# Information Theory and Rate Distortion Theory for Communications and Compression

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# Information Theory and Rate Distortion Theory for Communications and Compression

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*SYNTHESIS LECTURES ON COMMUNICATIONS #9*

## ABSTRACT

This book is very specifically targeted to problems in communications and compression by providing the fundamental principles and results in information theory and rate distortion theory for these applications and presenting methods that have proved and will prove useful in analyzing and designing real systems. The chapters contain treatments of entropy, mutual information, lossless source coding, channel capacity, and rate distortion theory; however, it is the selection, ordering, and presentation of the topics within these broad categories that is unique to this concise book.

While the coverage of some standard topics is shortened or eliminated, the standard, but important, topics of the chain rules for entropy and mutual information, relative entropy, the data processing inequality, and the Markov chain condition receive a full treatment. Similarly, lossless source coding techniques presented include the Lempel-Ziv-Welch coding method. The material on rate Distortion theory and exploring fundamental limits on lossy source coding covers the often-neglected Shannon lower bound and the Shannon backward channel condition, rate distortion theory for sources with memory, and the extremely practical topic of rate distortion functions for composite sources.

The target audience for the book consists of graduate students at the master's degree level and practicing engineers. It is hoped that practicing engineers can work through this book and comprehend the key results needed to understand the utility of information theory and rate distortion theory and then utilize the results presented to analyze and perhaps improve the communications and compression systems with which they are familiar.

## KEYWORDS

information theory, rate distortion theory, fundamental limits on communications, fundamental limits on compression

*To Tyler Dean Gibson.*

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# Preface

The disciplines of Information Theory and Rate Distortion Theory were introduced by Shannon more than 65 years ago, and there is likely little argument about the impact of these ideas on digital communication systems and source compression. However, to a broad audience of practicing engineers and even many deeply involved in digital communications and multimedia compression, the details of the connections may be obscure. Indeed, there are those who would vigorously debate the practical utility of information theory and rate distortion theory when it comes to the communications and compression systems that have evolved over the years.

One reason for the obscure connections of information theory and rate distortion theory to practice comes from the content of most courses on information theory and rate distortion theory, which often require considerable preparation in mathematics and statistics before they can be undertaken. As a result, many outstanding engineers do not have the time nor motivation to delve into these topics. Further, most research papers in the field do not address the specific connections of their models and results with practical systems, other than perhaps a cursory sentence or two stating some broad practical problem at a high level.

In order to provide greater access to these fields over the years, some authors have written textbooks aimed at undergraduate engineering students, and some very nice books have been written indeed. In fact, perhaps the very best of the group of widely accessible treatments of information theory is the early, and very influential, text by Abramson. However, these texts have not produced much momentum for the spreading of this knowledge. One reason is that the texts themselves almost always point out that information theory and rate distortion theory establish existence but do not provide design methods. Another reason is authors often say that the complexity required to achieve the performance promised by information theory and rate distortion theory is prohibitive, which certainly discourages students and practitioners alike.

To exploit the true promise of information theory and rate distortion theory, it is necessary to understand the theory at a basic level, which sounds obvious, and it is also necessary to understand deeply the particular physical problem being addressed. After all, fundamental limits in information theory and rate distortion theory require good models of the channels and sources, and meaningful mathematical expressions for distortion measures. Even the most advanced books on information theory and rate distortion theory acknowledge these facts, and unfortunately, often point to these requirements as bottlenecks to applications, not a particularly motivating way to convince students to master the material!

This book is very specifically targeted to problems in communications and compression, to providing the fundamental principles and results in information theory and rate distortion theory for these applications, and to presenting methods that have proved and will prove useful to analyze

and to design real systems. We hope to achieve these goals with a minimum of mathematical preparation, although considerable mathematical prowess is still needed, while still maintaining the inherent elegance and power of the theory and the results.

The target audience for the book consists of graduate students at the master's degree level and practicing engineers. As a result, the technical and mathematical sophistication required is likely beyond all but the most advanced undergraduate. This book is not intended to replace a careful theorem/proof development of information theory and rate distortion theory; however, it will provide a strong leg up on mastering the material in such a more advanced course, though.

It is hoped that practicing engineers can work through this book and comprehend the key results needed to understand the utility of information theory and rate distortion theory to practical problems in communications and compression, and further, to be able to utilize the results presented to analyze and perhaps improve the communications and compression systems with which they are familiar.

**For Readers/Students:** There are proofs in this book, and indeed, some proofs are necessary to establish key practical results, to allow the reader to obtain some basic proficiency, and to (hopefully) retain the elegance of the information theoretic approaches to practical problems. All results in this book have great practical relevance or they would not be included.

**For Instructors:** The book can be used for either a one-semester or a one-quarter course on information theory and rate distortion theory. For a semester, the proofs of the Kraft inequality and the Asymptotic Equipartition Property can be covered and the achievability proofs for the Channel Coding Theorem and the Rate Distortion Theorem can be expanded in full from the outlines presented. For one quarter, some proofs might be curtailed and the sections on the Gaussian autoregressive source and conditional rate distortion theory can be skipped.

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