Most of existing literatures on technology mapping use depth to refer to delay.

 $[\]star 4$ The problem for depth-minimization can be solved optimally in polynomial time using a dynamic programming $^{3),7)}.$

 $[\]star 5$ A K-feasible cone is a cone with K-input. Technology mapping is often treated as a problem to cover given Boolean network with K-feasible cones.

^{★6} For example, the most recent method based on Boolean matching ⁶⁾ consumes 69 minutes of run-time for a circuit "des" in MCNC benchmarks. For several larger circuits, it is experimentally confirmed to consume several thousands of minutes in Ref. 6).

area cost, but does actual area to be reduced. Cut Resubstitution runs fast because it uses only structures of Boolean network and LUT network without using Boolean matching. For details, Cut Resubstitution iterates the following process. At first, Cut Resubstitution extracts a LUT l and its fanout LUTs. Cut Resubstitution substitutes the fanout LUTs with the same number of LUTs in such a way that the LUT l is to be redundant. This substitution is executed with keeping the depth of LUT network. Then Cut Resubstitution eliminates the redundant LUT l. Experiments to compare Cut Resubstitution and one of the state of the arts depth-minimum mapper; DAOmap 2) are performed. LUT networks given for Cut Resubstitution are generated by a simple depthminimum mapper *1. Cut Resubstitution has generated LUT network whose area is 7%, 7%, 10% smaller on average in the case of K=4,5,6 than that generated by DAOmap respectively. The depth of LUT network generated by Cut Resubstitution and that by DAOmap are same. The run-time of the depthminimum mapper combined with Cut Resubstitution is only a few tens of seconds even for large circuits as ITC'99 benchmarks, which is similar or slightly shorter than that of DAOmap.

The rest of this paper is organized as follows. Section 2 presents some basic definitions and formulation. Cut Resubstitution is presented in Section 3. Section 4 presents overall technology mapping combined with Cut Resubstitution. Section 5 presents experimental results. Section 6 concludes this paper.

2. Preliminaries

The inputs of technology mapping are a DAG which is called **subject graph** and a natural number K. For each node v in subject graph, there is a constraint where the number of inputs is up to K^{*2} . The natural number K corresponds to the maximum number of inputs of LUTs. The output of technology mapping is a network whose nodes represent K-input LUTs. This network is called **LUT**

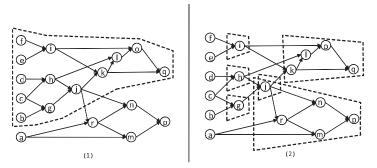


Fig. 1 (1) The transitive fanin graph of q. (2) An example of a realizable set.

network.

A node of a subject graph (V, E) represents a Boolean function and has up to K inputs. If a node $i \in V$ is an input of a node $j \in V$, there is an edge $(i, j) \in E$. The **fanin** of a node v, denoted by FI(v), is the set of immediate predecessors of v. The fanin of v is defined by $FI(v) = \{u \mid \exists (u, v) \in E\}$. The **fanout** of a node v, denoted by FO(v), is the set of immediate successors of v. The fanout of v is defined by $FO(v) = \{w \mid \exists (v, w) \in E\}$. A node v where $FI(v) = \phi$ is called a **primary input**. A node v where $FO(v) = \phi$ is called a **primary output**. PI and PO denote the set of primary inputs and the set of primary outputs respectively. The **transitive fanin** of a node v is the set of all nodes which lie on all paths from any PI to v. More exactly, a transitive fanin of v, denoted by TFI(v), is defined by the following expression.

$$TFI(v) = \{v\} \cup \bigcup_{u \in FI(v)} TFI(u)$$

The **transitive fanin graph** of a node v is the subgraph induced by TFI(v), denoted by TFI(v). **Figure 1**(1) is the example of a subject graph and a transitive fanin graph. The circles and arrows represent nodes and edges in the subject graph respectively. The dashed line in Fig. 1(1) illustrates the transitive fanin graph of node q. The **transitive fanout** of a node v is the set of all nodes which lie on all paths from v to any PO. More exactly, a transitive fanout of v, denoted by TFO(v), is defined by the following expression.

 $[\]star 1$ Cut Resubstitution is not applicable to an LUT network generated by DAOmap in a simple way because Cut Resubstitution needs an information which K-feasible cones are used for covering a given Boolean network to generate the LUT network.

 $[\]star 2$ This constraint guarantees that there is at least one LUT network derived by a subject graph.

$$TFO(v) = \{v\} \cup \bigcup_{u \in FO(v)} TFO(u)$$

A separator s for TFIG(v) is a set of nodes where any path from any primary input to v includes one or more nodes in s. For example, nodes $\{g, h, i, j\}$ in subject graph in Fig. 1 (1) is a separator for TFIG(q). A minimal separator s for TFIG(v) is a separator for TFIG(v) which does not contain any other separator for TFIG(v). For example, nodes $\{q, h, i\}$ in Fig. 1 (1) is a minimal separator for TFIG(q). $\{q, h, i, j\}$ is not minimal separator for TFIG(q) because $\{q, h, i, j\}$ contains the separator $\{q, h, i\}$. A cut (s, v) is a pair of node v and set of nodes s where s is a minimal separator for TFIG(v). For a node v, the cut $(v, \{v\})$ is called the *trivial* cut of v. For a cut (v, s), v is called the **root** of the cut, denoted by RT((v,s)). For a cut (v,s), s is called the **leaf** of the cut, denoted by LEAF((v,s)). For a cut c, |LEAF(c)| is called the cut-size of c. A K-feasible cut c is a cut where |LEAF(c)| is up to K. Because only K-feasible cuts are considered, a K-feasible cut is simply called as a cut in the rest of this paper. $\Phi_K(v)$ denotes the set of all of cuts whose roots are v. For a set of cuts C and a cut $c \in C$, the cut fanin CFI(c,C) and cut fanout CFO(c,C) are defined by the following equations.

$$CFI(c, C) = \{c' | c' \in C, RT(c') \in LEAF(c)\}$$

$$CFO(c, C) = \{c' | c' \in C, RT(c) \in LEAF(c')\}$$

For a feasible cut c, the **feasible cone** KFC(c) is the subgraph induced by the nodes between RT(c) and LEAF(c). A K-feasible cone is exactly defined as the subgraph induced by the set of nodes $V_{interv}(RT(c), LEAF(c))$, where $V_{interv}(v, V)$ is derived by the following equation.

$$V_{interv}(v, V) = \{v\} \cup \bigcup_{u \in FI(v) - V} V_{interv}(u, V)$$

For example, $(q, \{g, h, i\})$ in Fig. 1 (1) is a 3-feasible cut at q. 3-feasible cone of $(q, \{g, h, i\})$ is a subgraph induced by the nodes $\{j, k, l, o, q\}$. For the K-feasible cone KFC(c), the root of c is also called the root of KFC(c), and denoted by RT(KFC(c)). In above case, the leaf LEAF(c) is called the inputs of K-feasible cone KFC(c), and denoted by INPUT(KFC(c)).

