The Structure and Properties of Color Spaces and the Representation of Color Images

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ABSTRACT

This lecture describes the author's approach to the representation of color spaces and their use for color image processing. The lecture starts with a precise formulation of the space of physical stimuli (light). The model includes both continuous spectra and monochromatic spectra in the form of Dirac deltas. The spectral densities are considered to be functions of a continuous wavelength variable.

This leads into the formulation of color space as a three-dimensional vector space, with all the associated structure. The approach is to start with the axioms of color matching for normal human viewers, often called Grassmann's laws, and developing the resulting vector space formulation. However, once the essential defining element of this vector space is identified, it can be extended to other color spaces, perhaps for different creatures and devices, and dimensions other than three. The CIE spaces are presented as main examples of color spaces. Many properties of the color space are examined.

Once the vector space formulation is established, various useful decompositions of the space can be established. The first such decomposition is based on luminance, a measure of the relative brightness of a color. This leads to a direct-sum decomposition of color space where a two-dimensional subspace identifies the chromatic attribute, and a third coordinate provides the luminance. A different decomposition involving a projective space of chromaticity classes is then presented. Finally, it is shown how the three types of color deficiencies present in some groups of humans leads to a direct-sum decomposition of three one-dimensional subspaces that are associated with the three types of cone photoreceptors in the human retina. Next, a few specific linear and nonlinear color representations are presented. The color spaces of two digital cameras are also described. Then the issue of transformations between *different* color spaces is addressed.

Finally, these ideas are applied to signal and system theory for color images. This is done using a vector signal approach where a general linear system is represented by a three-by-three system matrix. The formulation is applied to both continuous and discrete space images, and specific problems in color filter array sampling and displays are presented for illustration.

The book is mainly targeted to researchers and graduate students in fields of signal processing related to any aspect of color imaging.

KEYWORDS

color, color space, color image, colorimetry, color Fourier transform, color filter array, color mosaic display, color image filtering, color image sampling

To my colorful grandchildren

Kiernan, Quinn, and Juliette

Contents

Preface

Color is everywhere and electronic color imaging is now pervasive. Color is an aspect of human perception and the attempt to describe it mathematically has been going on for centuries, with important contributions from great scientists like Newton, Maxwell, Helmholz, Schrödinger and many, many more [32]. There are numerous excellent, comprehensive and beautiful treatises on color theory, color science, colorimetry and so on ([61], [21], [28], [65] to name just a few). However, it is my belief that none of these adequately present the algebraic theory of color spaces and their applications in color imaging in a suitably comprehensive fashion for signal processing theorists. This lecture describes my approach to the representation of color spaces and their use for color image processing. It is mainly targeted to researchers and graduate students in fields of signal processing related to any aspect of color imaging. However, I will be happy if it finds use in other aspects of color science in general. I have attempted to establish a complete and consistent notation to deal with all aspects of this subject. Since there is no single universally adopted notation for many of the concepts used here, I had to introduce new notation for many aspects. However, I have tried to remain consistent with all standard nomenclature such as color matching functions, chromaticity, etc. All the notation has been summarized in the Notation section, along with a synthetic diagram illustrating the main sets, spaces and transformations that have been introduced.

I would like to thank several colleagues who have read various versions of this book and provided valuable corrections and suggestions for improvement. Specifically, my great thanks to Prof. Stéphane Coulombe of the Department of Software and IT Engineering, École de technologie supérieure, Montreal, Quebec; Prof. Abdol-Reza Mansouri of the Department of Mathematics and Statistics, Queen's University, Kingston Ontario; and Prof. Gaurav Sharma of the Department of Electrical and Computer Engineering, University of Rochester, Rochester, New York. Their input pushed me to make many improvements to the book, as well to correct a number of errors, and I have attempted to address all the issues they raised. Of course, all the remaining errors and inadequacies are entirely my responsibility. I also thank Mustafa Fanaswala for pointing out a number of errors.

