## Recommended Paper

# A Pairing-Based Anonymous Credential System with Efficient Attribute Proofs* 

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#### Abstract

To enhance user privacy, anonymous credential systems allow the user to convince a verifier of the possession of a certificate issued by the issuing authority anonymously. The typical application is the privacy-enhancing electronic ID (eID). Although a previously proposed system achieves the constant complexity in the number of finiteset attributes of the user, it requires the use of RSA. In this paper, we propose a pairing-based anonymous credential system excluding RSA that achieves the constant complexity. The key idea of our proposal is the adoption of a pairingbased accumulator that outputs a constant-size value from a large set of input values. Using zero-knowledge proofs of pairing-based certificates and accumulators, any AND and OR relation can be proved with the constant complexity in the number of finite-set attributes. We implement the proposed system using the fast pairing library, compare the efficiency with the conventional systems, and show the practicality in a mobile eID application.


Keywords: anonymity, anonymous credentials, attributes, pairings, accumulators

## 1. Introduction

Electronic identification has been widely applied to access authorization to buildings, use of facilities, Web services, etc. Currently, electronic identity (eID) such as eID card is often used. The eID is issued by a trusted organization such as the government, company, or university, and is used for its service. Trusted ID is very attractive for secondary use in commercial services. The eID includes attributes of the user such as the name, the address, the gender, the occupation, and the date of birth. In commercial cases, the attribute-based authentication is desired. For example, a service provider can deny access to kids, by checking the age in the eID.
One of serious issues in the existing eID systems is user's privacy. In the systems, the eID may reveal the user's identity. The service provider can collect the use history of each user. Anonymous credential systems [10], [12], [13] are one of the solutions.

An anonymous credential system basically consists of the setup algorithm, issuing protocol and proof protocol. The participants are an issuing authority, users, and verifiers. In the setup algorithm, an issuing authority generates his/her public key ipk and secret key isk. ipk is bound to the issuing authority. In the issuing protocol, the issuing authority with isk issues a certificate cert to a user, given common input $i p k$. Each certificate is a proof of membership, qualification, or privilege, and contains user's attributes. In the proof protocol, given the common input ipk, the

[^0]user with cert can prove to a verifier that the user has a certificate issued by the authority bound to ipk. This protocol is anonymous, i.e., the verifier cannot know who is the proving user. This means that only ipk is the public key and the user's public key is not used. In addition, the proving user can disclose some selected attributes without revealing any other information about the user's privacy. Furthermore, the user can prove complex relations of the attributes using AND and OR relations. AND relation is used when proving the possession of all of the multiple attributes. For example, the user can prove that he is a student, and has a valid student card, when entering the faculty building. OR relation represents the proof for possession of one of multiple attributes. For example, he can prove that he is either a staff or a teacher when using a copy machine in a laboratory. An implementation of eID on a standard java card is shown in Ref. [6].

In Ref. [13], Camenisch and Lysyanskaya firstly proposed an anonymous credential system based on RSA. Unfortunately, it suffers from a linear complexity in the number of user's attributes in proving AND and OR relations. Hence, this system is not suitable for small devices such as smart cards. In Ref. [10], Camenisch and Groß extended the scheme to solve the drawback. They classified attribute types into two categories: string attributes and finite-set attributes. The former can be represented as a string, such as name and ID number. The latter can be represented as an element from relatively small finite-set, such as gender and profession. There are much fewer string type of attributes, and thus the costs on finite-set attribute types impacts the total efficiency. In Camenisch-Groß system, by encoding a large number

[^1]of finite-set attributes into prime numbers, one value for the finiteset attributes can be embedded into the certificate. Then, the AND and OR relations are proved with the constant complexity in the number of finite-set attributes using zero-knowledge proofs of integer relations on prime numbers. However, this extended system also depends on RSA. Currently, 1,024-bit RSA modulus is obsolete, and we require 2,048 bits or more. As the modulus size increases, the data size in the authentication becomes larger, and the processing times also become larger. To shorten the data size, a pairing-based system excluding the RSA assumption is desired.
In this paper, for a pairing-based anonymous credential system using BBS+ signatures [8], we show how to prove AND and OR relations with constant complexity. It seems difficult to adopt the approach in Camenisch-Groß system. This is because their approach strictly depends on integer commitments on the strong RSA assumption to prove the integer relation over integers, as mentioned in Ref. [10]. This is why we adopt a different approach. The key idea of the construction is the adoption of a pairing-based accumulator [12]. The accumulator outputs a constant-size value from a large set of input values. We consider that the input values are assigned to attributes. Then, we utilize an extended BBS+ signatures to certify a set of attributes as the accumulator. Using zero-knowledge proofs of BBS+ signatures and accumulators, we can prove AND and OR relations with constant complexity in the number of finite-set attributes. The drawback is that the size of public key is depending on the number of attribute values. It varies from 1 MBytes to 10 MBytes for the number of attribute values 1,000 to 15,000 . In the current mobile environments, the data size is sufficiently practical, since the public key is not changed after it is distributed.

To show the efficiency of our system, we implemented the system using a pairing library, and compare the processing time to a conventional pairing-based anonymous credential system with linear complexity. In addition, we compare our system to the RSA-based Camenisch-Groß system. Furthermore, we implemented a prototype system of eID application and show the processing time including the communication to confirm practicality.

Remark 1 In the RSA-based anonymous credential system with efficient complexity [10], NOT relation is also equipped. Namely, the prover can prove that a specified attribute is not in his certificate. On the other hand, our system does not have the protocol to directly prove NOT relation. However, OR relation substitutes NOT relation. In an attribute type, let $S=\left\{a_{1}, \ldots, a_{k}\right\}$ be the set of all attribute values of the attribute type. Into the user's certificate, an element from $S$ is embedded. Here we assume that any user owns an attribute from $S$. In the case that the user does not own any attribute value from the attribute type, we can insert the value indicating no attribute value into $S$. Consider the proof that the user's attribute is not $a_{1}$. This proof can be performed by the OR proof that the user owns some attribute of $S-\left\{a_{1}\right\}$. For example, for proving that the user is not a student, we can prove that she has some of all other profession attribute values including the value indicating no profession.

## 2. Preliminaries

### 2.1 Bilinear Groups

Our scheme utilizes the following bilinear groups:
(1) $\mathbb{G}_{1}, \mathbb{G}_{2}$, and $\mathbb{G}_{T}$ are multiplicative cyclic groups of prime order $p$,
(2) $g$ and $h$ are randomly chosen generator of $\mathbb{G}_{1}$ and $\mathbb{G}_{2}$, respectively,
(3) $e$ is an efficiently computable bilinear map: $e: \mathbb{G}_{1} \times \mathbb{G}_{2} \rightarrow$ $\mathbb{G}_{T}$, namely, (1) for all $u, u^{\prime} \in \mathbb{G}_{1}, v, v^{\prime} \in \mathbb{G}_{2}, e\left(u u^{\prime}, v\right)=$ $e(u, v) e\left(u^{\prime}, v\right)$ and $e\left(u, v v^{\prime}\right)=e(u, v) e\left(u, v^{\prime}\right)$, and thus for all $u \in \mathbb{G}_{1}, v \in \mathbb{G}_{2}$ and $a, b \in Z, e\left(u^{a}, v^{b}\right)=e(u, v)^{a b}$, and (2) $e(g, h) \neq 1$.

### 2.2 Assumptions

The security of our scheme is based on the $q$-SDH assumption [7], the $q$-HSDH (Hidden SDH) assumption [9], and $q$-TDH (Triple DH ) assumption [5] for the underlying signatures, and $n$ DHE assumption [12] for the accumulator, where $q, n$ are nonnegative integer.

Definition 1 ( $q$-SDH assumption) For all PPT algorithm $\mathcal{A}$, the probability

$$
\operatorname{Pr}\left[\mathcal{A}\left(g, h, h^{a}, g^{a}, \ldots, g^{a^{a}}\right)=\left(b, g^{1 /(a+b)}\right) \wedge b \in Z_{p}\right]
$$

is negligible, where $g \in_{R} \mathbb{G}_{1}, h \in_{R} \mathbb{G}_{2}$ and $a \in_{R} Z_{p}$.
Definition 2 ( $q$-HSDH assumption) For all PPT algorithm $\mathcal{A}$, the probability

$$
\begin{aligned}
& \operatorname{Pr}\left[\mathcal{A}\left(g, h, \hat{h}, g^{a}, h^{a},\left(h^{1 /\left(a+b_{1}\right)}, g^{b_{1}}, \hat{h}^{b_{1}}\right), \ldots,\left(h^{1 /\left(a+b_{q}\right)}, g^{b_{q}}, \hat{h}^{b_{q}}\right)\right)\right. \\
& \left.\quad=\left(h^{1 /(a+b)}, g^{b}, \hat{h}^{b}\right) \wedge \forall i \in[1, q]: g^{b} \neq g^{b_{i}}\right]
\end{aligned}
$$

is negligible, where $g \in_{R} \mathbb{G}_{1}, h, \hat{h} \in_{R} \mathbb{G}_{2}, a \in_{R} Z_{p}$, and $b, b_{i} \in Z_{p}$.
Definition 3 ( $q$-TDH assumption) For all PPT algorithm $\mathcal{A}$, the probability

$$
\begin{aligned}
\operatorname{Pr} & {\left[\mathcal{A}\left(g, h, g^{a}, h^{a}, h^{b},\left(c_{1}, h^{1 /\left(a+c_{1}\right)}\right), \ldots,\left(c_{q}, h^{1 /\left(a+c_{q}\right)}\right)\right)\right.} \\
& \left.=\left(g^{r a}, h^{r b}, h^{r a b}\right) \wedge \forall i \in[1, q]: c \neq c_{i} \wedge r \neq 0\right]
\end{aligned}
$$

is negligible, where $g \in_{R} \mathbb{G}_{1}, h \in_{R} \mathbb{G}_{2}, a, b \in_{R} Z_{p}$, and $c_{i}, c \in Z_{p}$.
Definition 4 ( $n$-DHE assumption) For all PPT algorithm $\mathcal{A}$, the probability

$$
\begin{aligned}
& \operatorname{Pr}\left[\mathcal{A}\left(g, h,, g^{a}, \ldots, g^{a^{n}}, g^{a^{n+2}}, \ldots, g^{a^{2 n}}, h^{a}, \ldots, h^{a^{n}}, h^{a^{n+2}}, \ldots, h^{a^{2 n}}\right)\right. \\
& \left.\quad=h^{a^{n+1}}\right]
\end{aligned}
$$

is negligible, where $g \in_{R} \mathbb{G}_{1}, h \in_{R} \mathbb{G}_{2}$ and $a \in_{R} Z_{p}$.