For a K-feasible cone C, INPUT(C) is up to K. Thus, a K-input LUT can implement the Boolean function of any K-feasible cone. If a K-input LUT L implement the Boolean function of a K-feasible cone KFC(c), the output signal of L corresponds to RT(KFC(c)), i.e. RT(c), and the input signals of L correspond to INPUT(KFC(c)), i.e. LEAF(c). If a set S of cuts meets below three conditions, S is called as the **realizable set**.

- $\forall i \in PO, (\exists c \in S, i = RT(c)) \lor i \in PI$
- $\forall c \in S, \ \forall i \in LEAF(c), (\exists c' \in S, i = RT(c')) \lor i \in PI$
- There is no trivial cut in S.

An LUT network can be generated from a realizable set S by below operations.

- For each primary input v in the subject graph, generate a primary input which corresponds to v in the LUT network.
- For each cut c in S, generate an LUT which implements the Boolean function of KFC(c).
- For each $c \in S$, for each $i \in CFI(c, S)$, generate the edge (b, a) where a is the LUT which implements the function of KFC(c) and b is the LUT which implements the function of KFC(i).

The technology mapping problem can be defined as DAG covering problem which is the problem to find a realizable set of cuts.

For a node L in an LUT network, the **level** of L is the length of the longest path from any primary input to L. For a realizable set S and a cut $c \in S$, the **cut level** LEV(c,S) denotes the level of LUT which implement the function of KFC(c). LEV(c,S) can be calculated by the following equation.

$$LEV(c, S) = \begin{cases} \max_{c' \in CFI(c, S)} LEV(c', S) + 1 & (CFI(c, S) \neq \phi) \\ 1 & \text{otherwise} \end{cases}$$

The **depth** of an LUT network is the longest path from any primary input to any primary output. The depth of an LUT network is equal to the largest level in the LUT network. For a realizable set S, the depth D(S) is calculated by the following equation.

$$D(S) = \max_{c \in S} (LEV(c, S))$$

The area of an LUT network is the number of nodes in LUT network. For a

realizable set S, the area is calculated by |S|. The technology mapping problem to generate an area-minimum LUT network under depth constraint can be defined as the problem to find a realizable set S where |S| is the minimum and D(S) is equal or under the depth constraint d. For a realizable set S and a cut $c \in S$, the **cut required level** RLV(c,S) denotes the level of LUT which implement the function of KFC(c) required to make the depth minimum. RLV(c,S) can be calculated by the following equation.

$$RLV(c,S) = \begin{cases} \min_{c' \in CFO(c,S)} RLV(c',S) - 1 & (CFO(c,S) \neq \phi) \\ d & \text{otherwise} \end{cases}$$
 (1)

For example, Fig. 1 (2) shows a realizable set where K=3. For each cut c in the realizable set, KFC(c) is illustrated by the dashed trapezoid. $LEV(g,\{c,d\})$, $LEV(h,\{c,d\})$ and $LEV(i,\{e,f\})$ are 1. $LEV(j,\{g,h\})$ and $LEV(p,\{a,g,h\})$ are 2. $LEV(q,\{h,i,j\})$ is 3. Thus, D(S) is 3 in Fig. 1 (2).

3. Cut Resubstitution; a Method for Area Recovery under Detph Constraint

Technology mapping algorithm often consists of the following four phases.

- (1) Cut Enumeration
- (2) Cut Ranking
- (3) Covering
- (4) Post-processing

Cut enumeration enumerates cuts in given subject graph. Cut ranking examines how good each cut is for depth and area in a topological order from PI to PO. Covering picks a good cut for each node in a reverse topological order from PO to PI, and generate a depth-minimum LUT network. Post-processing recovers the area with keeping the depth.

Proposed method is a heuristic post-processing which is called as Cut Resubstitution. **Figure 2**(1) is a motivational example for Cut Resubstitution for K = 3. Cuts $(w, \{q, v\})$ and $(x, \{s, v, q\})$ are selected as a part of realizable set. If cuts $(w, \{q, y\})$ and $(x, \{s, y\})$ are selected as shown in Fig. 2 (2), cuts $(v, \{r, s\})$ would not be needed. Substituting $(w, \{q, v\})$ and $(x, \{s, v, q\})$ with $(w, \{q, y\})$ and $(x, \{s, y\})$ makes $(v, \{r, s\})$ be redundant without changing the area and the

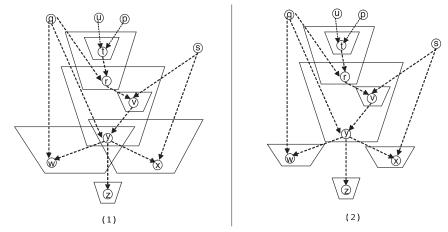


Fig. 2 A motivational example for Cut Resubstituion.

depth. Then, if $(v, \{r, s\})$ is removed from current realizable set, $(r, \{q, u, p\})$ can also be removed because it is also redundant. Therefore, area reduction is 2 in this example.

