

Fundamentals of Electromagnetics 1: Internal Behavior of Lumped Elements

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Fundamentals of Electromagnetics 1: Internal Behavior of Lumped Elements

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ABSTRACT

This book is the first of two volumes which have been created to provide an understanding of the basic principles and applications of electromagnetic fields for electrical engineering students. *Fundamentals of Electromagnetics Vol 1: Internal Behavior of Lumped Elements* focuses upon the DC and low-frequency behavior of electromagnetic fields within lumped elements. The properties of electromagnetic fields provide the basis for predicting the terminal characteristics of resistors, capacitors, and inductors. The properties of magnetic circuits are included as well. For slightly higher frequencies for which the lumped elements are a significant fraction of a wavelength in size the second volume of this set, *Fundamentals of Electromagnetics Vol 2: Quasistatics and Waves*, examines how the low-frequency models of lumped elements are modified to include parasitic elements. Upon completion of understanding the two volumes of this book, students will have gained the necessary knowledge to progress to advanced studies of electromagnetics.

KEYWORDS

Electromagnetics Introduction, Electromagnetic fields within lumped elements, Terminal characteristics of resistors, capacitors, and inductors, Properties of magnetic circuits

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Preface

“Learning about 0’s and 1’s is much easier than learning about Electromagnetics.”

—Robert MacIntosh

Most students begin their studies in electrical engineering with courses in computer logic and electric circuits. This is quite understandable since the students’ mathematical background is usually limited to scalar mathematics. But, the fundamental principles of electrical engineering upon which the majority of physical processes such as semiconductor devices, power generation, or wireless technology are represented mathematically by three-dimensional, vector calculus. Electromagnetics is a very challenging subject for students lacking a strong background in mathematics or three-dimensional visualization skills—even for students who have a thorough understanding of electric circuits. On the other hand, the principles of electromagnetics are relatively simple when the underlying vector calculus is understood by the students. This textbook is a non-traditional approach to bridge this gap that is based upon the similarities of lumped passive elements of circuits—resistors, capacitors, and inductors.

The terminal characteristics of lumped elements are derived from the observable quantities of voltage difference and current flow that are measured with instruments of finite size, i.e., dimensions on a macro-scale. But a more fundamental description of the underlying phenomena is on a micro-scale, infinitesimal in size, i.e., smaller than the smallest instrument. This behavior is described in terms of differential dimensions, with dimensions that shrink to zero; it is not measurable by man-made instruments and is much more complicated than two terminal properties of voltage drop and current flow. These inner workings of lumped elements on a micro-scale are related to their external, observable macro-scale (or terminal) characteristics.

This approach utilizes the following set of underlying restrictions:

- The terminal behavior of passive, lumped elements, $v_R = i_R R$, $i_C = C dv_C/dt$, and $v_L = L di_L/dt$, is known from circuits. Circuit theory experience, especially with KVL and KCL, is assumed and used.
- Each material type has only a single electromagnetic property—conductors have non-zero conductivity, σ , and allow current flow; dielectrics have permittivity, ϵ , and store only electric energy; and magnetic materials have permeability, μ , and store only magnetic energy.

- Lumped elements of finite size can be decomposed into discrete, curvilinear regions each of which behaves as an incremental lumped element. The total element value is calculated using the series/parallel equations of circuits.
- The electric or magnetic scalar potentials of all three types of lumped elements satisfies Laplace's equation, $\nabla^2 V = 0$. The same solution methods can be used for all elements.
- There are no fringing or leakage fields from any lumped elements. Consequently, all inductors have torroidal-shaped, magnetic cores.
- The frequency of operation is low enough that the elements are small compared to wavelength. Consequently, radiation and wave effects are ignored.
- Linearity and superposition apply with one notable exception—magnetic circuits.
- All electrodes and connecting wires are lossless with zero resistance and as such are called Perfect Electric Conductors (PEC).

These restrictions are imposed so that the basic principles of electromagnetics are not hidden by complicating geometric or mathematical details. Those details can come in following courses. Those students who take more advanced electromagnetic courses are not hindered by this approach. In fact, a case may be made that this approach provides a more complete understanding of the basics than traditional approaches.

Analytic expressions are used throughout this text as a calculation technique. Numeric and graphical methods for two dimensional structures are integrated within the textbook as well. In addition, circuit simulation is introduced as a viable solution method.

Finally, It is assumed that all students have studied electricity and magnetism in introductory physics and have an understanding of the concepts of force, displacement, work, potential energy, opposite charges attract, like charges repel, and current is usually assumed to be uniformly distributed currents in resistors. In addition, students should be familiar with vector algebra. Most work uses Cartesian coordinates with a few examples in cylindrical and spherical coordinates. The SI system of units is used throughout.

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