### 2.3 Extended Accumulator with Efficient Updates

In Ref. [12], the accumulator with efficient updates is proposed. The accumulator is generated from a set of values, and we can verify that a single value is accumulated. Thus, for $k$ values, we have to verify that each value is accumulated multiple times. This means that the complexity depends on the number of proved values, $k$. Here, we extend the accumulator to verify that $k$ values are accumulated with the constant complexity.

Here, we consider that some values in $\{1, \ldots, n\}$ with size $n$ are accumulated. Let $V$ be a set of accumulated values that is a subset of $\{1, \ldots, n\}$. Let $U=\left\{i_{1}, \ldots, i_{k}\right\}$ be a subset of $V$ with size $k$. The
accumulator allows us to confirm that all elements of $U$ belong to $V$, i.e., $U \subseteq V$, all at once.
AccSetup: This is the algorithm to output the public parameters. Select bilinear groups $\mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}$ with a prime order $p$ and a bilinear map $e$. Select $g \in_{R} \mathbb{G}_{1}$ and $h \in_{R} \mathbb{G}_{2}$. Select $\gamma \in_{R} Z_{p}$ and compute and publish $p, \mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}, e, g, g_{1}=$ $g^{\gamma^{1}}, \ldots, g_{n}=g^{\gamma^{n}}, g_{n+2}=g^{\gamma^{n+2}}, \ldots, g_{2 n}=g^{\gamma^{2 n}}, h_{1}=$ $h^{\gamma^{1}}, \ldots, h_{n}=h^{\gamma^{n}}, h_{n+2}=h^{\gamma^{n+2}}, \ldots, h_{2 n}=h^{\gamma^{2 n}}$ and $z=$ $e(g, h)^{\gamma^{n+1}}$ as the public parameters.
AccUpdate: This is the algorithm to compute the accumulator using the public parameters. The accumulator $\operatorname{acc}_{V}$ of $V$ is computed as $\operatorname{acc}_{V}=\prod_{i \in V} g_{n+1-i}$.
AccWitUpdate: This is the algorithm to compute the witness that values are included in an accumulator, using the public parameters. Given $V$ and the accumulator $\operatorname{acc}_{V}$, the witness of values $i_{1}, \ldots, i_{k}$ in $U$ is computed as $W=$ $\prod_{i \in U} \prod_{j \in V}^{j \neq \tilde{i}} h_{n+1-j+i}$.
AccVerify: This is the algorithm to verify that all values in $U$ are included in an accumulator, using the witness and the public parameters. Given $a c c_{V}, U$, and $W$, accept if

$$
\frac{e\left(a c c_{V}, \prod_{i \in U} h_{i}\right)}{e(g, W)}=z^{k} .
$$

Theorem 1 Under the $n$-DHE assumption, any adversary cannot output $(U, V, W)$ where $U \subseteq\{1, \ldots, n\}, V \subseteq\{1, \ldots, n\}$ s.t. AccVerify accepts $U, a c c_{V}, W$ and $U \backslash V \neq \emptyset$.

Proof. Assume an adversary which outputs $(U, V, W)$ s.t. AccVerify accepts $U, a c c_{V}, W$ and $U \backslash V \neq \emptyset$. Let $U_{1}=U \backslash V$ and $U_{2}=U \cap V . U \backslash V \neq \emptyset$ (i.e., $U_{1} \neq \emptyset$ ) implies $\left|U_{2}\right| \neq k$.

Since AccVerify accepts these,

$$
\frac{e\left(a c c_{V}, \prod_{\tilde{i} \in U} h_{\bar{i}}\right)}{e(g, W)}=z^{k}=e\left(g, h_{n+1}\right)^{k}
$$

where $h_{n+1}=h^{\gamma^{n+1}}$. From $\operatorname{acc}_{V}=\prod_{i \in V} g_{n+1-i}$,

$$
\begin{aligned}
& \frac{e\left(\prod_{i \in V} g_{n+1-i}, \prod_{\tilde{i} \in U} h_{i}\right)}{e(g, W)}=e\left(g, h_{n+1}\right)^{k}, \\
& e\left(g, \prod_{\tilde{i} \in U} \prod_{i \in V} h_{n+1-i+i}\right)=e\left(g, W h_{n+1}^{k}\right) .
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
& \prod_{\tilde{i} \in U} \prod_{i \in V} h_{n+1-i+\tilde{\imath}}=W h_{n+1}{ }^{k}, \\
& \prod_{\tilde{i} \in U_{1}} \prod_{i \in V} h_{n+1-i+\tilde{\imath}} \cdot \prod_{\tilde{i} \in U_{2}} \prod_{i \in V} h_{n+1-i+\tilde{l}}=W h_{n+1}{ }^{k}, \\
& \left(\prod_{\tilde{i} \in U_{1}} \prod_{i \in V} h_{n+1-i+\tilde{\imath}}\right) \cdot h_{n+1}{ }^{\left|U_{2}\right|} \prod_{\tilde{i} \in U_{2}} \prod_{i \in V, i \neq \tilde{\imath}} h_{n+1-i+\tilde{\imath}}=W h_{n+1}{ }^{k}, \\
& \prod_{\tilde{i} \in U_{1}} \prod_{i \in V} h_{n+1-i+\tilde{\imath}} \cdot \prod_{\tilde{i} \in U_{2}} \prod_{i \in V, i \neq \tilde{\imath}} h_{n+1-i+\tilde{\imath}}=W h_{n+1}{ }^{k-\left|U_{2}\right|} .
\end{aligned}
$$

We obtain

$$
h_{n+1}=\left(W^{-1} \cdot \prod_{\tilde{i} \in U_{1}} \prod_{i \in V} h_{n+1-i+\tilde{\imath}} \cdot \prod_{\tilde{i} \in U_{2}} \prod_{i \in V, i \neq \tilde{\imath}} h_{n+1-i+\tilde{\imath}}\right)^{1 /\left(k-\left|U_{2}\right|\right)},
$$

where $k-\left|U_{2}\right| \neq 0$ due to $\left|U_{2}\right| \neq k$.

For any $\tilde{\imath} \in U_{1}$ and any $i \in V, h_{n+1-i+\tilde{\imath}} \neq h_{n+1}$, due to $U_{1} \cap V=\phi$. Also, for any $\tilde{\imath} \in U_{2}$ and any $i \in V$ satisfying $i \neq \tilde{l}, h_{n+1-i+\tilde{i}} \neq h_{n+1}$. Therefore, we can compute $h_{n+1}$ given $g_{1}, \ldots, g_{n}, g_{n+2}, \ldots, g_{2 n}, h_{1}, \ldots, h_{n}, h_{n+2}, \ldots, h_{2 n}$, which contradicts $n$-DHE assumption.

### 2.4 Modified BBS+ Signatures

We utilize an extension of BB signature scheme [7], called BBS+ signatures. The extension is informally introduced in Ref. [8] and the concrete construction is shown in Refs. [2], [15]. This scheme allows us to sign a set of messages. Our system requires that the accumulator is signed. In the BBS+ signature, the messages to be signed are set in exponents (elements of $Z_{p}$ ), whereas the accumulator is the product of $g_{i}$ 's from $\mathbb{G}_{1}$. Thus, we modify the BBS+ signature to be able to sign on $g_{i}$ 's, as follows. $\mathbf{m B B S}+$ Setup: Select bilinear groups $\mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}$ with a prime order $p$ and a bilinear map $e$. Select $g, g_{0}, \tilde{g}_{1}, \ldots, \tilde{g}_{L} \in_{R} \mathbb{G}_{1}$. Select $h \in_{R} \mathbb{G}_{2}$. Select $\gamma \in_{R} \quad Z_{p}$ and compute $g_{1}=$ $g^{\gamma^{1}}, \ldots, g_{n}=g^{\gamma^{n}}, g_{n+2}=g^{\gamma^{n+2}}, \ldots, g_{2 n}=g^{\gamma^{2 n}}$.
mBBS+KeyGen: Select $X \quad \epsilon_{R} \quad Z_{p}$ and compute $Y=g^{X}$ and $Z=h^{X}$. The secret key is $X$ and the public key is $\left(p, \mathbb{G}_{1}, \mathbb{G}_{2}, e, g, g_{0}, g_{1}, \ldots, g_{n}, g_{n+2}, \ldots, g_{2 n}\right.$, $\left.\tilde{g}_{1}, \ldots, \tilde{g}_{L}, h, Y, Z\right)$.
mBBS+Sign: Given messages $m_{1}, \ldots, m_{n}, m_{n+2}, \ldots, m_{2 n} \in$ $\{0,1\}, M_{1}, \ldots, M_{L} \in Z_{p}$, select $w, r \in_{R} Z_{p}$ and compute

$$
A=\left(\prod_{1 \leq j \leq 2 n}^{j \neq n+1} g_{j}^{m_{j}} \prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}} g_{0}^{r} g\right)^{1 /(X+w)} .
$$

The signature is $(A, w, r)$.
mBBS+Verify: Given messages $m_{1}, \ldots, m_{n}, m_{n+2}, \ldots, m_{2 n}$, $M_{1}, \ldots, M_{L}$ and the signature $(A, w, r)$, check

$$
e\left(A, Z h^{w}\right)=e\left(\prod_{1 \leq j \leq 2 n}^{j \neq n+1} g_{j}^{m_{j}} \prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}} g_{0}^{r} g, h\right) .
$$

The modified BBS+ signature is unforgeable against adaptively chosen message attack under the $q$-SDH assumption. It is shown in a similar way to Ref. [3], as follows.
$B B$ signatures. Since the security is proved using the security of the underlying $B B$ signatures [7], we briefly show the scheme.
BBSetup: Select bilinear groups $\mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}$ with a prime order $p$ and a bilinear map $e$. Select $g \in_{R} \mathbb{G}_{1}, h \in_{R} \mathbb{G}_{2}$.
BBKeyGen: Select $X \quad \in_{R} \quad Z_{p}$ and compute $Y=g^{X}$ and $Z=h^{X}$. The secret key is $X$ and the public key is $\left(p, \mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}, e, g, h, Y, Z\right)$.
BBSign: Given message $m \in Z_{p}$, compute $B=g^{1 /(X+m)}$. The signature is $B$.
BBVerify: Given message $m$ and the signature $B$, check $e\left(B, Z h^{m}\right)=e(g, h)$.
BB signatures are existentially unforgeable against weak chosen message attack under the $q$-SDH assumption [7]. In this attack, the adversary must choose messages queried for the signing oracle, before the public key is given.