Cut Resubstitution recovers the area of an LUT network under a depth constraint $^{\star 1}$. Inputs of Cut Resubstitution are a subject graph, a depth constraint, all of cuts and a realizable set. Cut Resubstitution generates a realizable set whose size is local optimum based on iterative elimination of cuts. Cut Resubstitution iterates the following process. At first, Cut Resubstitution identifies potentially redundant cuts in current realizable set S. A potentially redundant cut is such a cut c which can be redundant with substituting CFO(c, S). Then, potentially redundant cuts are ranked with using a metric gain that denotes how good a cut is to be eliminated for area reduction. Then, for such a potentially redundant cut c_{best} whose gain is the maximum, Cut Resubstitution substitutes $CFO(c_{best}, S)$, and eliminates c_{best} . If the substitution of $CFO(c_{best}, S)$ or elimination of c_{best} make other cuts to be redundant, Cut Resubstitution also

 $[\]star 1$ The depth of an LUT network must be equal or smaller than the depth constraint to apply Cut Resubstitution.

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 \begin{aligned} & \text{Cut Resubstitution}(\ (V,E),C_{all},S_0,d\ ) \{ \\ & C_R := \text{RCE}(C_{all},S_0); \\ & S_i := S_0; \\ & \text{while (1) } \{ \\ & \text{CBEC}(\ (V,E),C_R,S_i,d\ ); \\ & S := \text{CE}(\ S_i,C_R\ ); \\ & \text{if } (S=S_i)\ \{ \\ & \text{break}; \\ \} \\ & S_i := S; \\ \} \\ & \text{return } S; \end{aligned}
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Fig. 3 Pseudo code for Cut Resubstitution.

eliminates them.

Figure 3 provides the pseudo code of Cut Resubstitution. In Fig. 3, (V, E) represents a subject graph. C_{all} represents the set of all cuts enumerated in (V, E). S_0 represents given realizable set. C_R represents the sets of cuts which are replaceable for each cut in S_0 . A replaceable cut for a cut c means a candidate for substitution of c. S_i and S represent realizable sets of cuts. d represents a depth constraint. RCE enumerates replaceable cuts for each cut in S_0 . CBEC examines each cut c in S_i whether c is a potentially redundant cut. Potentially redundant cuts are ranked with using gain. For a potentially redundant cut c whose gain is the maximum, CE substitutes cuts in $CFO(c, S_i)$ with other cuts in C_R , and eliminates c and other redundant cuts from S_i . Then the size of new realizable set S is at least one smaller than that of S_i . If there is no potentially redundant cut in S_i , the process of Cut Resubstitution finishes.

3.1 Enumeration of Replaceable Cuts

At this phase, the set of all replaceable cuts are enumerated. A cut $c' \in C_{all}$ is a replaceable cut of $c \in S_0$ ($c \neq c'$) if c' meets the following conditions.

$$RT(c) = RT(c') \land (\forall i \in LEAF(c'), \exists c'' \in S_0, RT(c'') = i)$$
(2)

3.2 Enumeration of Potentially Redundant Cuts and Ranking

At this phase, each cut $c \in S_i$ is examined whether c is a potentially redundant cut. Each potentially redundant cut is ranked by an indicator gain. The gain of cut c means an estimated number of cuts to be redundant if c is redundant.

For a cut $c \in S_i$, a substitutable set of cuts is defined. A substitutable set C

for $c \in S_i$ is a subset of C_R which meets the following three conditions. At first, there is only a cut $c'' \in C$ which meets the condition RT(c'') = RT(c') for each $c' \in CFO(c, S_i)$. At second, $SW(c, C, S_i)$ is a realizable set, where $SW(c, C, S_i)$ denotes a set which is obtained by removing $CFO(c, S_i)$ from S_i , and adding C to S_i . Finally, $D(SW(c, C, S_i))$ must be equal or smaller than d. For example in Fig. 2 (1), $\{(w, \{q, y\}), (x, \{y, s\})\}$ is a substitutable set for $(v, \{r, s\})$.

A cut $c \in S_i$ is a potentially redundant cut if there is a substitutable set for c. Each cut $c \in S_i$ is checked whether there is a substitutable set for c. But examining all combinations to select a repleceable cut for each $c' \in CFO(c, S_i)$ may consume large run-time because the number of all combinations can increase exponentially with $|CFO(c, S_i)|$. For example, if the number of replaceable cuts for each $c \in CFO(c, S_i)$ are the same with s, and if $|CFO(c, S_i)| = t$, then the number of all combinations is s^t . To avoid this problem, a heuristic technique is introduced to check whether there is a substitutable set of $c \in S_i$. At first, each $c' \in CFO(c, S_i)$ is sorted in such a way that roots of cuts are in topological order from PI to PO, and copy S_i to S_i' . Then, each $c' \in CFO(c, S_i)$ is checked whether c' can be substituted with any replaceable cut of c' in the sorted order. If c' and a replaceable cut c'' meet the following condition (3) (4), c' can be substituted with c''.

$$RT(c') = RT(c'') \land (\forall j \in LEAF(c''), (j \in PI \lor (j \neq RT(c) \land (\exists p \in S'_i, RT(p) = j))))$$

$$\max_{j \in CFI(c'', S'_i)} LEV(j, S'_i) + 1 \leq RLV(c', S'_i))$$

$$(3)$$

If the condition (3) is met, the set of cuts obtained with substituting c' in S_i' with c'' is a realizable set. The condition (4) is for holding the depth constraint. If c' can be substituted by c'', update $S_i' = SW(c', \{c''\}, S_i')$. If there is a cut which can substitute c' for each $c' \in CFO(c, S_i)$, c is determined as a potentially redundant cut.

An example in **Fig. 4** shows how to check whether there is a substitutable set of c. Current realizable set S'_i is illustrated in the solid trapezoids. FI(q), FI(t), FO(x), FO(z) are omitted in Fig. 4. The pair of numbers (α, β) beside a node v means the pair of cut level and cut required level for cuts whose roots are v. In this example, $CFO(c, S_i)$ is $\{(x, \{q, t, s\}), (y, \{q, t, s\})\}$. At first, $CFO(c, S_i)$

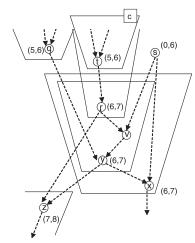


Fig. 4 Checking a substitutable set for a cut c.