In addition, I would like thank several people who provided data or clarifications. I thank Cong Phuoc Huynh, of the Department of Information Engineering, College of Engineering and Computer Science, Australian National University, Canberra, Australia, for his data on the spectral sensitivity of digital cameras; Charles Poynton, noted expert on color imaging systems of Toronto, Ontario, for data on the spectral density of red, green and blue phosphors in a CRT display; and Xuemei Zhang, senior researcher at HP labs in Palo Alto, California, for explanations concerning

computation of the S-CIELAB error measure. Finally, I thank Joel Claypool and Al Bovik for their encouragement to complete this work and their patience in waiting for the final result while I was busy with my administrative duties.

Eric Dubois
October 2009

Notation

Radiometry and Photometry

P_e, $P_e(\lambda)$	Radiant power (W), radiant power spectral density (W/nm)
E_e, $E_e(\lambda)$	Irradiance ($\mathrm{W/m^2}$), ($\mathrm{W/m^2 nm}$)
M_e, $M_e(\lambda)$	Radiant exitance ($\mathrm{W/m^2}$), ($\mathrm{W/m^2 nm}$)
I_e, $I_e(\lambda)$	Radiant intensity (W/sr), (W/sr nm)
L_e, $L_e(\lambda)$	Radiance ($\mathrm{W/sr\,m^2}$), ($\mathrm{W/sr\,m^2\,nm}$)
L_v	Luminance ($\mathrm{lm/sr\,m^2}$)

Equivalence Relations

$\triangleq$	Trichromatic physical metameric equivalence on $\mathcal{P}$
$\boxed{=}$	Trichromatic extended metameric equivalence on $\mathcal{A}$
$\overset{\triangle}{\sim}$	Brightness equivalence on $\mathcal{P}$
$\boxed{\sim}$	Extended brightness equivalence on $\mathcal{A}$
$\boxed{\approx}$	Dichromatic extended metameric equivalence on $\mathcal{A}$

Sets and Spaces

$\mathcal{P}$	The set of physical light stimuli
$\mathcal{A}$	The vector space of differences of physical light stimuli
$\mathcal{A}_c$, $\mathcal{A}_d$	The continuous and discrete subspaces of $\mathcal{A}$ in the decomposition $\mathcal{A} = \mathcal{A}_c \oplus \mathcal{A}_d$
$\mathcal{A}^*$	Dual space of $\mathcal{A}$, space of linear functionals from $\mathcal{A}$ to $\mathbb{R}$
$\mathcal{A}^*_{\mathscr{C}}$	Subspace of $\mathcal{A}^*$ consisting of elements continuous on $\mathcal{V}$
$\mathcal{V}$	The set of wavelengths comprising the visible spectrum, e.g., [350 nm, 780 nm]
$\mathcal{C}$	Color vector space
$\mathcal{D}_P$, $\mathcal{D}_D$, $\mathcal{D}_T$	Generic dichromat color spaces for protanopes, deuteranopes and tritanopes, respectively
$[C(\lambda)]_{\sim}$	Equivalence class of spectral densities under the equivalence relation $\sim$
$\mathcal{B}$	A generic basis for the color space $\mathcal{C}$
$\mathcal{K}$	Black space, kernel (or null space) of the mapping $\mathcal{S}$
$\mathcal{VS}_{\mathcal{C}}$	Subspace of $\mathcal{A}^*_{\mathscr{C}}$ that defines the color space $\mathcal{C}$, spanned by any set of color matching functions
$\mathcal{C}_R$	The subset of $\mathcal{C}$ of physically realizable colors, $\mathcal{C}_R = \mathcal{S}(\mathcal{P})$
$\mathcal{M}$	The subset of $\mathcal{C}$ consisting of monochromatic lights
$\mathcal{H}$	The convex hull of $\mathcal{M}$
$\mathcal{F}([\mathbf{Q}])$	Physical metamer set, the subset of $\mathcal{P}$ consisting of all the physically realizable metamers of $[\mathbf{Q}]$ in a given color space