Theorem $2 \mathrm{mBBS}+$ signature is unforgeable against adaptively chosen message attack under the $q$-SDH assumption.
Proof. This proof is derived from Ref. [3].

Assume that $\mathcal{A}$ breaks the unforgeability of mBBS+ signatures, and we construct the following simulator $\mathcal{B}$ breaking BB signatures that are secure under the $q$-SDH assumption.
$\mathcal{B}$ chooses random messages $w_{1}, \ldots, w_{q-1}$ for BB signatures, and is given the corresponding BB signatures $B_{i}=g^{1 /\left(X+w_{i}\right)}$ with the public key $\left(p, \mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}, e, g, h, Y, Z\right)$. Then, $\mathcal{B}$ selects $w^{*}, k^{*}, a^{*} \in_{R} Z_{p}$, and compute $g_{0}=\left(\left(Y g^{w^{*}}\right)^{k^{*}} g^{-1}\right)^{1 / a^{*}}=$ $g^{\left(\left(X+\omega^{*}\right) k^{*}-1\right) / a^{*}}$. Also, $\mathcal{B}$ selects $\gamma, \mu_{1}, \ldots, \mu_{L} \quad \in_{R} \quad Z_{p}$, and compute $g_{1}=g_{0}^{\gamma^{1}}, \ldots, g_{n}=g_{0}^{\gamma^{n}}, g_{n+2}=g_{0}^{\gamma^{n+2}}, \ldots, g_{2 n}=$ $g_{0}^{\gamma^{2 n}}$, and $\tilde{g}_{1}=g_{0}^{\mu_{1}}, \ldots, \tilde{g}_{L}=g_{0}^{\mu_{L}}$. $\mathcal{B}$ sets the public key of mBBS+ signatures $\left(p, \mathbb{G}_{1}, \mathbb{G}_{2}, e, g, g_{0}, g_{1}, \ldots, g_{n}, g_{n+2}, \ldots, g_{2 n}\right.$, $\left.\tilde{g}_{1}, \ldots, \tilde{g}_{L}, h, Y, Z\right)$, and runs $\mathcal{A}$. Out of $q$ signing queries from $\mathcal{A}$, $\mathcal{B}$ randomly selects a query, which called $*$ query. For messages $\left(m_{1, i}, \ldots, m_{n, i}, m_{n+2, i}, \ldots, m_{2 n, i}, M_{1, i}, \ldots, M_{L, i}\right)$ of the $i$-th query, define

$$
t_{i}=\sum_{1 \leq j \leq 2 n}^{j \neq n+1} m_{j, i} \gamma^{j}+\sum_{1 \leq j \leq L} M_{j, i} \mu_{j} .
$$

To the queries except $*, \mathcal{B}$ responds using the BB signature $\left(B_{i}, w_{i}\right)$ as follows. $\mathcal{B}$ selects $r_{i} \in_{R} Z_{p}$, and compute $a_{i}=r_{i}+t_{i}$ and the following $A_{i}$.

$$
\begin{aligned}
& A_{i}=B_{i}^{\left(1-\frac{a_{i}+\left(w_{i}-w^{*}\right) a_{i} k^{*}}{a^{*}}\right)} g^{\frac{a_{i}}{a^{*}} k^{*}} \\
& =B_{i}^{\left(1-\frac{a_{i}}{a^{*}}\right)} g^{\frac{-\left(w_{i}-w^{*}\right) a_{i} k^{*}+i_{i} k^{*}\left(x+w_{i}\right)}{\left(X+w_{i} \alpha^{*}\right.}} \\
& =B_{i}^{\left(1-\frac{a_{i}}{\left.a^{*}\right)}\right.}\left(g^{\frac{a_{i}}{a_{i}}}\right)^{\frac{-w_{i}+w^{*}+X+w_{i}}{X+w_{i}}} \\
& =B_{i} g^{\frac{-a_{i}+a_{i}^{*}+\left(X+w^{*}\right)}{a^{*}\left(X+t_{i}\right)}} \\
& =B_{i} g_{0}^{\frac{a_{i}}{\left.X+w_{i}\right)}}=\left(g g_{0}^{\left.a_{i}\right)^{\frac{1}{X+w_{i}}}}\right.
\end{aligned}
$$

$\mathcal{B}$ returns $\left(A_{i}, w_{i}, r_{i}\right)$.
To the $*$ query, $\mathcal{B}$ sets $r^{*}=a^{*}-t_{i}$, computes $A^{*}=g^{k^{*}}=$ $\left(g g_{0}^{a^{*}}\right)^{1 /\left(X+w^{*}\right)}$ and returns $\left(A^{*}, w^{*}, r^{*}\right)$.
Finally, $\mathcal{A}$ outputs the forged signature $\left(A^{\prime}, w^{\prime}, r^{\prime}\right)$ on message $\left(m_{1}^{\prime}, \ldots, m_{n}^{\prime}, m_{n+2}^{\prime}, \ldots, m_{2 n}^{\prime}, M_{1}^{\prime}, \ldots, M_{L}^{\prime}\right)$. There are three cases. Define

$$
a^{\prime}=r^{\prime}+\sum_{1 \leq j \leq 2 n}^{j \neq n+1} m_{j}^{\prime} \gamma^{j}+\sum_{1 \leq j \leq L} M_{j}^{\prime} \mu_{j} .
$$

- Case I $\left[w^{\prime} \notin\left\{w_{1}, \ldots, w_{q}, w^{*}\right\}\right]: \mathcal{B}$ computes the following $B^{\prime}$.

$$
\begin{aligned}
& B^{\prime}=\left(A^{\prime} g^{\frac{-k^{*}}{a^{*}} a^{\prime}}\right)^{\overline{a^{*}-a^{\prime}-k^{*} a^{\prime}} a^{\prime}\left(w^{\prime}-w^{*}\right)} \\
& =\left(\left(g g^{\frac{\left(X+w^{*}\right) \alpha^{*} a^{\prime}-a^{\prime}}{a^{*}}}\right)^{\frac{1}{X+w^{\prime}}} g^{\frac{-k^{*}}{a^{*}} a^{\prime}}\right) \frac{a^{*}}{a^{*}-a^{\prime}-k^{*} a^{\prime}\left(\omega^{\prime}-w^{*}\right)} \\
& =\left(g^{\frac{a^{*}+\left(X+w^{*}\right) k^{*} a^{\prime}-a^{\prime}-k^{*} a^{\prime}\left(X+w^{\prime}\right)}{a^{*}\left(X+w^{\prime}\right)}}\right)^{\frac{a^{*}-a^{*}}{a^{*}-k^{*} a^{\prime}\left(w^{\prime}-w^{*}\right)}} \\
& =\left(g^{\frac{a^{*}-a^{\prime}-k^{*}\left(a^{\prime}\left(w^{\prime}-w^{*}\right)\right.}{a^{*}\left(X+w^{\prime}\right)}}\right)^{\frac{a^{*}}{a^{*}-a^{\prime}-k^{*} a^{\prime}\left(w^{\prime}-w^{*}\right)}}=g^{\frac{1}{X+w^{\prime}}}
\end{aligned}
$$

This means that a BB signature for a new message $w^{\prime}$ is forged, which contradicts $q$-SDH assumption.

- Case II [ $\left(w^{\prime}=w_{i}\right.$ and $A^{\prime}=A_{i}$ for some $i$ ) or $\left(w^{\prime}=w^{*}\right.$ and $\left.\left.A^{\prime}=A^{*}\right)\right]$ : Consider $w^{\prime}=w_{i}$ and $A^{\prime}=A_{i}$ (The other case is similar). From $A^{\prime X+w^{\prime}}=A_{i}{ }^{X+w_{i}}, g g_{0}^{a^{\prime}}=g g_{0}^{a_{i}}$ holds and we obtain $a^{\prime}=a_{i}$. Thus, letting $\Delta r=r^{\prime}-r_{i}, \Delta m_{j}=m_{j}^{\prime}-m_{j, i}$, and $\Delta M_{j}=M_{j}^{\prime}-M_{j, i}$,

$$
\Delta r+\sum_{1 \leq j \leq 2 n}^{j \neq n+1} \Delta m_{j} \gamma^{j}+\sum_{1 \leq j \leq L} \Delta M_{j} \mu_{j}=0
$$

Some $\Delta m_{j}$ is not 0 or some $\Delta M_{j}$ is not 0 . If $\Delta M_{j} \neq 0$, the above equation means that we can compute $\mu_{j}$ in case that $\mu_{j}=\log _{g_{0}} \tilde{g}_{j}$ is unknown. This contradict the DL assumption and then the $q$-SDH assumption.
If $\Delta m_{j} \neq 0$, we can compute $\gamma^{j} \bmod p$ and thus $\gamma$, given $g_{0}, g_{0}^{\gamma}, \ldots, g_{0}^{\gamma^{n}}, g_{0}^{\gamma^{n+2}}, \ldots, g_{0}^{\gamma^{2 n}}$. This means that, given $g, g^{\gamma}, \ldots, g^{\gamma^{2 n}}$, we can compute $\left(c, g^{1 /(\gamma+c)}\right)$ for any $c \in Z_{p}$, which contradicts the $q$-SDH assumption, where $q=2 n$.