are sorted in such a way that roots of cuts are in topological order from PI to PO, and the order $((y,\{q,t,s\}),(x,\{q,t,s\}))$ is obtained. Then, $(y,\{q,t,s\})$ is checked whether $(y,\{q,t,s\})$ can be substituted with any replaceable cut of $(y,\{q,t,s\})$. Let c' and c'' denote $(y,\{q,t,s\})$ and $(y,\{q,r,s\})$. c'' is a reaplaceable cut of c'. $s \in LEAF(c'')$ is in PI. $q \in LEAF(c'') \neq RT(c)$ and there is a cut whose root is q. $r \in LEAF(c'')$ is similar to q. Thus, c' and c'' meets Eq. (3). $\max_{j \in CFI(c'')} LEV(j,S'_i) + 1$ is 7. $RLV(c',S'_i)$ is 7. Thus, c' and c'' meets Eq. (4). Then, S'_i is updated with substituting c' with c''. Next, $(x,\{q,t,s\})$ is checked in a similar way. The replaceable cuts of $(x,\{q,t,s\})$ are $(x,\{q,r,s\})$ and $(x,\{y,s\})$. Although the pair of $(x,\{q,t,s\})$ and $(x,\{y,s\})$ meets Eq. (3), it does not meet Eq. (4). That is because $LEV((y,r,s),S'_i)$ is 7, and cut level for $(x,\{y,s\})$ is 8, while the cut required level for $(x,\{y,s\})$ is 7. On the other hand, the pair of $(x,\{q,t,s\})$ and $(x,\{q,r,s\})$ meets Eqs. (3) and (4). Therefore, $\{(y,\{q,r,s\}),(x,\{q,r,s\})\}$ is a substitutable set of c.

In Eq. (4), $\max_{j \in CFI(c'',S'_i)} LEV(j,S'_i) + 1$ is the same with the cut level for c'' after substituting $CFO(c,S_i)$. That is because a cut to substitute each $j \in CFO(c,S_i)$ is decided in such an order that roots of cuts are in a topological order from PI to PO, and cut fanin of c'' have already been decided. On

the other hand, $RLV(c', S'_i)$ in Eq. (4) might be different from cut required level for c'' after substituting $CFO(c, S_i)$, because the cuts to substitute $CFO(c', S'_i)$ have not decided yet. Thus, Cut Resubstitution might fail to find some few substitutable sets. But it is guaranteed that Cut Resubstitution does not substitute $CFO(c, S_i)$ with other cuts in such a way that violates the depth constraint. That is because $RLV(c', S'_i)$ is the same with cut required level for c'' after substituting $CFO(c, S_i)$ if the following condition for c' is met.

$$(TFO(RT(c')) - \{RT(c')\}) \cap \left(\bigcup_{j \in CFO(c,S_i)} RT(j)\right) = \phi$$

A cut which meets the above condition is called a rearmost cut in $CFO(c, S_i)$. There are one or more rearmost cuts in $CFO(c, S_i)$. For example in Fig. 4, $(x, \{q, t, s\})$ is the rearmost cut in $CFO(c, S_i)$. Because $RLV(c', S'_i)$ is accurate cut required level for c'' after substituting a rearmost c' with c'', if a pair of c' and c'' meets Eq. (4), substituting $CFO(c, S_i)$ with the current set of cuts does not violate the depth constraint. Cut Resubstitution avoids to examine all combinations to select a replaceable cut for each $c' \in CFO(c, S_i)$ with employing the above heuristic approach $^{\star 1}$.

For each cut $c \in S_i$, gain is calculated. The cut whose gain is the maximum among potentially redundant cuts is called best-gain cut. One of best-gain cuts is recorded. The $gain\ GAIN(c,S_i)$ for each $c \in S_i$ is calculated by the following equation in topological order from PI to PO in linear time $^{\star 2}$.

$$GAIN(c, S_i) = 1 + \sum_{j \in CFI(c, S_i)} GAIN'(c, j, S_i)$$

$$GAIN'(c, j, S_i) = \begin{cases} GAIN(j, S_i) & (CFO(j, S_i) = \{c\}) \\ 0 & \text{otherwise} \end{cases}$$
(5)

For example in Fig. 2(1), $(v, \{s, r\})$ is a potentially redundant cut. If $(v, \{s, r\})$

 $[\]star 1$ For all curcuits used in experiments in this paper, there is no difference for the area of an LUT network between using the above heuristic approach and using exact substitutable sets.

^{*2} The computing complexity for gain for each cut $c \in S_i$ is O(|LEAF(c)|). The computing complexity for gain of all cuts in S_i is $O(|S_i|)$ because $|LEAF(c)| \le K$.

is eliminated, $(r, \{q, t\})$ can also be eliminated because $(r, \{q, t\})$ becomes redundant. Thus, the *gain* of $(v, \{s, r\})$ is 2.

3.3 Elimination of the Best Potentially Redundant Cut

At this phase, the *best-gain* cut c is eliminated. At first, S_i is copied to S. If there is no potentially redundant cut found in the previous phase, return S to finish Cut Resubstitution. Otherwise, each cut in $CFO(c, S_i)$ is deleted from S. For each $c \in CFO(c, S_i)$, the substitutional cut calculated in the immediately preceding phase is added to S. Then, each cut c' in S where $CFO(c', S) = \phi$ is deleted from S. The set S is returned.

The set of the replaceable cuts C_R is not needed to be updated after elimination of the best-gain cut. For a cut $c \notin C_R$, LEAF(c) contains a node v which meets the condition $\forall s \in S_0, v \neq RT(s)$. In above case, if and only if a cut s whose root is v is added to S_0 for each $v \in LEAF(c)$ where $\forall s \in S_0, v \neq RT(s), c$ becomes a replaceable cut for a cut $s' \in S_0 + \{s\}$ where RT(c) = RT(s'). No cut c whose root is v where $\forall s \in S_0, v \neq RT(s)$ is added to current realizable set S in any step of Cut Resubstitution. Thus, any cut which is not a replaceable cut for any cut in current realizable set never become a replaceable cut. On the other hand, there is a case where a replaceable cut c for a cut in current realizable set S becomes no-replaceable for any cut in next realizable set S'. But such cut c is not used to substitute any cut in S' because c does not meet Eq. (3). Therefore, the set of the replaceable cuts C_R is not needed to be updated after elimination of the best-gain cut.

