

# An Introduction to Verification of Visualization Techniques

# Synthesis Lectures on Visual Computing

## Computer Graphics, Animation, Computational Photography, and Imaging

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# An Introduction to Verification of Visualization Techniques

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*SYNTHESIS LECTURES ON VISUAL COMPUTING: COMPUTER GRAPHICS, ANIMATION, COMPUTATIONAL PHOTOGRAPHY, AND IMAGING #22*

## ABSTRACT

As we increase our reliance on computer-generated information, often using it as part of our decision-making process, we must devise tools to assess the correctness of that information. Consider, for example, software embedded on vehicles, used for simulating aircraft performance, or used in medical imaging. In those cases, software correctness is of paramount importance as there's little room for error. Software verification is one of the tools available to attain such goals. Verification is a well known and widely studied subfield of computer science and computational science and the goal is to help us increase confidence in the software implementation by verifying that the software does what it is supposed to do.

The goal of this book is to introduce the reader to software verification in the context of visualization. In the same way we became more dependent on commercial software, we have also increased our reliance on visualization software. The reason is simple: visualization is the lens through which users can understand complex data, and as such it must be verified. The explosion in our ability to amass data requires tools not only to store and analyze data, but also to visualize it.

This book is comprised of six chapters. After an introduction to the goals of the book, we present a brief description of both worlds of visualization (Chapter 2) and verification (Chapter 3). We then proceed to illustrate the main steps of the verification pipeline for visualization algorithms. We focus on two classic volume visualization techniques, namely, Isosurface Extraction (Chapter 4) and Direct Volume Rendering (Chapter 5). We explain how to verify implementations of those techniques and report the latest results in the field of verification of visualization techniques. The last chapter concludes the book and highlights new research topics for the future.

## KEYWORDS

visualization, verification, isosurfaces, volume rendering, geometry processing, verifiable visualization

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# Preface

The term *verification* has become ubiquitous in both the computer science and engineering communities as denoting a process that somehow convinces the user that verified tools, whether those be circuits, algorithms, implementations, etc. are more safe, accurate, or complete than other tools that have not been verified. Although the term verification has a common root usage within both communities, it has evolved to mean something specific to each subarea of computer science and of engineering. For instance, within computer science, the verification of a circuit denotes either the exhaustive testing or proof that under all possible inputs, the circuit will produce the correct (specified) outputs. Similarly, for software, verification relates how well an implementation represents the behavior of its specification under all possible inputs. Within the engineering world, verification takes on a different, more nuanced meaning. One assumes that there exists an “exact solution” or “exact representation” resulting from the solution of a mathematical system of equations. In all but the most trivial circumstances, this exact solution is not attainable, and approximate solutions must be formed. The process of quantifying how well a numerical scheme or representation approximates the exact solution is referred to as verification. Verification may involve looking at how well (or quickly) an approximate solution converges (in an appropriate norm) to the exact solution, or may involve identifying features or invariants of the solution that should be maintained regardless of the approximate representation. As visualization models, algorithms and implementations lie at the interface of these two branches, what does it mean to produce *verifiable visualizations*?

This question motivated the research work that has become the foundation of this book. To answer such a broad question, we started as most researchers would: by examining a concrete example in which our ideas could be refined. We started with isosurface extraction. Many tests and a few software bugs (which our process found) later, we realized that not only were our results worth communicating to the community, but that there was much work still to do. We moved to verifying different techniques used within the visualization community—in turn learning new things along the way. We began to appreciate that verification is a process, and that articulating the guiding principles of that process was itself a contribution to our community. The various papers we reference outline the specific contributions of our work. This book is meant to make that work accessible to the general reader in a pedagogical way. We hope the reader will take away not just a particular technique, but a way of approaching and testing visualization algorithms and their implementations. In the end, we hope that all successful visualization techniques will produce verifiable visualizations.

Any work of this size and scope has benefitted by many people both indirectly and directly. We wish to thank our collaborators that helped to shape this work, in particular Luis

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