$\mathscr{M}(\cdot)$	Metamer mismatch set

Elements of Sets and Spaces

$[\mathbf{C}]$	Element of a color space, called simply a color
$\mathbf{C}_{\mathcal{B}}$	Column matrix of coefficients of color $[\mathbf{C}]$ with respect to basis $\mathcal{B}$ (tristimulus values)
$\boldsymbol{x}$	Independent variables, $(x, y) \in \mathbb{R}^2$ for still images, or $(x, y, t) \in \mathbb{R}^2 \times \mathbb{R}$ for time-varying images
$\boldsymbol{u}$	Frequency variables, (u, v) for still images, or (u, v, w) for time-varying images
$[\mathbf{C}](\boldsymbol{x})$	Element of a color signal space, or value of a color signal at $\boldsymbol{x}$ (according to context)
$\mathbf{C}_{\mathcal{B}}(\boldsymbol{x})$	Color signal expressed with respect to basis $\mathcal{B}$
$[\widehat{\mathbf{C}}](\boldsymbol{u})$	Fourier transform of $[\mathbf{C}](\boldsymbol{x})$
$\widehat{\mathbf{C}}_{\mathcal{B}}(\boldsymbol{u})$	Fourier transform of $\mathbf{C}_{\mathcal{B}}(\boldsymbol{x})$ expressed in basis $\mathcal{B}$

Transformations

$\mathcal{S}$	Mapping from the vector space $\mathcal{A}$ of radiometric functions to the color space $\mathcal{C}$
$\mathbf{A}_{\mathcal{B}\to\tilde{\mathcal{B}}}$	Matrix to convert tristimulus values from basis $\mathcal{B}$ to basis $\tilde{\mathcal{B}}$
$\mathcal{T}_{\boldsymbol{d}}$	Shift system
$\mathbf{H}_{\mathcal{B}}(\boldsymbol{u})$	Transfer matrix of a linear shift-invariant system expressed in basis $\mathcal{B}$

Special Functions

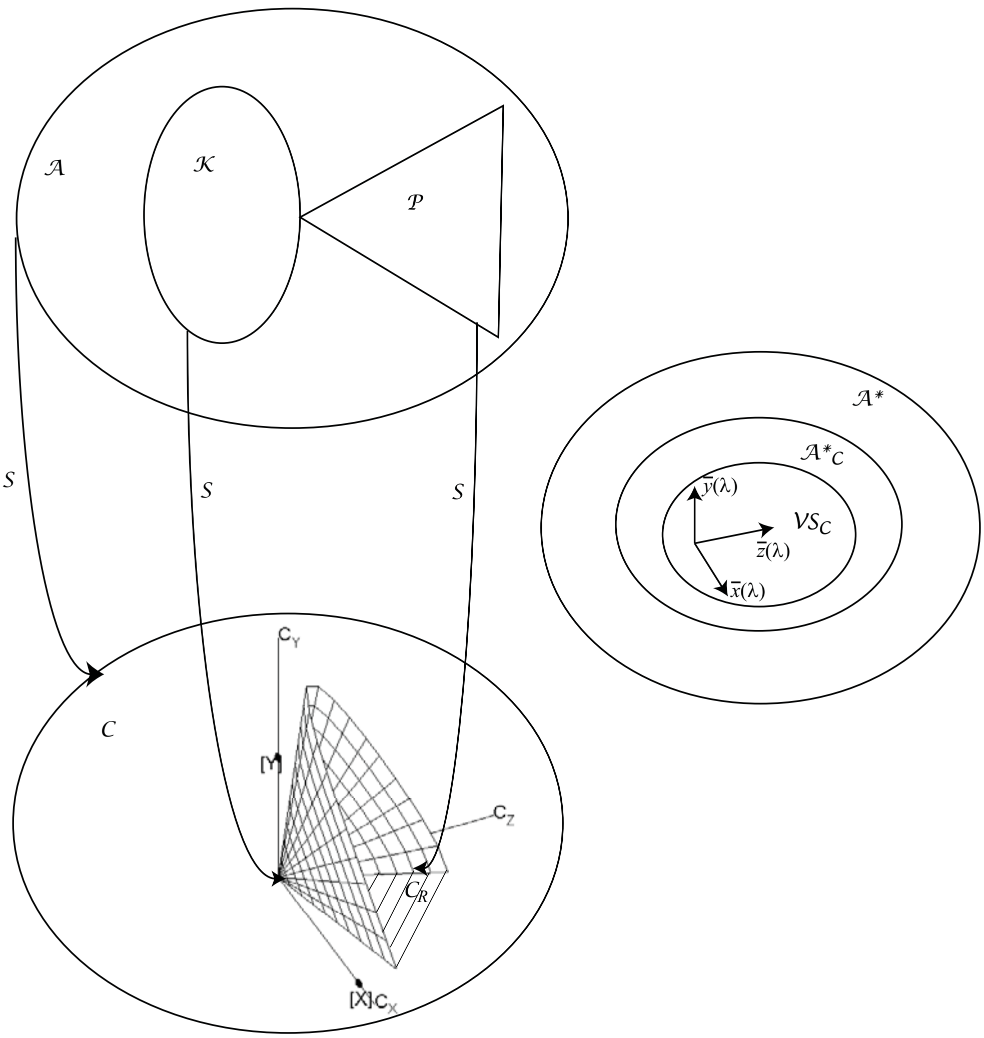
$\{\bar{p}_i(\lambda)\}$	Set of color matching functions relative to primaries $\{[\mathbf{P}_i]\}$ that specify the mapping $\mathcal{S}$
$V(\lambda)$	CIE photopic spectral luminous efficiency curve