4. Overall Technology Mapping combined with Cut Resubstituion

Overall Technology mapping combined with Cut Resubstitution consists of cut enumeration, cut ranking, covering and Cut Resubstitution. In this section, cut enumeration, cut ranking and covering are described. The algorithms described in this section are classic. They are introduced to make clear the overall technology mapping and help to understand the experiments following this section.

4.1 Cut Enumeration

All of cuts are enumerated with using an existing technique shown in Ref. 5). The computing complexity for calculating each $\Phi_K(v)$ is proportional to the size of Cartesian products of $\Phi_K(u_1)$ and $\Phi_K(u_2)$, where $FI(v) = \{u_1, u_2\}$. If K

is 4,5 or $6^{\star 1}$, $\Phi_K(v)$ for each node v is not so large in most cases. Thus, the run-time of the cut enumeration is practical as far as K is up to 6.

4.2 Cut Ranking

Cut Ranking examines how good each cut is for such objectives as depth and area. Depth-cost and area-cost of each cut are calculated. Depth-cost of a cut c means the minimum cut level in cut levels LEV(c, S) for all of realizable sets S which include c. Depth-cost can be calculated exactly in polynomial time in topological order from PI to PO based on dynamic programming 3). For a cut c at v, the depth-cost DC(c) is calculated by the following expression (6), where BD(v) denotes the minimum depth-cost of all cuts at v. BD(v) of each PI is 0.

$$DC(c) = \max_{i \in LEAF(c)} BD(i) + 1$$

$$BD(i) = \begin{cases} 0 & (i \in PI) \\ \min_{j \in \Phi_K(i)} DC(j) & \text{otherwise} \end{cases}$$

$$(6)$$

Area-cost of a cut c means the minimum size of realizable set S for TFIG(v) where S includes c. Because area minimization for DAG covering problem has been shown to be NP-hard $^{10)}$, it is likely that there is no efficient way to compute area-cost accurately. The difficulty of area estimation before covering is mainly due to the existence of nodes with multiple fanout and their reconvergence. For example in Fig. 1 (1), it is difficult to make a decision during cut ranking whether the nodes i, h, j, k, r would be better to be duplicated in covering. The node j may be covered by a single LUT. On the other hand, duplicating j in covering, j may be covered by $2 \sim 3$ LUTs. In Fig. 1 (2), j is covered by 2 LUTs.

Several existing algorithms $^{1),2),5),9),13)}$ calculate approximate area-cost based on a heuristic technique; area flow. The key idea of area flow is to distribute the area-cost of each node to its fanout nodes with taking into account the effect of input sharing. For a cut c, area-cost $AC_{af}(c)$ based on area flow is calculated by Eq. (7), where BA(v) denotes the minimum area-cost of all cuts of v and U(c) denotes the area contributed by c itself $^{\star 2}$.

 $[\]star 1$ 4,5 and 6 are popular numbers for the maximum number of inputs of LUT in commercial FPGAs.

 $[\]star 2 \ U(c)$ is usually 1.

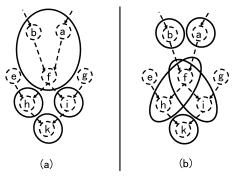


Fig. 5 An example of two realizable sets.

$$AC_{af}(c) = U(c) + \sum_{i \in LEAF(c)} BA(i)/|FO(i)|$$
(7)

$$AC_{af}(c) = U(c) + \sum_{i \in LEAF(c)} BA(i)/|FO(i)|$$

$$BA(i) = \begin{cases} 0 & (i \in PI) \\ \min_{j \in \Phi_K(i)} AC(j) & \text{otherwise} \end{cases}$$
(8)

Equation (7) is based on the heuristics dividing BA(v) by |FO(v)| to avoid summing up BA(v) redundantly.

Area-cost based on area flow is accurate if there is no duplication of any node. But if not so, area-cost based on area flow can be inaccurate. Figure 5 shows an example of two realizable sets. Graphs by dashed lines illustrate subject graphs. and circles in solid lines illustrate the K-feasible cones corresponding to a cuts in the realizable set. The predecessors of a and b are omitted, and assume that BA(i) for each $i \in FI(a) \cup FI(b)$ is 0. In Fig. 5 (a), the area for TFIG(f) is 1, and both areas estimated for TFIG(h) and that for TFIG(i) are 3/2. The area estimated for TFIG(k) is 4. Therefore the area based on area flow is accurate in Fig. 5 (a). In Fig. 5 (b), both areas estimated for TFIG(h) and that for TFIG(i)are 3. The area estimated for TFIG(k) is 6. Therefore the area estimated based on area flow is inaccurate because the correct area is 5 in Fig. 5 (b).

weighted area flow is another heuristic which is implemented in Magus^{*1}. The key idea of weighted area flow is to avoid that an area-cost is redundantly summed along the reconvergent paths with considering the effect of fanouts from nodes in K-feasible cone KFC(c) to other nodes not in KFC(c). Area-cost $AC_{waf}(c)$ of cut c is calculated by Eq. (9).

$$AC_{waf}(c) = 1 + \sum_{i \in LEAF(c)} BA(i) \times NW(i, v)$$
(9)

NW(i,v) is an inverse number of times that mean how many the area for TFIG(i) distributed to v is summed up redundantly. The weight w((v,w)) for an edge (v, w) is defined as w((v, w)) = 1/|FO(v)|. NW(i, v) for a pair of node v and $i \in TFI(v)$ is calculated by summing up each product $w((i,v)) \times NW(i,j)$ for each $j \in FI(v)$ where j is in any path from i to v and NW(i, i) is 1. Area estimated based on weighted area flow is accurate in Fig. 5(b). w((e,h)), w((b,f))and w((a, f)) are 1, and w((f, h)) is 1/2. NW(b, h) and NW(a, h) are both 1/2. NW(e,h) is 1. The area estimated for TFIG(h) is 2. Then, NW(f,i) is 1/2, and NW(b,i), NW(a,i) are both 1/2. NW(q,i) is 1. The area estimated for TFIG(i) is 2. Then, the area estimated for TFIG(k) is 5. Therefore, the area-cost based on weighted area flow is correct in Fig. 5 (b).