- Case III $\left[w^{\prime} \in\left\{w_{1}, \ldots, w_{q}, w^{*}\right\}\right.$ and $\left.A^{\prime} \notin\left\{A_{1}, \ldots, A_{q}, A^{*}\right\}\right]$ : With the probability $1 / q, w^{\prime}=w^{*}$. Then, from

$$
A^{\prime}=\left(g g_{0}^{a^{\prime}}\right)^{1 /\left(X+w^{*}\right)}=g^{\left(a^{*}+a^{\prime}\left(X+w^{*}\right) k^{*}-a^{\prime}\right) /\left(a^{*}\left(X+w^{*}\right)\right),}
$$

compute the following $B^{\prime}$.

$$
\begin{aligned}
B^{\prime} & =\left(A^{\prime} g^{\frac{-k^{*} a^{\prime}}{a^{\prime}}}\right) \\
& =\left(g^{\frac{a^{*}+a^{\prime}}{a^{*}\left(a^{*}+w^{*}\right.}}\right) \frac{a^{a^{*}} a^{a^{*}}}{a^{*}-a^{\prime}} \\
& =g^{\frac{1}{X+w^{*}}}
\end{aligned}
$$

This means that a BB signature for a new message $w^{*}$ is forged, which contradicts $q$-SDH assumption.
The security proof assumes that valid $g_{j}$ 's are signed, instead of any element from $\mathbb{G}_{1}$. Thus, for proving the knowledge of this signature, we have to ensure the correctness of $g_{j}$ 's by other technique, the following F-secure BB signatures.

## 2.5 $\quad \boldsymbol{F}$-secure BB Signatures

We also adopt another variant of BB signature scheme, called $F$-secure signature [5].
$\boldsymbol{F B B S e t u p : ~ S e l e c t ~ b i l i n e a r ~ g r o u p s ~} \mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}$ with a prime order $p$ and a bilinear map $e$. Select $g \in_{R} \mathbb{G}_{1}$ and $h, \hat{h} \in_{R} \mathbb{G}_{2}$.
FBBKeyGen: Select $\tilde{X}, \hat{X} \in_{R} Z_{p}$ and compute $\tilde{Y}=g^{\tilde{X}}, \hat{Y}=g^{\hat{X}}$, $\tilde{Z}=h^{\tilde{x}}, \hat{Z}=h^{\hat{X}}$. The secret key is ( $\left.\tilde{X}, \hat{X}\right)$ and the public key is $\left(p, \mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}, e, g, h, \hat{h}, \tilde{Y}, \hat{Y}, \tilde{Z}, \hat{Z}\right)$.
The public key can be checked by $e(\tilde{Y}, h)=e(g, \tilde{Z})$ and $e(\hat{Y}, h)=e(g, \hat{Z})$.
FBBSign: Given message $M \in Z_{p}$, select $\mu \in_{R} Z_{p}-\left\{\frac{\tilde{X}-M}{\hat{X}}\right\}$ and compute $S=h^{1 /\left(\tilde{X}+M+\chi_{\mu}\right)}, T=\hat{Y}^{\mu}, U=\hat{h}^{\mu}$. The signature is ( $S, T, U$ ).
FBBVerify: Given the signature $(S, T, U)$ on message $M$, check $e\left(\tilde{Y} g^{M} T, S\right)=e(g, h)$ and $e(T, \hat{h})=e(\hat{Y}, U)$.
Define bijection $F$ as $F(M)=\left(g^{M}, \hat{h}^{M}\right)$ for message $M$. The correctness of $F(M)$ can be checked by $e\left(g^{M}, \hat{h}\right)=e\left(g, \hat{h}^{M}\right)$. The $F$-security means that no adversary can output $(F(M), \sigma)$ where $\sigma$ is the signature on message $M$ s.t. he has never previously obtained the signature after his adaptive chosen message attacks. The security is proved under the $q$-HSDH and $q$-TDH assumptions [5].

### 2.6 Proving Relations on Representations

We adopt zero-knowledge proofs of knowledge ( $P K \mathrm{~s}$ ) on representations, which are the generalization of the Schnorr identification protocol [11]. Concretely we utilize a $P K$ proving the knowledge of a representation of $C \in \mathbb{G}_{1}$ to the bases $g_{1}, g_{2}, \ldots, g_{t} \in \mathbb{G}_{1}$, i.e., $x_{1}, \ldots, x_{t}$ s.t. $C=g_{1}^{x_{1}} \cdots g_{t}^{x_{t}}$. This can be also constructed on groups $\mathbb{G}_{2}$ and $\mathbb{G}_{T}$. The $P K$ can be extended to proving multiple representations with equal parts.

Since we use only prime-order groups, we can extract the proved secret knowledge given two accepting protocol views whose commitments are the same and whose challenges are different.

## 3. Proposed System

### 3.1 Construction Idea

As in Ref. [10], we categorize finite-set attributes and string attributes. In the finite-set attributes, the values are binary or from a pre-defined finite set, for example, gender, degree, nationality, etc. On the other hand, name and identification number are the string attributes.

Our proposal is based on the pairing-based anonymous credential system using the BBS+ signatures, which is described in Ref. [12] for example. In the underlying system, the certificate is a BBS+ signature [8], where each attribute type is expressed as an exponent on a base assigned to the attribute type, such as $\tilde{g}_{j}{ }_{j}$, and all parts of $\tilde{g}_{j}^{M_{j}}$ have to be signed. Namely, the certificate is $(A, w, r)$ s.t.

$$
A=\left(\prod_{1 \leq j \leq L^{\prime}} \tilde{g}_{j}^{M_{j}} \tilde{g}_{L^{\prime}+1}^{x} g_{0}^{r} g\right)^{1 /(X+w)}
$$

where $x$ is a secret identity that only the user with the certificate knows. Then, proving the knowledge of the signature needs the cost depending on the number of attribute types.
To express the finite-set attributes (For the string type, we still use the exponent), we use a pairing-based accumulator in Ref. [12]. Let all attribute values in all finite-set attribute types be numbered. The $j$-th attribute value is assigned to an input value $g_{j}$ 's in the accumulator. The multiple inputs (i.e., attribute values) are accumulated into a single value. When $V$ is the set of indexes of the attribute values for a user, they are accumulated to $a c c_{V}=\prod_{j \in V} g_{n+1-j}$. We consider that the accumulated value is signed by a modified BBS+ signature,

$$
A=\left(a c c_{V} \cdot \prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}} \tilde{g}_{L+1}^{x} g_{0}^{r} g\right)^{1 /(X+w)}
$$

where the original representation $\tilde{g}_{j}^{M_{j}}$ is still used for the string type.

However, in the $P K$ of the modified BBS+ signature, $a c c_{V}$ is committed for secrecy. That is, the validity of the committed value (i.e., it is the form of $a c c_{V}$ ) is unknown to the verifier. The $P K$ for representations only proves the form of $A=$ $\left(R \cdot \prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}} \tilde{g}_{L+1}^{x} g_{0}^{r} g\right)^{1 /(X+w)}$, for some $R \in \mathbb{G}_{1}$. However, the security proof of the modified BBS+ signatures assumes that the message is the product of $g_{j}$ 's, i.e., $\prod_{1 \leq j \leq 2 n}^{j \neq n+1} g_{j}^{m_{j}}$. For example, we can show the following forge by manipulating the value of $\operatorname{acc}_{V}$ :

$$
\operatorname{acc}_{V}=\prod_{1 \leq j \leq 2 n}^{j \neq n+1} g_{j}^{m_{j}} \cdot\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{-M_{j}}\right) \tilde{g}_{L+1}^{-x} \cdot g_{0}^{-r} g^{-1} Y g^{w}, \quad A=g .
$$

It is unknown whether this forge is meaningful or not. However, we cannot prove the security of our protocols, if the validity of $\operatorname{acc}_{V}$ is unknown and the modified BBS+ signature is forgeable. Thus, we add another signature on $a c c_{V}$ by signing the exponent $\sum_{j \in V} \gamma^{n+1-j}$. This approach is also used in Ref. [12] to ensure
the $g_{j}$ in the membership certificate. They use a weakly secure BB signature [7], based on interactive HSDH assumption [4] or HSDHE assumption [12]. We consider that it is a rather strong assumption. This is why we use the $F$-secure BB signature [5] derived from fully secure BB signature, based on the better assumptions (HSDH assumption and TDH assumption).
AND relation. For AND relation $\left(a_{1} \wedge \cdots \wedge a_{k}\right)$, it is needed to prove that a specified set of attributes $\left(a_{1}, \ldots, a_{k}\right)$ are all embedded into the user's certificate. Using AccVerify in the extended accumulator, we can prove that multiple values are accumulated to the accumulator in the certificate with constant complexity. By the similar way to Ref. [12], we can obtain the $P K$ of AccVerify with constant complexity.
OR relation. For OR relation $\left(a_{1} \vee \cdots \vee a_{k}\right)$, it is needed to prove that one (denoted as $\tilde{a})$ of a specified set of attributes $\left(a_{1}, \ldots, a_{k}\right)$ is embedded into the user's certificate. Similarly to AND relation, using AccVerify, a signer can prove that a value $\tilde{a}$ is accumulated to the accumulator in the certificate. Furthermore, the verifier prepares another accumulator acc ${ }^{\prime}$ from specified attributes $a_{1}, \ldots, a_{k}$. Then, the signer proves that the same value $\tilde{a}$ is accumulated to the additional accumulator $a c c^{\prime}$.

### 3.2 Proposed Construction

The following construction uses the asymmetric pairings defined in Section 2.1. Our implementation shown later is based on a library of asymmetric pairings. By letting $\mathbb{G}_{1}=\mathbb{G}_{2}$, our construction can be also implemented by symmetric pairings.

