Jordan Canonical Form: Application to Differential Equations

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ABSTRACT

Jordan Canonical Form (JCF) is one of the most important, and useful, concepts in linear algebra. In this book we develop JCF and show how to apply it to solving systems of differential equations. We first develop JCF, including the concepts involved in it–eigenvalues, eigenvectors, and chains of generalized eigenvectors. We begin with the diagonalizable case and then proceed to the general case, but we do not present a complete proof. Indeed, our interest here is not in JCF per se, but in one of its important applications. We devote the bulk of our attention in this book to showing how to apply JCF to solve systems of constant-coefficient first order differential equations, where it is a very effective tool. We cover all situations–homogeneous and inhomogeneous systems; real and complex eigenvalues. We also treat the closely related topic of the matrix exponential. Our discussion is mostly confined to the 2-by-2 and 3-by-3 cases, and we present a wealth of examples that illustrate all the possibilities in these cases (and of course, a wealth of exercises for the reader).

KEYWORDS

Jordan Canonical Form, linear algebra, differential equations, eigenvalues, eigenvectors, generalized eigenvectors, matrix exponential

Contents

Preface

Jordan Canonical Form (JCF) is one of the most important, and useful, concepts in linear algebra. In this book, we develop JCF and show how to apply it to solving systems of differential equations.

In Chapter 1, we develop JCF. We do not prove the existence of JCF in general, but we present the ideas that go into it—eigenvalues and (chains of generalized) eigenvectors. In Section 1.1, we treat the diagonalizable case, and in Section 1.2, we treat the general case. We develop all possibilities for 2-by-2 and 3-by-3 matrices, and illustrate these by examples.

In Chapter 2, we apply JCF. We show how to use JCF to solve systems $Y' = AY + G(x)$ of constant-coefficient first-order linear differential equations. In Section 2.1, we consider homogeneous systems $Y' = AY$. In Section 2.2, we consider homogeneous systems when the characteristic polynomial of A has complex roots (in which case an additional step is necessary). In Section 2.3, we consider inhomogeneous systems $Y' = AY + G(x)$ with $G(x)$ nonzero. In Section 2.4, we develop the matrix exponential e^{Ax} and relate it to solutions of these systems. Also in this chapter we provide examples that illustrate all the possibilities in the 2-by-2 and 3-by-3 cases.

Appendix A has background material. Section A.1 gives background on coordinates for vectors and matrices for linear transformations. Section A.2 derives the basic properties of the complex exponential function. This material is relegated to the Appendix so that readers who are unfamiliar with these notions, or who are willing to take them on faith, can skip it and still understand the material in Chapters 1 and 2.

Our numbering system for results is fairly standard: Theorem 2.1, for example, is the first Theorem found in Section 2 of Chapter 1.

As is customary in textbooks, we provide the answers to the odd-numbered exercises here. *Instructors* may contact me at shw2@lehigh.edu and I will supply the answers to all of the exercises.

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