

# **Accurate Computation of Mathieu Functions**

# Synthesis Lectures on Computational Electromagnetics

Editor

**Constatine A. Balanis**, *Arizona State University*

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## **ABSTRACT**

This lecture presents a modern approach for the computation of Mathieu functions. These functions find application in boundary value analysis such as electromagnetic scattering from elliptic cylinders and flat strips, as well as the analogous acoustic and optical problems, and many other applications in science and engineering. The authors review the traditional approach used for these functions, show its limitations, and provide an alternative “tuned” approach enabling improved accuracy and convergence. The performance of this approach is investigated for a wide range of parameters and machine precision. Examples from electromagnetic scattering are provided for illustration and to show the convergence of the typical series that employ Mathieu functions for boundary value analysis.

## **KEYWORDS**

elliptic cylinder, electromagnetic scattering, boundary value problems



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