### 3.2.1 Setup

The inputs of this algorithm are $\ell, n$, and $L$, where $\ell$ is the security parameter, $n$ is the maximum number of finite-set attribute values, and $L$ is the maximum number of string attribute types. The outputs are issuer's public key $i p k$ and issuer's secret key isk.
(1) Select bilinear groups $\mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}$ with the same order $p$ with length $\ell$ and the bilinear map $e$.
(2) Select $g, g_{0}, \hat{g}, \tilde{g}_{1}, \ldots, \tilde{g}_{L+1} \in_{R} \mathbb{G}_{1}$ and $h, \hat{h}, \tilde{h} \in_{R} \mathbb{G}_{2}$. Select $X, \tilde{X}, \hat{X}, \tilde{X}^{\prime}, \hat{X}^{\prime}, \gamma \in_{R} Z_{p}^{*}$, compute $Y=g^{X}, Z=h^{X}, \tilde{Y}=g^{\tilde{X}}$, $\hat{Y}=g^{\hat{X}}, \tilde{Z}=h^{\tilde{X}}, \hat{Z}=h^{\hat{X}}, \tilde{Y}^{\prime}=g^{\tilde{X}^{\prime}}$ and $\hat{Y}^{\prime}=g^{\hat{X}^{\prime}}, \tilde{Z^{\prime}}=h^{\tilde{X}^{\prime}}$ and $\hat{Z}^{\prime}=h^{\hat{X}^{\prime}}$. Compute $g_{1}=g^{\gamma^{1}}, \ldots, g_{n}=g^{\gamma^{n}}, g_{n+2}=$ $g^{\gamma^{n+2}}, \ldots, g_{2 n}=g^{\gamma^{2 n}}, h_{1}=h^{\gamma^{1}}, \ldots, h_{n}=h^{\gamma^{n}}, h_{n+2}=$ $h^{\gamma^{n+2}}, \ldots, h_{2 n}=h^{\gamma^{2 n}}$, and $z=e(g, h)^{\gamma^{n+1}}$. Select hash function $H:\{0,1\}^{*} \rightarrow Z_{p}$.
(3) For $1 \leq j \leq n$, select $\mu_{j} \in_{R} Z_{p}-\left\{\frac{\tilde{X}^{\prime}-\gamma^{j}}{X^{\prime}}\right\}$ and compute the $F$-secure BB signature on $\gamma^{j}\left(F\left(\gamma^{j}\right)=\left(g^{\gamma^{j}}, h^{\gamma^{j}}\right)\right)$ as follows:

$$
\tilde{S}_{j}=\tilde{h}^{1 /\left(\tilde{X}^{\prime}+\gamma^{j}+\mu_{j} \hat{X}^{\prime}\right)}, \quad \tilde{T}_{j}=\hat{Y}^{\mu_{j}}, \quad \tilde{U}_{j}=h^{\mu_{j}} .
$$

(4) Output the issuer public key ipk $=\left(p, \mathbb{G}_{1}, \mathbb{G}_{2}, \mathbb{G}_{T}\right.$, $e, H, g, \hat{g}, h, \hat{h}, \tilde{h}, g_{0}, g_{1}, \ldots, g_{n}, g_{n+2}, \ldots, g_{2 n}, h_{1}, \ldots, h_{n}$, $h_{n+2}, \ldots, h_{2 n}, \tilde{g}_{1}, \ldots, \tilde{g}_{L+1}, z,\left(\tilde{S}_{1}, \tilde{T}_{1}, \tilde{U}_{1}\right), \ldots,\left(\tilde{S}_{n}, \tilde{T}_{n}, \tilde{U}_{n}\right)$, $\left.Y, Z, \tilde{Y}, \hat{Y}, \tilde{Z}, \hat{Z}, \tilde{Y}^{\prime}, \hat{Y}^{\prime}, \tilde{Z}^{\prime}, \hat{Z}^{\prime}\right)$, and the issuer secret key $i s k=\left(X, \tilde{X}, \hat{X}, \tilde{X}^{\prime}, \hat{X}^{\prime}, \gamma\right)$.

### 3.2.2 Issuing Certificate

This is an interactive protocol between the issuer Issuer and user User. The common inputs of this protocol consist of ipk, and (SA, FA) that are sets of string attribute values and finiteset attribute values of the user, respectively. Each string attribute
value of the $j$-th attribute type in SA is represented by an element $M_{j}$ from $Z_{p}^{*}$ (If the user does not have any attribute value in the attribute type, we assign an attribute value implying not applicable). Each finite-set attribute value is represented by an index in $\{1, \ldots, n\}$. Thus, set SA consists of attribute values and set FA consists of indexes of attribute values (sets TSA and TFA shown later are similar). The input of Issuer is isk. The output of User is the certificate cert.
(1) [User] Select $x, r^{\prime} \in_{R} Z_{p}^{*}$. Compute $A^{\prime}=\tilde{g}_{L+1}^{x} g_{0}^{r^{\prime}}$. Send $A^{\prime}$ to Issuer. In addition, prove the validity of $A^{\prime}$ using $P K$ for representations, i.e., prove the knowledge of $x, r^{\prime}$ s.t. $A^{\prime}=\tilde{g}_{L+1}^{x} g_{0}^{r^{\prime}}$.
(2) [Issuer] Given the user's attributes (SA, FA), compute the accumulator of the finite-set attributes as acc $=\prod_{a \in \mathrm{FA}}$ $g_{n+1-a}$. Select $w, r^{\prime \prime} \in_{R} Z_{p}^{*}$. Compute the modified BBS+ signature as follows:

$$
\begin{aligned}
A & =\left(\operatorname{acc}\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}}\right) A^{\prime} g_{0}^{r^{\prime \prime}} g\right)^{1 /(X+w)} \\
& =\left(\operatorname{acc}\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}}\right) \tilde{g}_{L+1}^{x} g_{0}^{r^{\prime}+r^{\prime \prime}} g\right)^{1 /(X+w)} .
\end{aligned}
$$

In addition, select $\mu \in_{R} Z_{p}-\left\{\frac{\tilde{X}-\sum_{a \in \mathbb{F A}} \gamma^{n+1-a}}{\chi}\right\}$ and compute an $F$-secure BB signature ensuring acc as follows:

$$
\begin{aligned}
& S=h^{1 /\left(\tilde{X}+\sum_{a \in \mathrm{EA}} \gamma^{n+1-a}+\mu \hat{X}\right)}, \quad T=\hat{Y}^{\mu}, \quad U=\tilde{h}^{\mu}, \\
& F=\tilde{h}^{\sum_{a \in \mathrm{EA}} \gamma^{n+1-a}} .
\end{aligned}
$$

Return ( $A, S, T, U, F, w, r^{\prime \prime}$ ) to User.
(3) [User] Compute $r=r^{\prime}+r^{\prime \prime}$, verify:

$$
\begin{aligned}
& e\left(A, Z h^{w}\right) \stackrel{?}{=} e\left(a c c\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}}\right) \tilde{g}_{L+1}^{x} g_{0}^{r} g, h\right) \\
& \wedge e(\tilde{Y} \cdot a c c \cdot T, S) \stackrel{?}{=} e(g, h) \wedge e(T, \tilde{h}) \\
& \stackrel{?}{=} e(\hat{Y}, U) \wedge e(a c c, \tilde{h}) \stackrel{?}{=} e(g, F) .
\end{aligned}
$$

Output cert $=(A, S, T, U, F, x, w, r)$.

### 3.2.3 Attribute Proofs

This is a protocol between the user and the verifier. The common inputs are $i p k$, and (TSA, TFA) are subsets of string attributes and finite-set attributes respectively, which are referenced in proofs, and user's secret inputs are cert and (SA, FA).

## Proving AND Relation.

For TFA $=\left\{a_{1}, \ldots, a_{k}\right\}$ with $a_{j} \in\{1, \ldots, n\}$, the user shows his possession of the certificate which includes all of the attributes, i.e., $a_{1} \wedge a_{2} \wedge \ldots \wedge a_{k}$.
(1) The user computes the witness that $a_{1}, \ldots, a_{k}$ are included in the accumulator of FA as: $W=\prod_{1 \leq j \leq k}\left(\prod_{a \in \mathrm{FA}}^{a \neq a_{j}} h_{n+1-a+a_{j}}\right)$. Set $D=\prod_{1 \leq j \leq k} h_{a_{j}}$.
(2) The user selects $\rho_{A}, \rho_{S}, \rho_{T}, \rho_{U}, \rho_{F}, \rho_{a}, \rho_{W} \in_{R} Z_{p}^{*}$, and compute commitments $C_{A}=A \hat{g}^{\rho_{A}}, C_{S}=S \hat{h}^{\rho_{S}}, C_{T}=T \hat{g}^{\rho_{T}}$, $C_{U}=U \hat{h}^{\rho_{U}}, C_{F}=F \hat{h}^{\rho_{F}}, C_{a}=a c c \cdot \hat{g}^{\rho_{a}}$, and $C_{W}=W \hat{h}^{\rho_{W}}$.
(3) The user selects $\rho_{w}, \rho^{\prime} \in_{R} Z_{p}^{*}$, sets $\alpha=w \rho_{A}, \zeta=\rho_{S} \rho_{a}$ and $\xi=\rho_{S} \rho_{T}$. The user computes auxiliary commitments $C_{w}=h^{w} \hat{h}^{\rho_{w}}$ and $C_{\rho_{S}}=h^{\rho_{s}} \hat{h}^{\rho^{\prime}}$. Then, the user sets $\rho_{\alpha}=\rho_{w} \rho_{A}$, $\rho_{\zeta}=\rho^{\prime} \rho_{a}$, and $\rho_{\xi}=\rho^{\prime} \rho_{T}$.
(4) The user sends the commitments $\left(C_{A}, C_{S}, C_{T}, C_{U}, C_{F}, C_{a}, C_{W}, C_{w}, C_{\rho_{s}}\right)$ to the verifier.
(5) By using the proof of knowledge ( $P K$ ) for representations, the user proves the knowledge of $x, w, r, \rho_{A}, \rho_{S}, \rho_{T}, \rho_{U}, \rho_{F}, \rho_{a}, \rho_{W}, \rho_{w}, \rho^{\prime}, \alpha, \zeta, \xi, \rho_{\alpha}, \rho_{\zeta}, \rho_{\xi}$, and $M_{j}$ for $M_{j} \notin$ TSA s.t.