Specific Color Spaces and Bases

$\mathcal{XYZ}$	Standard basis for the CIE 1931 colorimetric observer
$\mathcal{RGB}31$	Standard monochromatic RGB basis for the CIE 1931 colorimetric observer
$\mathcal{XYZ}_{10}$	Standard basis for the CIE 1964 10° colorimetric observer
$\mathcal{UCS}76$	CIE 1976 Uniform Chromaticity Scale (UCS) basis
$\mathcal{R}709$	RGB basis for CIE 1931 colorimetric observer standardized by ITU-R Rec. BT.709 for HDTV and also sRGB for multimedia
$\mathcal{OZW}$	Opponent-colors basis for the CIE 1931 colorimetric observer used on the S-CIELAB error metric
$\mathcal{C}_{\mathrm{SB10}}$	The Stiles and Burch 10° colorimetric observer
$\mathcal{SB}10$	The RGB primaries for the Stiles and Burch 10° colorimetric observer
$\mathcal{SS}10$	The Stockman and Sharp LMS primaries for the Stiles and Burch 10° colorimetric observer

Acronyms

CIE	Commission International d'Éclairage, or in English, International Commission on Illumination
sRGB	Default nonlinear RGB representation for multimedia, defined in IEC 61966-2-1
IEC	International Electrotechnical Commission
ITU	International Telecommunication Union
NTSC	National Television System Committee



$\mathcal{P}$ is a subset of $\mathcal{A}$ (not a subspace), which is a convex cone
$\mathcal{K}$ is a vector subspace of $\mathcal{A}$
$\mathcal{K} \cap \mathcal{P} = 0$
$\mathcal{S}(\mathcal{A}) = \mathcal{C}, \mathcal{S}(\mathcal{P}) = \mathcal{C}_R, \mathcal{S}(\mathcal{K}) = 0$
$\mathcal{A}$ and $\mathcal{K}$ are infinite dimensional; $\dim(\mathcal{C}) = 3$
$\mathcal{A}^*$ is the dual space of $\mathcal{A}$; $\mathcal{A}^*_{\mathscr{C}}$ is a vector subspace of $\mathcal{A}^*$; $\mathcal{VS}_{\mathcal{C}}$ is a vector subspace of $\mathcal{A}^*_{\mathscr{C}}$
$\mathcal{A}^*$ and $\mathcal{A}^*_{\mathscr{C}}$ are infinite dimensional; $\dim(\mathcal{VS}_{\mathcal{C}}) = 3$