4.3 Covering

Covering selects the best cuts to cover each node. A best cut is the cut whose area-cost is the minimum where the depth-cost is not up to the cut required level. Covering is executed as follows. At first, the set of cut $S = \phi$ is prepared. At each $o \in PO$, a best cut c is selected and added to S. Next, the best cut of each node in LEAF(c) is selected and added to S^{*2} . Above process executed iteratively in reverse topological order from PO to PI.

5. Experimental Results

Experiments are performed to compare Cut Resubstitution with one of the state of the arts depth-minimum mapper; DAOmap ²⁾. DAOmap generates a depth-minimum LUT network with using area cost based on area flow, and recovers area by iterating global transformation of LUT network with modifying

^{*1} Magus is a Logic Synthesis system developed in Kyushu University

^{*2} The set S may be not realizable set yet. But the cut require level RLV(c,S) can be calculated by the equation (1) because all nodes in FO(RT(c)) has been covered and CFO(c,S)has been determinate.

area cost. Cut Resubstitution has been implemented using C++ within the Magus. Subject graphs are generated by decomposing each node of networks on MCNC benchmark set and ITC'99 benchmark set to nodes whose number of inputs are up to 2. The maximum number of inputs of LUT K is assumed to be 4,5 or 6. A machine whose CPU is Intel Xeon 3.00 GHz and memory size is 16 GB is used.

The initial LUT networks given for Cut Resubstitution are generated by FMap and WMap those are simple depth-minimum mappers implemented in Magus $^{\star 1}$. FMap and WMap are based on the method described in section 4 to generate LUT network. FMap calculates area-cost based on area flow. WMap calculates area-cost based on weighted area flow. The depth constraints given for Cut Resubstitution are the depths of LUT networks derived by FMap and WMap. The combination of FMap and Cut Resubstitution is called as FRmap, and the combination of WMap and Cut Resubstitution is called as WRmap. If given subject graph and K are the same, the depths generated by DAOmap and FRmap and WRmap are the same because they are guaranteed to generate a depth-minimum LUT network. Area of LUT network and run-time for each algorithm are evaluated.

Table 1, **Table 3** and **Table 5** show the results on MCNC benchmark set in K=4,5,6, respectively. **Table 2**, **Table 4** and **Table 6** show the results on ITC'99 benchmark set in K=4,5,6, respectively. The results for small benchmarks are omitted. DAOmap cannot generate an LUT network for "des" in MCNC benchmark set in the case of K=6 in 4 days. Thus, the result for "des" in the case of K=6 is also omitted. "D" in tables means DAOmap. "F" and "FR" means FMap and FRmap, respectively. "W" and "WR" means WMap and WRmap, respectively. "AVG" means average.

Area of LUT network generated by FRmap is 5%, 7%, 8% smaller for K=4,5,6 respectively than that generated by DAOmap on average. Area of LUT network

Table 1 The results for MCNC benchmark set in the case of K = 4.

| | Area | | | | | | Rur | Run Time (sec) | | |
|-------|------|------|------|------|------|------|------|----------------|-----|-----|
| Name | D | F | FR | FR/D | W | WR | WR/D | D | FR | WR |
| C5315 | 580 | 588 | 568 | 98% | 590 | 568 | 98% | 0.2 | 0.3 | 0.3 |
| C6288 | 554 | 676 | 537 | 97% | 605 | 508 | 92% | 0.5 | 0.9 | 0.7 |
| C7552 | 773 | 791 | 747 | 97% | 779 | 731 | 95% | 0.3 | 0.4 | 0.4 |
| att10 | 1190 | 1163 | 1161 | 98% | 1230 | 1161 | 98% | 0.2 | 0.3 | 0.3 |
| att15 | 868 | 794 | 791 | 91% | 801 | 790 | 91% | 0.2 | 0.2 | 0.2 |
| att16 | 560 | 554 | 548 | 98% | 574 | 541 | 97% | 0.1 | 0.2 | 0.2 |
| att21 | 5481 | 5346 | 5346 | 98% | 5348 | 5346 | 98% | 3.0 | 1.2 | 1.2 |
| att6 | 547 | 520 | 518 | 95% | 544 | 528 | 97% | 0.1 | 0.2 | 0.2 |
| att8 | 858 | 832 | 832 | 97% | 832 | 832 | 97% | 0.2 | 0.2 | 0.2 |
| des | 2807 | 2623 | 2615 | 93% | 2702 | 2649 | 94% | 0.9 | 0.7 | 0.7 |
| rot | 520 | 496 | 489 | 94% | 503 | 486 | 93% | 0.1 | 0.1 | 0.1 |
| vda | 547 | 520 | 518 | 95% | 544 | 528 | 97% | 0.1 | 0.2 | 0.1 |
| AVG | | | | 96% | | | 95% | 0.5 | 0.4 | 0.4 |

Table 2 The results for ITC'99 benchmark set in the case of K = 4.

| | Area | | | | | Rur | Run Time (sec) | | | |
|-------|-------|-------|-------|------|-------|-------|----------------|-----|-----|-----|
| Name | D | F | FR | FR/D | W | WR | WR/D | D | FR | WR |
| b14 | 3097 | 3033 | 2945 | 95% | 3248 | 2394 | 77% | 1.1 | 1.2 | 1.2 |
| b14_1 | 2289 | 2169 | 2089 | 91% | 2261 | 2076 | 91% | 0.7 | 0.8 | 0.8 |
| b15 | 3586 | 3571 | 3393 | 95% | 3715 | 3355 | 94% | 1.0 | 1.1 | 1.1 |
| b15_1 | 4260 | 4185 | 4110 | 96% | 4298 | 4026 | 95% | 1.5 | 1.3 | 1.3 |
| b17 | 11796 | 11548 | 11139 | 94% | 12139 | 10995 | 93% | 3.7 | 3.7 | 3.6 |
| b17_1 | 13032 | 12676 | 12447 | 96% | 13075 | 12228 | 94% | 4.6 | 4.1 | 4.1 |
| b20 | 6255 | 6049 | 5874 | 94% | 6321 | 5776 | 92% | 2.2 | 2.4 | 2.4 |
| b20_1 | 4468 | 4313 | 4161 | 93% | 4484 | 4067 | 91% | 1.5 | 1.7 | 1.7 |
| b21 | 6437 | 6200 | 6043 | 94% | 6428 | 5932 | 92% | 2.3 | 2.5 | 2.5 |
| b21_1 | 4586 | 4430 | 4283 | 93% | 4614 | 4228 | 92% | 1.5 | 1.7 | 1.7 |
| b22 | 9291 | 8944 | 8735 | 94% | 9413 | 8588 | 92% | 3.4 | 3.6 | 3.6 |
| b22_1 | 6802 | 6560 | 6362 | 94% | 6859 | 6229 | 92% | 2.3 | 2.6 | 2.6 |
| AVG | | | | 94% | | | 91% | 2.2 | 2.2 | 2.2 |