$$
\begin{align*}
& C_{w}=h^{w} \hat{h}^{\rho_{w}}, 1=C_{w}^{\rho_{A}} h^{-\alpha} \hat{h}^{-\rho_{a}},  \tag{1}\\
& e\left(C_{A}, Y\right) e\left(C_{a} \prod_{1 \leq j \leq L, M} \in \tilde{M}_{j} \in \mathrm{TSA}\right. \\
& \left.\left.\quad \tilde{g}_{j}^{M_{j}}\right) g, h\right)^{-1} \\
& \left.\quad \prod_{1 \leq j \leq L, M_{j} \notin \mathrm{TSA}} e\left(\tilde{g}_{j}, h\right)^{M_{j}}\right) e\left(\tilde{g}_{L+1}, h\right)^{x} e\left(g_{0}, h\right)^{r}  \tag{2}\\
& \quad e(\hat{g}, Y)^{\rho_{A}} e(\hat{g}, h)^{\alpha} e\left(C_{A}, h\right)^{-w} e(\hat{g}, h)^{-\rho_{a}},  \tag{3}\\
& C_{\rho_{S}}=h^{\rho_{S}} \hat{h}^{\rho^{\prime}}, 1=C_{\rho_{S}}^{\rho_{a}} h^{-\zeta} \hat{h}^{-\rho_{\zeta}}, 1=C_{\rho_{S}}^{\rho_{T}} h^{-\xi} \hat{h}^{-\rho_{\xi}}, \\
& e\left(\tilde{Y} C_{a} C_{T}, C_{S}\right) e(g, h)^{-1}  \tag{4}\\
& \quad=e\left(\tilde{Y} C_{a} C_{T}, \hat{h}\right)^{\rho_{S}} e\left(\hat{g}, C_{S}\right)^{\rho_{a}+\rho_{T}} e(\hat{g}, \hat{h})^{-\zeta-\xi},  \tag{5}\\
& e\left(C_{T}, \tilde{h}\right) e\left(\hat{Y}, C_{U}\right)^{-1}=e(\hat{g}, \tilde{h})^{\rho_{T}} e(\hat{Y}, \hat{h})^{-\rho_{U}},  \tag{6}\\
& e\left(C_{a}, \tilde{h}\right) e\left(g, C_{F}\right)^{-1}=e(\hat{g}, \tilde{h})^{\rho_{a}} e(g, \hat{h})^{-\rho_{F}},  \tag{7}\\
& e\left(C_{a}, D\right) e\left(g, C_{W}\right)^{-1} z^{-k}=e(\hat{g}, D)^{\rho_{a}} e(g, \hat{h})^{-\rho_{W}} .
\end{align*}
$$

## Proving OR Relation.

For TFA $=\left\{a_{1}, \ldots, a_{k}\right\}$, the user shows his possession of the certificate which includes one of the attributes, i.e., $a_{1} \vee a_{2} \vee \ldots \vee$ $a_{k}$. Assume that $\tilde{a}$ is the proved attribute.

Before the protocol, the user and the verifier prepare another accumulator acce $^{\prime}=\prod_{a_{j} \in \text { TFA }} g_{n+1-a_{j}}$. This protocol is obtained by modifying the protocol of the AND relation, as follows.
(1) Similarly, the user computes $W=\prod_{a \in \mathrm{FA}}^{a \neq \tilde{a}} h_{n+1-a+\tilde{a}}$ for acc. Furthermore, the user computes the new witness $W^{\prime}=$ $\prod_{a_{j} \in \mathrm{TFA}}^{a_{j} \neq \tilde{a}} h_{n+1-a_{j}+\tilde{a}}$ for $a c c^{\prime}$.
(2) In addition to step 2 in AND relation, the user selects $\rho_{g}, \rho_{W^{\prime}}, \rho_{\tilde{S}}, \rho_{\tilde{T}}, \rho_{\tilde{U}}, \rho_{h} \in_{R} \mathbb{Z}_{p}^{*}$, and compute the new commitment $C_{g}=g_{\tilde{a}} \hat{g}^{\rho_{g}}, C_{W^{\prime}}=W^{\prime} \hat{h}^{\rho_{w^{\prime}}}, C_{\tilde{S}}=\tilde{S}_{\tilde{a}} \hat{h}^{\rho_{\tilde{s}}}, C_{\tilde{T}}=\tilde{T}_{\tilde{a}} \hat{g}^{\rho_{\tilde{T}}}$, $C_{\tilde{U}}=\tilde{U}_{\tilde{a}} \hat{h}^{\rho_{\tilde{U}}}$, and $C_{h}=h_{\hat{a}} \hat{h}^{\rho_{h}}$.
(3) In addition to step 3 in AND relation, the user selects $\tilde{\rho}, \tilde{\rho^{\prime}} \epsilon_{R}$ $Z_{p}^{*}$, sets $\delta=\rho_{h} \rho_{a}, \tilde{\zeta}=\rho_{\tilde{S}} \rho_{g}$ and $\tilde{\xi}=\rho_{\tilde{S}} \rho_{\tilde{T}}$. The user computes auxiliary commitments $C_{\rho_{h}}=h^{\rho^{\mu}} \hat{h^{\tilde{\rho}}}$ and $C_{\rho_{\tilde{S}}}=h^{\rho_{\bar{s}}} \hat{h}^{\rho^{\prime}}$. Then, the user sets $\rho_{\delta}=\tilde{\rho} \rho_{a}, \rho_{\tilde{\xi}}=\tilde{\rho}^{\prime} \rho_{g}$, and $\rho_{\tilde{\xi}}=\tilde{\rho}^{\prime} \rho_{\tilde{T}}$.
(4) The user sends the commitments $\left(C_{A}, C_{S}, C_{T}, C_{U}, C_{F}, C_{g}, C_{a}, C_{W}, C_{W^{\prime}}, \quad C_{\tilde{S}}, C_{\tilde{T}}, C_{\tilde{U}}, C_{h}\right.$, $C_{w}, C_{\rho_{s}}, C_{\rho_{h}}, C_{\rho_{s}}$ ) to the verifier.
(5) Similarly to the AND relation, the user conducts the PK, where Eq. (7) is replaced by

$$
\begin{align*}
& C_{\rho_{h}}=g^{\rho_{h}} \hat{h}^{\tilde{\rho}}, 1=C_{\rho_{h}}{ }^{\rho_{a}} g^{-\delta} \hat{h}^{-\rho_{\delta}},  \tag{8}\\
& e\left(C_{a}, C_{h}\right) e\left(g, C_{W}\right)^{-1} z^{-1} \\
& \quad=e\left(\hat{g}, C_{h}\right)^{\rho_{a}} e\left(C_{a}, \hat{h}\right)^{\rho_{h}} e(\hat{g}, \hat{h})^{-\delta} e(g, \hat{h})^{-\rho_{W}}, \tag{9}
\end{align*}
$$

and the following equations are added:

$$
\begin{align*}
& C_{\rho_{\tilde{\tilde{}}}}=h^{\rho_{\tilde{S}}} \hat{h}^{\tilde{\rho}^{\prime}}, 1=C_{\rho_{\tilde{s}}}^{\rho_{g}} h^{-\tilde{\zeta}} \hat{h}^{-\rho_{\tilde{\zeta}}}, 1=C_{\rho_{\tilde{S}}}^{\rho_{\tilde{T}}} h^{-\tilde{\xi}} \hat{h}^{-\rho_{\tilde{\xi}}},  \tag{10}\\
& e\left(\tilde{Y}^{\prime} C_{g} C_{\tilde{T}}, C_{\tilde{S}}\right) e(g, \tilde{h})^{-1} \\
& \quad=e\left(\tilde{Y}^{\prime} C_{g} C_{\tilde{T}}, \hat{h}\right)^{\rho_{\tilde{S}}} e\left(\hat{g}, C_{\tilde{S}}\right)^{\rho_{g}+\rho_{\tilde{T}}} e(\hat{g}, \hat{h})^{-\tilde{\zeta}-\tilde{\xi}},  \tag{11}\\
& e\left(C_{\tilde{T}}, h\right) e\left(\hat{Y}^{\prime}, C_{\tilde{U}}\right)^{-1}=e(\hat{g}, h)^{\rho_{\tilde{T}}} e\left(\hat{Y}^{\prime}, \hat{h}\right)^{-\rho_{\tilde{U}}}, \tag{12}
\end{align*}
$$

$$
\begin{align*}
& e\left(C_{g}, h\right) e\left(g, C_{h}\right)^{-1}=e(\hat{g}, h)^{\rho_{g}} e(g, \hat{h})^{-\rho_{h}},  \tag{13}\\
& e\left(a c c^{\prime}, C_{h}\right) e\left(g, C_{W^{\prime}}\right)^{-1} z^{-1}=e\left(a c c^{\prime}, \hat{h}\right)^{\rho_{h}} e(\hat{g}, \hat{h})^{-\rho_{W^{\prime}}} . \tag{14}
\end{align*}
$$

## 4. Security

Here, we show that the proposed protocols are the $P K \mathrm{~s}$ for AND and OR relations on the finite-set attributes. The security on the string attributes can be proved in the similar way to the underlying protocols.
Theorem 3 The protocol of AND relation is a proof of knowledge of a modified BBS+ signature $(A, w, r)$ on secret $x$, the string type of attributes $M_{1}, \ldots, M_{L}$, and the finite-set type of attributes indicated by accumulator $a c c$, s.t. all attributes in TFA are accumulated to $a c c$.
Proof. From the $P K$, we have an extractor of knowledge satisfying Eqs. (1)-(7). Using Eq. (1), we obtain $1=\left(h^{w} \hat{h}_{\omega}\right)^{\rho_{A}} h^{-\alpha} \hat{h}^{-\rho_{\alpha}}$, and thus $1=h^{w \rho_{A}-\alpha} \hat{h}^{\rho_{w} \rho_{A}-\rho_{\alpha}}$. Since the discrete $\log$ of $\hat{h}$ to base $h$ is unknown under the DL assumption (due to $q$-SDH assumption), this means $\alpha=w \rho_{A}$. By substituting this to Eq. (2), we have

$$
\begin{aligned}
& \left.e\left(C_{A}, Y\right) e\left(C_{a} \prod_{1 \leq j \leq L, M_{j} \in \mathrm{TSA}} \tilde{g}_{j}^{M_{j}}\right) g, h\right)^{-1} \\
& \quad=\left(\prod_{1 \leq j \leq L, M j \notin \mathrm{TSA}} e\left(\tilde{g}_{j}, h\right)^{M_{j}}\right) e\left(\tilde{g}_{L+1}, h\right)^{x} e\left(g_{0}, h\right)^{r} e(\hat{g}, Y)^{\rho_{A}} \\
& \quad \cdot e(\hat{g}, h)^{w \rho_{A}} e\left(C_{A}, h\right)^{-w} e(\hat{g}, h)^{-\rho_{a}} \\
& e\left(C_{A}, Y\right) e\left(\hat{g}^{-\rho_{A}}, Y\right) e\left(\hat{g}^{-\rho_{A}}, h^{w}\right) e\left(C_{A}, h^{w}\right) \\
& \quad=e\left(C_{a}\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}}\right) g, h\right) e\left(\tilde{g}_{L+1}^{x}, h\right) e\left(g_{0}^{r}, h\right) e\left(\hat{g}^{-\rho_{a}}, h\right) \\
& e\left(C_{A} \hat{g}^{-\rho_{A}}, Y h^{w}\right)=e\left(C_{a} \hat{g}^{-\rho_{a}}\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}}\right) \tilde{g}_{L+1}^{x} g_{0}^{r} g, h\right)
\end{aligned}
$$

