

Boolean Differential Equations

Synthesis Lectures on Digital Circuits and Systems

Editor

Mitchell A. Thornton, *Southern Methodist University*

The Synthesis Lectures on Digital Circuits and Systems series is comprised of 50- to 100-page books targeted for audience members with a wide-ranging background. The Lectures include topics that are of interest to students, professionals, and researchers in the area of design and analysis of digital circuits and systems. Each Lecture is self-contained and focuses on the background information required to understand the subject matter and practical case studies that illustrate applications. The format of a Lecture is structured such that each will be devoted to a specific topic in digital circuits and systems rather than a larger overview of several topics such as that found in a comprehensive handbook. The Lectures cover both well-established areas as well as newly developed or emerging material in digital circuits and systems design and analysis.

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ABSTRACT

The Boolean Differential Calculus (BDC) is a very powerful theory that extends the structure of a Boolean Algebra significantly. Based on a small number of definitions, many theorems have been proven. The available operations have been efficiently implemented in several software packages. There is a very wide field of applications. While a Boolean Algebra is focused on values of logic functions, the BDC allows the evaluation of *changes* of function values. Such changes can be explored for pairs of function values as well as for whole subspaces. Due to the same basic data structures, the BDC can be applied to any task described by logic functions and equations together with the Boolean Algebra. The BDC can be widely used for the analysis, synthesis, and testing of digital circuits.

Generally speaking, a *Boolean differential equation* (BDE) is an equation in which elements of the BDC appear. It includes variables, functions, and derivative operations of these functions. The solution of such a BDE is a *set of Boolean functions*. This is a significant extension of Boolean equations, which have sets of Boolean vectors as solutions. In the simplest BDE a derivative operation of the BDC on the left-hand side is equal to a logic function on the right-hand side. The solution of such a simple BDE means to execute an operation which is inverse to the given derivative. BDEs can be applied in the same fields as the BDC, however, their possibility to express sets of Boolean functions extends the application field significantly.

KEYWORDS

Boolean differential equation, Boolean Differential Calculus, Boolean Algebra, Boolean function, Boolean equation, set of Boolean functions, XBOOLE

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