generated by WRmap is 7%, 7%, 10% smaller for K=4,5,6 respectively than that generated by DAOmap on average. For all of the circuits but att15 and att16 for K=6, FRmap generated LUT networks with less area than those generated by DAOmap. For all of the circuits but C6288 for K=5, WRmap generated LUT networks with less area than those generated by DAOmap. Cut Resubstitution generates better LUT network than DAOmap, because Cut Resubstitution performs local transformations with considering actual area reduc-

^{*1} The experiments do not include applying Cut Resubstitution to LUT networks generated by DAOmap. That is because the realizable set used to generate an LUT network in DAOmap is not known. If functionally equivalent points between each node in an LUT network and nodes in a subject graph are specified, the realizable set can be derived with using the functionally equivalent points and the set of all cuts.

Table 3 The results for MCNC benchmark set in the case of K=5.

| | | | | Area | | | | Rur | Run Time (sec) | | |
|----------------------|------|------|------|------|------|------|------|-----|----------------|-----|--|
| Name | D | F | FR | FR/D | W | WR | WR/D | D | FR | WR | |
| C5315 | 438 | 445 | 428 | 98% | 448 | 425 | 97% | 0.5 | 0.5 | 0.5 | |
| C6288 | 560 | 659 | 540 | 96% | 894 | 632 | 113% | 2.1 | 4.0 | 3.1 | |
| C7552 | 634 | 652 | 605 | 95% | 662 | 608 | 96% | 1.0 | 1.0 | 1.2 | |
| att10 | 935 | 886 | 884 | 95% | 932 | 882 | 94% | 0.5 | 0.4 | 0.4 | |
| att15 | 651 | 523 | 522 | 80% | 533 | 522 | 80% | 0.4 | 0.3 | 0.4 | |
| att16 | 436 | 437 | 428 | 98% | 439 | 404 | 93% | 0.3 | 0.3 | 0.3 | |
| att21 | 4391 | 4207 | 4207 | 96% | 4204 | 4202 | 96% | 5.1 | 1.4 | 1.4 | |
| att6 | 455 | 424 | 424 | 93% | 455 | 422 | 93% | 0.2 | 0.2 | 0.3 | |
| att8 | 619 | 598 | 598 | 97% | 598 | 598 | 97% | 0.2 | 0.2 | 0.2 | |
| des | 2079 | 1975 | 1967 | 95% | 2140 | 1971 | 95% | 1.5 | 1.3 | 1.3 | |
| rot | 385 | 359 | 358 | 93% | 379 | 359 | 93% | 0.2 | 0.2 | 0.2 | |
| vda | 455 | 424 | 424 | 93% | 455 | 422 | 93% | 0.2 | 0.2 | 0.2 | |
| AVG | | | | 94% | | | 95% | 1.0 | 0.8 | 0.8 | |

Table 4 The results for ITC'99 benchmark set in the case of K = 5.

| | Area | | | | | | | Run Time (sec) | | |
|------------|-------|-------|-------|------|-------|------|------|----------------|------|------|
| Name | D | F | FR | FR/D | W | WR | WR/D | D | FR | WR |
| b14 | 2265 | 2260 | 2163 | 95% | 2390 | 2097 | 93% | 3.0 | 3.2 | 3.1 |
| b14_1 | 1749 | 1721 | 1653 | 95% | 1824 | 1633 | 93% | 1.9 | 2.1 | 2.1 |
| b15 | 2944 | 3021 | 2788 | 95% | 3175 | 2693 | 91% | 2.4 | 3.1 | 3.0 |
| $b15_{-}1$ | 3571 | 3339 | 3264 | 91% | 3438 | 3145 | 88% | 4.4 | 3.1 | 3.3 |
| b17 | 9457 | 9410 | 8993 | 95% | 10206 | 8726 | 92% | 9.9 | 9.9 | 9.7 |
| $b17_{-}1$ | 10747 | 10250 | 10018 | 93% | 10576 | 9676 | 90% | 12.9 | 9.8 | 10.4 |
| b20 | 4932 | 4562 | 4366 | 89% | 4825 | 4276 | 87% | 6.5 | 6.8 | 6.8 |
| $b20_{-}1$ | 3437 | 3275 | 3138 | 91% | 3486 | 3087 | 90% | 4.3 | 4.8 | 4.9 |
| b21 | 5069 | 4788 | 4560 | 90% | 5067 | 4499 | 89% | 6.6 | 7.0 | 7.0 |
| b21_1 | 3553 | 3415 | 3247 | 91% | 3605 | 3219 | 91% | 4.3 | 5.0 | 5.0 |
| b22 | 6860 | 6740 | 6489 | 95% | 7151 | 6310 | 92% | 9.2 | 10.1 | 10.1 |
| $b22_{-1}$ | 5084 | 5029 | 4815 | 95% | 5366 | 4731 | 93% | 6.4 | 7.5 | 7.5 |
| AVG | | | | 93% | | | 91% | 6.0 | 6.0 | 6.1 |

tion while DAOmap performs only global transformations with considering area cost. For LUT network generated based on area cost, local transformations with considering actual area reduction is likely to work well to reduce area. Cut Resubstitution has been experimentally found to work better in the case of large K. Cut Resubstitution reduced 3%, 3%, 6% area of LUT network generated by FMap on average for K=4,5,6. Cut Resubstitution reduced 7%, 9%, 10% area of LUT network generated by WMap on average for K=4,5,6. WMap is consid-

Table 5 The results for MCNC benchmark set in the case of K = 6.