Thus, we can extract $A=C_{A} \hat{g}^{-\rho_{A}}$ and acc $=C_{a} \hat{g}^{-\rho_{a}}$ s.t.

$$
e\left(A, Y h^{w}\right)=e\left(\operatorname{acc}\left(\prod_{1 \leq j \leq L} \tilde{g}_{j}^{M_{j}}\right) \tilde{g}_{L+1}^{x} g_{0}^{r} g, h\right)
$$

Similarly, using Eq. (3), we have $\zeta=\rho_{S} \rho_{a}$ and $\xi=\rho_{S} \rho_{T}$. By substituting them to Eq. (4), we have

$$
\begin{aligned}
& e\left(\tilde{Y} C_{a} C_{T}, C_{S}\right) e(g, h)^{-1} \\
& \quad=e\left(\tilde{Y} C_{a} C_{T}, \hat{h}\right)^{\rho_{S}} e\left(\hat{g}, C_{S}\right)^{\rho_{a}+\rho_{T}} e(\hat{g}, \hat{h})^{-\rho_{S} \cdot \rho_{a}-\rho_{S} \cdot \rho_{T}} \\
& e\left(\tilde{Y} C_{a} C_{T}, C_{S}\right) e\left(\tilde{Y} C_{a} C_{T}, \hat{h}^{-\rho_{S}}\right) e\left(\hat{g}^{-\rho_{a}-\rho_{T}}, C_{S}\right) e\left(\hat{g}^{-\rho_{a}-\rho_{T}}, \hat{h}^{-\rho_{S}}\right) \\
& \quad=e(g, h) \\
& e\left(\tilde{Y} C_{a} \hat{g}^{-\rho_{a}} C_{T} \hat{g}^{-\rho_{T}}, C_{S} \hat{h}^{-\rho_{S}}\right)=e(g, h)
\end{aligned}
$$

Thus, for the extracted $a c c=C_{a} \hat{g}^{-\rho_{a}}$, we can extract $S=C_{S} \hat{h}^{-\rho_{S}}$ and $T=C_{T} \hat{g}^{-\rho_{T}}$ s.t. $e(\tilde{Y} \cdot \operatorname{acc} \cdot T, S)=e(g, h)$. Similarly, using Eqs. (5), (6), we obtain $U=C_{U} \hat{h}^{-\rho_{U}}$ and $F=C_{F} \hat{h}^{-\rho_{F}}$ s.t. $e(T, \tilde{h})=e(\hat{Y}, U)$ and $e(a c c, \tilde{h})=e(g, F)$. Since $F$-secure BB signatures w.r.t. the public key $\tilde{Y}, \hat{Y}$ is issued on only accumulators, it means $a c c=\prod_{a \in \mathrm{FA}} g_{n+1-a}$ for FA of a user (otherwise, the signature is forgeable).

On the other hand, using Eq. (7), we can similarly extract $W=$ $C_{W} \hat{h}^{-\rho_{W}}$ s.t. $e(a c c, D) e(g, W)^{-1}=z^{k}$ for $D=\prod_{1 \leq j \leq k} h_{a_{j}}$. From
the security of the extended accumulator, all values $a_{1}, \ldots, a_{k}$ are accumulated into acc.

Theorem 4 The protocol of OR relation is a proof of knowledge of a modified BBS+ signature $(A, w, r)$ on secret $x$, the string type of attributes $M_{1}, \ldots, M_{L}$, and the finite-set type of attributes indicated by accumulator acc, s.t. one of attributes in TFA is accumulated to acc.
Proof. From the $P K$, we have an extractor of knowledge satisfying the equations. Similarly to AND relation, we can extract a modified $\mathrm{BBS}+$ signature $(A, w, r)$ as the certificate including $a c c=\prod_{a \in \mathrm{FA}} g_{n+1-a}$.

Similarly to the extraction of $F$-secure BB signature in the AND relation, using Eqs. (10)-(13), we can extract the $F$-secure BB signature $(\tilde{S}, \tilde{T}, \tilde{U})$ on $\tilde{G}=C_{g} \hat{g}^{-\rho_{g}}$ and $\tilde{F}=C_{h} \hat{g}^{-\rho_{h}}$ s.t. $e\left(\tilde{Y^{\prime}} \tilde{G} \tilde{T}, \tilde{S}\right)=e(g, \tilde{h}), e(\tilde{T}, h)=e\left(\hat{Y}^{\prime}, \tilde{U}\right)$ and $e(\tilde{G}, h)=e(g, \tilde{F})$. Since $F$-secure BB signatures w.r.t. the public key $\tilde{Y}^{\prime}, \hat{Y}^{\prime}$ is issued on only $\left(g_{j}=g^{\gamma^{j}}, h_{j}=h^{\gamma^{j}}\right)$ 's, it means $\tilde{G}=g_{\tilde{a}}$ and $\tilde{F}=h_{\tilde{a}}$ (otherwise, the signature is forgeable).

Using Eq. (8), we can obtain $\delta=\rho_{a} \rho_{h}$. By substituting this into Eq. (9), we can extract $W=C_{W} \hat{g}^{-\rho_{W}}$ s.t. $e\left(\right.$ acc, $\left.h_{\tilde{a}}\right) e(g, W)^{-1}=z$ for the extracted $\left(g_{\tilde{a}}, h_{\tilde{a}}\right)$. This means that attribute $\tilde{a}$ is accumulated into acc. Using Eq. (14), we can extract $W^{\prime}=C_{W^{\prime}} \hat{g}^{-\rho_{W^{\prime}}}$ s.t. $e\left(a c c^{\prime}, h_{\tilde{a}}\right) e\left(g, W^{\prime}\right)^{-1}=z$ for $\left(g_{\tilde{a}}, h_{\tilde{a}}\right)$. This means that attribute $\tilde{a}$ is also accumulated into $a c c^{\prime}$, that is, attribute $\tilde{a}$ is one of attributes $a_{1}, \ldots, a_{k}$.

## 5. Implementation and Experiments

### 5.1 Used Pairing Library

In the implementation of our system (and the compared conventional system), we utilize the fast pairing library called "Cross-twisted $\chi$-based Ate (Xt-Xate) pairing" [1] with 254-bit group order and the embedding degree is 12, as in Ref. [20].

### 5.2 Comparisons to Conventional System

We compare the efficiency between our system and the conventional pairing-based system [12] using the BBS+ signatures. The anonymous credential system described in Ref. [12] does not equip proofs of AND and OR relations. However, similarly to the conventional RSA-based systems described in Ref. [10], we can construct the proof protocols for AND and OR relations, which are described in Appendix A.1. We introduce the following parameters.

- $L$ : the total number of string attribute types
- $\tilde{L}$ : the total number of finite-set attribute types (e.g., gender, profession)
- $n$ : the total number of finite-set attribute values (e.g., male, female, student, teacher)
- $k$ : the number of attributes referenced in a proof.

We measured the computation time of the systems using a laptop PC with the specifications shown in Table 1.

In this environment, the computation times for a pairing computation, an exponentiation on $\mathbb{G}_{1}$, an exponentiation on $\mathbb{G}_{2}$, and an exponentiation on $\mathbb{G}_{T}$ are about $13.69 \mathrm{~ms}, 1.84 \mathrm{~ms}, 3.52 \mathrm{~ms}$, and 4.63 ms , respectively. We set $L=3, \tilde{L}$ is varied from 5 to 100 , and $k$ is varied from 10 to 100 . In the implementation, we use pre-computations for pairing calculations of fixed values such

Table 1 Specifications of PC in experiments.

| $\mathrm{O} / \mathrm{S}$ | Ubuntu Linux kernel-2.6.35-30-generic |
| :---: | :--- |
| CPU | AMD E-350 Dual-Core Processor 1.6 GHz Cache size 512 KB |
| RAM | 2 GB |
| Compiler | gcc-4.4 GNU C Compiler |
| Softwares | GMP Library 5.0.2, Openssl-0.9.8o |



Fig. 1 AND Relation - processing times for $\tilde{L}$ (number of finite-set attribute types).


Fig. 2 OR Relation - processing times for $\tilde{L}$ (number of finite-set attribute types).
as public key and certificate, as in Ref. [17].
Figure 1 shows the comparisons for proof generation and verification times in case of AND relation. Figure 2 shows them in case of OR relation. We vary $\tilde{L}$ from 5 to 100 , and fix $k=10$ and $n=15,000$ (in the case of proposed system). In the proof generation, for $\tilde{L}<30$, the time of the conventional system is better than the proposed system. In contrast, when $\tilde{L}>30$, our proposed system becomes more efficient than the conventional one. The proof generation time of proposed system is constant at about 165 ms and 345 ms , for AND relation and OR relation, respectively. Table 2 shows the example of attributes in eID. In the conventional system, if a user may own multiple attribute values from an attribute type, we have to prepare bases for the possible multiple values, namely $\tilde{L}$ increases by the number of possible multiple values. For example, a user can have multiple profession attributes such as student and technician in a company, and a user may own 5 or more language ability. As the results, $\tilde{L}$ becomes relatively large. Therefore, in the general case that $\tilde{L}>30$, proving AND relation in our system has more efficiency.

Figure 3 shows the comparison for the OR relation proof in the

Table 2 Example of string and finite-set attributes.

| String | Finite-set | Example Values |
| :--- | :--- | :--- |
| 1) name | 3) day of issuance | $1-31$ |
| 2) identity number | 4) month of issuance | $1-12$ |
|  | 5) year of issuance | $2000-2011$ |
|  | 6) day of expiration | $1-31$ |
|  | 7) month of expiration | $1-12$ |
| 8) year of expiration | $2000-2011$ |  |
|  | 9) gender | male,female |
|  | 10) day of birth | $1-31$ |
|  | 11) month of birth | $1-12$ |
|  | 12) year of birth | $1930-2005$ |
|  | 13) marital status | single,marriage |
|  | 14-16) nationality | 193 recognized states |
|  | 17) hometown | 200 allocated cities |
|  | 18) city living | 200 allocated cities |
|  | 19) residence status | citizen,immigrant,... |
|  | $20)$ religion | Moslem,Christian,... |
|  | $21)$ blood type | A,B,O,AB |
| $22-27)$ profession | student,teacher,... |  |
|  | $28-30)$ academic degree | B.S.,M.S,Ph.D.,... |
|  | $31-35)$ major | science,economic,... |
|  | $36-45)$ language | 100 allocated lang. |
|  | $46-48)$ social benefit status | none, unemployed, ... |
|  | $49-51)$ eye and hair color | 6 hair colors, 8 eye colors |
|  | $52-54)$ minority status | blind, deaf, ... |
|  | $\ldots$ |  |



Fig. 3 OR Relation - impact of $k$ (number of attributes referenced in a proof).
variety of $k$. We vary $k$ from 10 to 100 , and set $L+\tilde{L}=100$ and $n=15,000$. As the result, our proposed system is not influenced by $k$, unlike the conventional system, which make it much more efficient.

