

Applications of Zero-Suppressed Decision Diagrams

Synthesis Lectures on Digital Circuits and Systems

Editor

Mitchell A. Thornton, *Southern Methodist University*

The *Synthesis Lectures on Digital Circuits and Systems* series is comprised of 50- to 100-page books targeted for audience members with a wide-ranging background. The Lectures include topics that are of interest to students, professionals, and researchers in the area of design and analysis of digital circuits and systems. Each Lecture is self-contained and focuses on the background information required to understand the subject matter and practical case studies that illustrate applications. The format of a Lecture is structured such that each will be devoted to a specific topic in digital circuits and systems rather than a larger overview of several topics such as that found in a comprehensive handbook. The Lectures cover both well-established areas as well as newly developed or emerging material in digital circuits and systems design and analysis.

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Tsutomu Sasao and Jon T. Butler

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Applications of Zero-Suppressed Decision Diagrams

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ABSTRACT

A zero-suppressed decision diagram (ZDD) is a data structure to represent objects that typically contain many zeros. Applications include combinatorial problems, such as graphs, circuits, faults, and data mining. This book consists of four chapters on the applications of ZDDs.

The first chapter by Alan Mishchenko introduces the ZDD. It compares ZDDs to BDDs, showing why a more compact representation is usually achieved in a ZDD. The focus is on sets of subsets and on sum-of-products (SOP) expressions. Methods to generate all the prime implicants (PIs), and to generate irredundant SOPs are shown. A list of papers on the applications of ZDDs is also presented. In the appendix, ZDD procedures in the CUDD package are described.

The second chapter by Tsutomu Sasao shows methods to generate PIs and irredundant SOPs using a divide and conquer method. This chapter helps the reader to understand the methods presented in the first chapter.

The third chapter by Shin-Ichi Minato introduces the “frontier-based” method that efficiently enumerates certain subsets of a graph.

The final chapter by Shinobu Nagayama shows a method to match strings of characters. This is important in routers, for example, where one must match the address information of an internet packet to the proper output port. It shows that ZDDs are more compact than BDDs in solving this important problem.

Each chapter contains exercises, and the appendix contains their solutions.

KEYWORDS

logic function, prime implicant, sum-of-products expression, binary decision diagram, zero-suppressed decision diagram, graph enumeration, CUDD package, frontier-based method, data-structure, non-deterministic automata, regular expression matching, one-hot code, intrusion detection

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Preface

This book focuses on ZDDs or zero-suppressed decision diagrams. Originally called zero-suppressed BDDs, ZDDs have since developed into a research tool for combinatorial algorithms, including sets of combinations, symbolic logic, probability theory, and string matching for computer virus detection, text retrieval from databases, and DNA matching. D. E. Knuth has described ZDDs as *the most beautiful construct in computer science*.

This book is a tutorial description designed to educate a reader with no previous background in decision diagrams. While the target audience consists of graduate students, established researchers with little background in decision diagrams will benefit from the book's tutorial approach. The first chapter is based on a technical report by a well-known expert in logic synthesis. The last two chapters were selected from the papers presented at the Reed-Muller Workshop (RM-2013) on May 23–24, 2013, in Toyama, Japan. These chapters are rewritten for the book so that non-experts can understand them. Each chapter

1. is self-contained,
2. has examples and illustrations to explain the concepts presented,
3. introduces important previous work with simple examples, and
4. contains exercises for the reader to solve.

Structure of the book.

Preface: T. Sasao and J. T. Butler

Chapter 1: A. Mishchenko, “An introduction to zero-suppressed decision diagrams.”

Chapter 2: T. Sasao, “Efficient generation of prime implicants and irredundant sum-of-products expressions.”

Chapter 3: S. Minato, “The power of enumeration BDD/ZDD-based algorithms for tackling combinatorial explosion.”

Chapter 4: S. Nagayama, “Regular expression matching using zero-suppressed decision diagrams.”

Appendix: Solutions to the exercises.

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Authors' and Editors' Biographies

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Yuta Urano helped in the editing of this book.

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