| | Area | | | | | | | | Run Time (sec) | | |
|-------|------|------|------|------|------|------|------|------|----------------|------|--|
| Name | D | F | FR | FR/D | W | WR | WR/D | D | FR | WR | |
| C5315 | 315 | 325 | 312 | 99% | 320 | 305 | 97% | 1.9 | 1.9 | 2.0 | |
| C6288 | 568 | 741 | 523 | 92% | 718 | 504 | 89% | 16.8 | 21.4 | 18.3 | |
| C7552 | 510 | 521 | 478 | 94% | 523 | 449 | 88% | 4.6 | 3.8 | 4.6 | |
| att10 | 837 | 765 | 759 | 91% | 823 | 773 | 92% | 1.3 | 0.8 | 0.8 | |
| att15 | 490 | 538 | 513 | 105% | 486 | 460 | 94% | 1.2 | 0.6 | 0.6 | |
| att16 | 352 | 358 | 355 | 101% | 369 | 339 | 96% | 0.6 | 0.4 | 0.4 | |
| att21 | 3900 | 3683 | 3679 | 94% | 3679 | 3678 | 94% | 8.1 | 2.0 | 2.0 | |
| att6 | 396 | 359 | 353 | 89% | 372 | 363 | 92% | 0.6 | 0.5 | 0.5 | |
| att8 | 525 | 480 | 480 | 91% | 480 | 480 | 91% | 0.3 | 0.3 | 0.3 | |
| rot | 326 | 310 | 297 | 91% | 315 | 295 | 90% | 0.3 | 0.3 | 0.4 | |
| vda | 396 | 359 | 353 | 89% | 372 | 363 | 92% | 0.6 | 0.4 | 0.4 | |
| AVG | | | | 94% | | | 93% | 3.3 | 2.7 | 2.5 | |

Table 6 The results for ITC'99 benchmark set in the case of K = 6.

| | Area | | | | | | | Run Time (sec) | | |
|------------|------|------|------|------|------|------|------|----------------|------|------|
| Name | D | F | FR | FR/D | W | WR | WR/D | D | FR | WR |
| b14 | 1925 | 1797 | 1706 | 89% | 1966 | 1705 | 89% | 14.3 | 12.4 | 12.4 |
| b14_1 | 1445 | 1487 | 1365 | 94% | 1520 | 1331 | 92% | 9.3 | 8.5 | 7.8 |
| b15 | 2423 | 2437 | 2178 | 90% | 2576 | 2144 | 88% | 11.2 | 12.8 | 12.5 |
| $b15_{-1}$ | 3173 | 2724 | 2590 | 82% | 2838 | 2569 | 81% | 20.4 | 11.5 | 15.6 |
| b17 | 8127 | 7937 | 7304 | 90% | 8457 | 7127 | 88% | 45.6 | 39.7 | 42.0 |
| $b17_{-1}$ | 9319 | 8354 | 7958 | 85% | 8681 | 7894 | 85% | 57.8 | 35.9 | 48.4 |
| b20 | 3887 | 3839 | 3568 | 92% | 4089 | 3476 | 89% | 31.6 | 29.9 | 29.4 |
| $b20_{-}1$ | 2938 | 2844 | 2650 | 90% | 3001 | 2590 | 88% | 22.0 | 20.6 | 20.7 |
| b21 | 3980 | 3983 | 3698 | 93% | 4131 | 3566 | 90% | 33.3 | 30.8 | 30.1 |
| b21_1 | 2979 | 2908 | 2701 | 91% | 3078 | 2656 | 89% | 23.0 | 21.7 | 21.7 |
| b22 | 5795 | 5670 | 5316 | 92% | 6003 | 5113 | 88% | 48.1 | 45.7 | 44.3 |
| $b22_{-}1$ | 4386 | 4339 | 4040 | 92% | 4585 | 3939 | 90% | 32.9 | 32.8 | 31.8 |
| AVG | | | | 90% | | | 88% | 29.1 | 25.2 | 26.4 |

ered to fit together with Cut Resubstitution better than FMap in most case. For example in Table 6, WMap generated LUT networks with more area than those generated by DAOmap or FMap for almost all of the circuits. Cut Resubstitution reduced 13% area of LUT network generated by WMap on averages, and the generated LUT network has 12% less area than that generated by DAOmap and 2% less area than that generated by FRmap on average. Furthermore, Cut Resubstitution with WMap works better for large circuit. The networks in ITC'99 benchmark set tend to be larger compared to the networks in MCNC benchmark

set. For LUT networks generated by WMap, the average rate of area reduction of Cut Resubstitution in ITC'99 benchmark set is 6%, 4%, 6% larger than that in MCNC benchmark set for K=4,5,6, respectively. On the other hand, for FMap, there is no great difference in the average rate of area reduction between MCNC benchmark set and ITC'99 benchmark set. Both the run-time of FRmap and that of WRmap are only a few tens of seconds even for large circuits as ITC'99 benchmarks, which is similar or slightly shorter than that of DAOmap.

6. Conclusions

In this paper Cut Resubstitution which is a heuristic post-processing of technology mapping to minimize area under depth constraint is proposed. Cut Resubstitution generates a local optimum solution because it considers not area cost but actual area to be reduced. Cut Resubstitution runs fast because it uses only structures of Boolean network and LUT network without using Boolean matching. For details, Cut Resubstitution iterates the following process. At first. Cut Resubstitution substitutes several LUTs in current network in such a way that another LUT is to be redundant. Then Cut Resubstitution eliminates the redundant LUT from network. Experimental results show that Cut Resubstitution fits together with a mapper using area-cost based on weighted area flow better than that using area-cost based on area flow. A simple depth-minimum mapper using weighted area flow followed by Cut Resubstitution generated network whose average area is 7%, 7%, 10% smaller than DAOmap²⁾ for K = 4, 5, 6respectively. The depth of LUT network generated by Cut Resubstitution and that by DAOmap are the same if given Boolean network and K are the same. The run-time of Cut Resubstitution is similar or slightly shorter than that of DAOmap. In conclusion, Cut Resubstitution is efficient to reduce area of LUT network generated with considering only area cost.

Our future work is to find initial LUT network which work well with Cut Resubstitution. Examinations of area of LUT networks generated by Cut Resubstitution combined with DAOmap or other algorithms are necessary.

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