On the other hand, the proposed system has a drawback that the size of public key varies from 1 MBytes to 10 MBytes when $n$ is from 1,000 to 15,000, as shown in Fig. 4. However, in the current mobile environments, the data size is sufficiently practical, since the public key is not changed after it is distributed.

### 5.3 Comparisons to Camenisch-Groß System

Next, we compare our pairing-based system to RSA-based Camenisch-Groß system. For the implementation of CamenischGroß System, the idemix (Identity Mixer) library is available [16]. The library is implemented by Java, while our implementation uses C language. Thus, this is not a fair comparison, but for reference we show the comparisons. The specifications are the same as Table 1, where Java version is 1.6.0_26 and idemix library version is 2.3 .3 . The size of RSA modulus is set as 3,072 bits, to adjust the our implementation based on 254-bit


Fig. 4 Public key sizes for $n$ (number of attribute values).

Table 3 Comparisons to Camenisch-Groß system (AND proof).

| System | Proof generation time | Verification time | Proof size |
| :--- | ---: | ---: | ---: |
| Camenisch-Groß | $8,901 \mathrm{~ms}$ | $6,815 \mathrm{~ms}$ | 2,517 Bytes |
| Ours | 165 ms | 325 ms | 1,256 Bytes |

Table 4 Comparisons to Camenisch-Groß system (OR proof).

| System | Proof generation time | Verification time | Proof size |
| :--- | ---: | ---: | ---: |
| Camenisch-Groß | $17,699 \mathrm{~ms}$ | $14,589 \mathrm{~ms}$ | 6,009 Bytes |
| Ours | 345 ms | 618 ms | 2,184 Bytes |

ECC. Tables 3, $\mathbf{4}$ show the comparisons for AND proof and OR proof, respectively. Note that both systems achieve the constant complexity. As for the processing times, our system is extremely better, although the implementation languages are different. As for the proof sizes, our system is also better, since data size for one element in the proof is shorter than the RSA-based system.

### 5.4 Application to eID

### 5.4.1 Summary of Implementation

To confirm the practicality of our system, we implemented a prototype system of eID application. At first, the issuer publishes its public key $i p k$. Then, the user registers himself along with particular attributes (SA, FA) to the issuer for certification by using the Issuing protocol via a secure channel. Based on the issued certificate, he requests a service to the Service Provider (SP). Then, the SP specifies attributes that the SP wants to know. This specification forms AND or OR relation, depending on the SP's requirement. Then, the user generates a proof for the possession of certificate w.r.t. the specified attribute(s) and shows it to the SP (verifier) anonymously by using the attribute proof protocol. If and only if the verification of user's proof is valid, the SP grants the user to access a requested service.
We implemented this prototype system using Java through the Java GUI and Java applet at the user and Java servlet at the Service Provider (SP) and the issuer, since the Java applet and Java servlet communications are often used for the web-based applications. The communication between the user and the servers (i.e., the issuer and SP) is over http connections. In our implementation, since algorithms of our anonymous credential system are implemented by C language as the middle-ware, we use Java Native Interface (JNI) as the interface between C and Java.

### 5.4.2 Experimental Results

The devices and software specifications used in this exper-

Table 5 Specification of devices in experiments.

| Verifier/ | Intel Core i5 CPU 650 3.2 GHz, 2 GB RAM <br> Issuer |
| :--- | :--- |
|  | Ubuntu Linux 10.10 kernel-2.6.35 <br> gcc-4.4.4, OpenSSL-0.9.8o, GMP-5.0.2 <br> Java-1.6.0_26, apache-tomcat-7.0.20 |
| Prover/ | Intel Atom 1.5 GHz, 1 GB RAM |
| Joining-User | Windows 7 |
|  | Atheros AR9285 Wireless Adapter |
|  | MinGW-5.1.6, OpenSSL-0.9.8g, |
| GMP-4.3.1, Java-1.6.0_24 |  |



Fig. 5 Total processing times.
iment are shown in Table 5. We used a laptop PC with an atom CPU for the user to simulate the mobile situation. The user PC is connected to the SP and issuer PCs via wireless LAN (IEEE802.11 b/g).
Figure 5 shows the total processing times of the issuing protocol and proof protocol. The total time of the proof protocol includes the proof generation time, the verification time, and the communication time during the process. This time varies from 714 ms to 738 ms for AND relation, when $n$ is from 1,500 to 15,000 . It is from $1,103 \mathrm{~ms}$ to $1,152 \mathrm{~ms}$ for the OR relation. The communication time is measured from sending the proof until receiving the response of the proof process from the verifier without verification time. The communication time is about 120 ms . The time of the issuing protocol varies from 654 ms to 690 ms when $n$ increases from 1,500 to 15,000 . In every case, we can confirm the practicality of our system for a mobile PC.

## 6. Conclusion

In this paper, for a pairing-based anonymous credential system, we have showed how to prove AND and OR relations on attributes with constant complexity in the number of finite-set attributes. From the experiments for the implementations, we showed the more efficiency than the conventional system and the RSA-based system, and the practicality in the mobile applications. The compensation is the increase of the public key size, although the public key is not changed after it is distributed.

Our future works include the implementations in smartphones and their applications to network services.

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## Appendix

## A. 1 Proving AND and OR Relations in Conventional System

For the reference, we describe proving AND and OR relations in the conventional system.

## A.1.1 Certificate

Let $L^{\prime}$ be the total number of attribute types. Then, the certificate is as follows.

$$
A=\left(\left(\prod_{1 \leq j \leq L^{\prime}} \tilde{g}_{j}^{M_{j}}\right) \tilde{g}_{L^{\prime}+1}^{x} g_{0}^{r} g\right)^{1 /(X+w)}
$$

## A.1.2 Proving AND relation

Let TA be the set of attributes referenced in the proof. Similarly to the proposed system, compute $C_{A}, C_{w}$. Then, prove the knowledge of $x, w, r, \rho_{A}, \rho_{w}, \alpha, \rho_{\alpha}$ and $M_{j}$ for $M_{j} \notin \mathrm{TA}$ s.t.

$$
\begin{aligned}
& C_{w}=h^{w} \hat{h}^{\rho_{w}}, 1=C_{w}^{\rho_{A}} h^{-\alpha} \hat{h}^{-\rho_{\alpha}}, \\
& \left.e\left(C_{A}, Y\right) e\left(\prod_{1 \leq j \leq L^{\prime}, M_{j} \in \mathrm{TA}} \tilde{g}_{j}^{M_{j}}\right) g, h\right)^{-1} \\
& \quad=\left(\prod_{1 \leq j \leq L^{\prime}, M_{j} \notin \mathrm{TA}} e\left(\tilde{g}_{j}, h\right)^{M_{j}}\right) e\left(\tilde{g}_{L^{\prime}+1}, h\right)^{x} \\
& \quad e\left(g_{0}, h\right)^{r} e(\hat{g}, Y)^{\rho_{A}} e(\hat{g}, h)^{\alpha} e\left(C_{A}, h\right)^{-w} .
\end{aligned}
$$

## A.1.3 Proving OR relation

Let TA $=\left\{M_{j_{1}}^{\prime}, \ldots, M_{j_{k}}^{\prime}\right\}$ be the set of attributes referenced in the proof, where $j_{i}$ means the $j_{i}$-th attribute types. Let STA be the set of $j_{i}$. Similarly to the proposed system, compute $C_{A}, C_{w}$, and additionally $C_{j}=h^{M_{j}} \hat{h}^{\rho_{j}}$ for $\rho_{j} \in_{R} Z_{p}^{*}$ with $j \in \mathrm{STA}$. Then, prove the knowledge of $x, w, r, \rho_{A}, \rho_{w}, \alpha, \rho_{\alpha}$, all $M_{j}$, and $\rho_{j^{\prime}}$ for $j^{\prime} \in$ STA s.t.

$$
\begin{aligned}
& C_{w}=h^{w} \hat{h}^{\rho_{\omega}}, 1=C_{w}^{\rho_{A}} h^{-\alpha} \hat{h}^{-\rho_{\alpha}}, \\
& e\left(C_{A}, Y\right) e(g, h)^{-1} \\
& \left.\quad=\left(\prod_{1 \leq j \leq L^{\prime}, j \in S T A} e\left(\tilde{g}_{j}, h\right)^{M_{j}}\right) \prod_{1 \leq j \leq L^{\prime}, j \notin S T A} e\left(\tilde{g}_{j}, h\right)^{M_{j}}\right) \\
& \quad \cdot e\left(\tilde{g}_{L^{\prime}+1}, h\right)^{x} e\left(g_{0}, h\right)^{r} e(\hat{g}, Y)^{\rho_{A}} e(\hat{g}, h)^{\alpha} e\left(C_{A}, h\right)^{-w}, \\
& C_{j}= \\
& h^{M_{j} h^{\rho_{j}}(\text { for } j \in \mathrm{STA}),}
\end{aligned}
$$

Additionally, prove

$$
C_{j_{1}} / h^{M_{j_{1}}^{\prime}}=\hat{h}^{\rho_{j_{1}}} \vee \cdots \vee C_{j_{k}} / h^{M_{j_{k}}^{\prime}}=\hat{h}^{\rho_{j_{k}}}
$$

This $P K$ for OR relation on representations is described in Ref. [14].

## Editor's Recommendation

Scalability of an anonymous credential system is a critical factor for wide and practical use of the system. By using a pairingbased accumulator, this paper proposes an efficient scheme which achieves the constant complexity regarding the number of user attributes. In addition to this theoretical efficiency, this paper shows a real implementation of a mobile eID system based on the proposed scheme. Thus the proposal has both theoretical and practical impacts on privacy-enhancing technologies.
(Program chair of CSS2010, Kanta Matsuura)


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